ԲԱՆԲԵՐ ԵՐԵՎԱՆԻ ՀԱՄԱԼՍԱՐԱՆԻ. ՏՆՏԵՍԱԳԻՏՈՒԹՅՈՒՆ
2021, № 3, 56-72 *Տնտեսամաթեմաթիկական մոդելավորում*
https://doi.org/10.46991/BYSU:G/2021.12.3.056

MODEL OF TERRITORIAL CONFLICT AND INTERNATIONAL MILITARY COOPERATION

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The current paper is devoted to the study of the extension of the Lottka-Volterra model as a means for the modeling of military expenditures for countries interested in one economic region. The problem studied in this paper involves multiple tasks such as problems of military spending, cooperation, interactions between states, steps leading to negotiations or war. As the main model, the extending of the Lottka-Volterra model is studied.

The numerical method for the solution of spline approximation is studied. The system of equations is given as a means for the presentation of countries' military expenditures behavior. The model given for countries' military expenditures behavior allows getting trajectories of countries military spending curves during three–five years' period. As the numerical example for the study of the research outcome the simulation model for five countries interested in one economic region is presented.

Key words: *Lottka-Volterra model, economic policy, military spending, cooperative game, ordinary differential equations*

1. Introduction

The Multinational Operations, Alliances, and International Military Cooperation as a key option for the modeling of conflict relations between countries is considered in present research.

Empirical researches implemented as an example of conflict relations showed that conflicts today are very diverse. Their natures, causes, and performance are also very complex in that they take place in different regions. Today's parties involved in conflicts are grappling with global political and economic forces which have economic interests in considering economic regions. This issue presents a kind of approach for the modeling of territorial conflict between groups of countries having economic interests in the region. As a basis for this model the Lottka - Volterra model is considered. During the modeling, we are considering the country's military spending amount because conflicts development showed that this factor successfully carries out country's missions to lead the conflict to war or resolve conflict using diplomacy efforts.

The problem highlighted in this research has multiple tasks. Therefore, we are presenting findings as follows. The first finding is the study of the feature of territorial conflict involving different countries which are mutually interacting. As a model to study the feature of territorial conflict we considered the Lottka – Volterra model.

The next finding is as follows. The complexity of conflict, diversity of features, relations between neighboring countries and their relations with countries which have interests for the conflicting economic region, geographically are far and don't have borders with countries of the conflicting region, the existence of economic pressure to countries involved in the conflict immediately is considered also in current research.

The third finding is related to the assessment of the degree of the interaction between countries through the consideration of coalitions between countries. To assess the degree of the interaction between countries studied in the Lottka – Volterra model we considered coalitions formed by countries involved in the conflict as coalitional structures. Further, for each coalition considered as a set of players the degree of the interaction is assessed through the concept of the vector Shapley.

The problem of the solution of the studied model is the fourth finding. The solution of the model of territorial conflict is implemented through the spline approximation of the system of differential equations using quadratic splines. The system of nonlinear equations and the solution of the system of ordinary differential equations is given. As the outcome of the solution of the model, the behavior of military spending for each conflicting country during three years period is given.

The study involves chapters as follows. The literature review as the basis for the substantiation of the research problem is presented. It involves researches implemented in the area of territorial conflicts, gives features, nature of the conflict, presents main parties involved in the conflict, relations between studied countries, possibilities to cooperate with immediately conflicting countries by countries out of the conflicting economic region. The examples of territorial conflicts are studied through the study of empirical research related to the conflicts for the corridor, involvement of territorial defense troops, support to conflicting countries as the means to increase military spending.

Further, definitions and notations involve the definition of the quadratic spline through the presentation of the equations and give the formula of the quadratic spline.

Taking into consideration that the Lottka – Volterra model is a key model to study territorial conflict the system of ordinary differential equations is given. Further, using spline approximation of the system of the ordinary differential equations the solution of the model is given through the system of nonlinear equations.

A practical simulation model of territorial conflict is studied to substantiate the research results.

Literature review

We revisit the problem of territorial conflict between countries interested in the territorial economic regions occupied historically from ancient times.

During the historic challenges that region appeared in the sphere of interests of the powerful countries seeking opportunities to solve their own geographic, economic, and political interests.

The literature devoted to studying the problems of territorial conflicts, stages of conflicts, processes of the development of conflicts, military consequences of conflicts leading to war and armaments is diverse, rich, and involves economic, social, military, and demography problems.

The literature review has features according to the problems solved in current research. It involves the researches devoted to the problems related to territorial conflicts. Taking into consideration that the model of the territorial conflict is presented using the Lottka –Volterra[1] model as the next feature the literature review presents the research devoted to the study of extending of Lottka – Volterra[2] model.

The approach of the solution of the extension of the Lottka – Volterra model based on the numerical method for the solution of the system of an ordinary differential equation is given. The numerical method that has been used is based on the spline approximation method, and consequently, numerical methods of the solution of ordinary differential equations are given in the literature review.

Literature devoted to the study of territorial conflicts.

An example of the territorial conflict between countries searching for ways to get as more as possible was studied by **David B. Carter[3].** The author, studying various empirical investigations devoted to the territorial conflicts discovered that conflicts related to territorial claims are causing interstate clashes and can end either military or through peaceful diplomacy. The diversity of empirical researches of territorial conflicts showed that countries involved in conflicts are pursuing interests related to the strategic location of the territory and consequently to the development of efficient strategy. In similar empirical researches after **Goertz, Gary, and Paul Diehl[4], Hensel, Paul R.[5]** is **Hill, Norman** research[6]. Similar to previous works the study after **Holsti, Kalevi J**.[7] highlights problems of conflicts leading to peace or war. Territorial disputes and conflicts leading to war are studied by **Kocs, Stephen A**[8]. The problems of international society through

[1] **Chauvet, Erica, Joseph E. Paullet, Joseph P. Previte, and Zac Walls.** "A Lottka-Volterra Three-Species Food Chain." Mathematics Magazine 75.4 (2002): 243-55

[2] **Lalith Devireddy** https://sites.math.washington.edu/~morrow/336_16/2016papers/ lalith.pdf

[3] **David B. Carter The** Strategy of Territorial Conflict https://onlinelibrary.wiley.com/ doi/abs/10.1111/j.1540-5907.2010.00471.x

[4] **Goertz, Gary, and Paul Diehl** Territorial Changes and International Conflict. London: Routledge. 1992, 392 pg

[5] **Hensel, Paul R.** "Territory: Theory and Evidence on Geography and Conflict." In What Do We Know about War?ed. John Vasquez. Lanham, MD: Rowman and Littlefield,57–64, 2000.

[6] **Hill, Norman** Claims to Territory in International Law and Relations. New York: Oxford University Press 1945 429 pg.

[7] **Holsti, Kalevi J. Peace and War: Armed Conflicts and International Order. New York: Cambridge University Press, 1991, 371 pg**

[8] **Kocs, Stephen A**. 1995. "Territorial Disputes and Interstate War, 1945–1987." Journal of Politics 57(1): 159–75.

the investigation of interstate war were studied by **Luard, Evan** [9] as well by **Vasquez, John A.**[10]. Given researches suggested that the territory is the central subject of the claim and it leads countries to war, armaments, an increase in the military budget, and consequently a reduction in social spending.

Further, **David B. Carter**[11] considers The Territorial Dispute Game and developed perfect information game-theoretic model providing the presentation of the situation of both the territorial status quo or not.

The paper **Dmitry Streltsov, Anna Kireeva, and Ilya Dyachkov**[12] is devoted to the study of Russian policy to resolve the problem of international security in Northeast Asia. Authors are suggesting that Russia showed a neutral position in the conflicting territorial process. Through this behavior, Russia gets a chance to enhance relations with countries of the region, to promote cooperation with all East Asian states. This research showed that Russia, seeking an opportunity to enhance cooperation with conflicting countries got a chance to become "as one of the major powers of East Asia in order to develop its Far East relations and help create a polycentric regional order, prefers to stand away from regional security conflicts."

The fifth Workshop of the Partnership for Peace Consortium's Military History[13] is interesting from the point of view that is viewing territorial conflicts associated with participation of various states, alliances, and the existence of military cooperation as the center of interests. Papers, published in the proceedings are devoted to the consideration of coalitions between conflicting states. Authors are concluding that coalitions between states are the main units appearing within all military operations.

The research presented in the Proceedings covers different national experiences during four centuries of history. **Erwin A. Schmidl** suggests that economic-political cooperation leads to political– military cooperation and has long historic experience.

In many cases, one of the "aims was to deal with issues currently relevant at a time when international associations and other alliance structures, such as the European Union (EU) and the North Atlantic Treaty Organization (NATO), are undergoing a period of strain and transformation. In this way, the MHWG is able to contribute usefully to the overall success of the Partnership for Peace Consortium".

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[9] **Luard, Evan.** War in International Society. London: I.B. Tauris and Company. 368 pg., 1986

[10] **Vasquez, John A.**. "Why Do Neighbors Fight? Proximity, Interaction, or Territoriality." Journal of Peace Research 32(3): 277–93, 1995

[11] **David B. Carter** The Strategy of Territorial Conflict https://onlinelibrary.wiley.com /doi/abs/10.1111/j.1540-5907.2010.00471

[12] **Dmitry Streltsov, Anna Kireeva, and Ilya Dyachkov,** Russia's View on the International Security in Northeast Asia The Korean Journal of Defense Analysis Vol. 30, No. 1, March 2018, 115－134https://sites.math.washington.edu/~morrow/336_16/2016papers/lalith.pdf

[13] Multinational Operations, Alliances, and International Military Cooperation Past and Future Proceedings of the Fifth Workshop of the Partnership for Peace Consortium's Military History Working Group Vienna, Austria 4–8 April 2005, Edited by Robert S. Rush and William W. Epley Center of Military History United States Army Washington, D.C., 2006, https://history.army.mil/html/books/multinational_operations/CMH_70-101-1.pdf

The significance of the cooperation between territorial defense troops (TDT) is noted in[14] . Authors of the report suggest that the cooperation between operating troops, adjustment of their number depending on the problems are to be solved increases the efficiency and capabilities of military forces.

The empirical study of the cooperation between operating troops proofed that armed forces, including Territorial Defense Troops, showed that TDT is capable to solve problems of the protection of the sovereignty of the territory.

The role of military forces in the recovering of failed states has been examined by **Susumu Takai**[15]. Author suggests that "The UN Security Council has the primary responsibility for maintaining international peace and security". The brigade of peacekeeping is based on the resolution given in "The Scandinavian countries, Canada, Austria, and other countries with a broad range of experience and advanced level of sophistication in the area of peacekeeping, established a research group to consider an emergency deployment force, and in 1996, drew up the Multinational Stand-by High Readiness Brigade for United. The paper[16] considers problems of conflict as follows: "1. What is the conflict about? 2. Causes and backgrounds 3. Possible solutions 4. Perspectives What is the conflict about? "Nagorno – Karabakh conflict is characterizing as a "hyper-complex" conflict. This conflict is dissimilar to the existing currently conflict in South Tyrol. Author suggests that the conflict in Nagorno – Karabakh has limited perspectives. Currently, different countries are involved in conflict resolution. The author notes that there is one truth related to Nagorno – Karabakh: "History is a superhuman process made by human beings." The experience of Nagorno – Karabakh conflict resolution brings to appearing of tension between Armenia and Azerbaijan and could have an influence on interested states. Consequently, the conflict resolution process lasts.

Kaliningrad conflict[17] is dissimilar to existing territorial conflicts in that it is linked to the CORRIDOR principle. The reason for the conflict is that the territory of the Kaliningrad region is outside Russia. Therefore, Russian citizens must apply to the Lithuanian government to enter Kaliningrad. This feature makes it difficult for Russia to visit Kaliningrad. «In November 2002, the EU and Russia agreed to a joint statement encouraging Russian citizens traveling to/from Kaliningrad to replace their visas with Facilitated Rail Transit Docu-

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[14] **Report Realized for the Buerau of Research Territorial Defense Troops. Present State forward Direction changes 44 pages** https://www.stratpoints.eu/wp-content/uploads/2018/03/WOT-Raport_ENG.pdf

[15] **Susumu Takai** Support for Conflict Resolution and the Role of Military Power http://www.nids.mod.go.jp/english/event/symposium/pdf/2002/sympo_e2002_11.pdf

[16] **Roland Benedikter Nagorno-Karabakh:** The Endless Conflict in the Black Garden https://www.e-ir.info/2021/07/14/nagorno-karabakh-the-endless-conflict-in-the-black-garden/ROLAND BENEDIKTER, JUL 14 2021, https://www.e-ir.info/2021/07/14/nagorno-karabakh-the-endless-conflict-in-the-black-garden/ Roland Benedikter July, 14 2021, 7pages

[17] **The Carter Center One Copenhill 453 Freedom Parkway Atlanta, GA 30307** TRANSPORTATION CORRIDORS Corridors in General Kaliningrad. Approaches to Solving Territorial Conflicts Sources, Situations, Scwww.cartercenter.orgenarios, and Suggestions May 2010, www.cartercenter.org

ments (FRTD) and Facilitated Transit Documents (FTD). Once such documents were obtained, eligible Russian residents traveling to/from the Kaliningrad region would no longer have to contact Lithuanian consulates»

Literature devoted to the study of the numerical methods for the solution of ordinary differential equations.

Among numerical methods for solving ordinary differential equations (ODE) the **Euler** method[18] (also called forward Euler method) is the procedure known as a first-order numerical procedure that provides solving ordinary differential equations given with initial value. Among numeric methods, it is the most basic explicit method for solving the ODE. The main idea of the Euler method is that while the curve is initially unknown, it uses starting point, and then, starting from the differential equation, computes the slope to the curve and so on. The Euler method is per step solution of the ODE. Moreover, this method means that the local error corresponding to each step is proportional to the square of the step size. The resulting (global) error is proportional to the step size. The advantage of the Euler method is that in the field of solution of ODE it serves as the basis for many more complex methods.

Adaptive Runge – Kutta (ARG)[19] method is the extension of Euler method. It is an implicit and explicit iterative method based on a well-known routine following from the Euler method. It is method is a numerical technique devoted to solving the ODE. ARG method is based on the temporal discretization approach and provides the approximation solution of ODE. It produces an estimate of the local truncation error of a single step. ARG method has two common steps. Due to this, estimating the error has a little or negligible computational cost. The feature of the ARG method is that the step, during the integration, is adapted such that the estimated error stays below a user.

Literature devoted to the study of the extension of Lottka-Volterra model.

It is known that the Lottka – Volterra model[20] is a pair of differential equations The Lotka-Volterra model is a pair of differential equations representing the populations consisting of a predator and prey species. The feature of the modeling is the study of the interactions between species. The model was independently proposed in 1925 by American statistician Alfred J. Lottka and Italian mathematician Vito Volterra. **E. Chauvet, J. Paullet, J. Previte,** extending the model of Lottka – Volterra studied the extension of Lottka – Volterra model considering three species. They added to the traditional system of two equations of Lottka – Volterra model one more equation. Further[21], **Lalith Devireddy** studied an extending the Lotka-Volterra model. The model consists of three and

[18] **Butcher, John C.** (2008), Numerical Methods for Ordinary Differential Equations, New York: John Wiley & Sons, ISBN 978-0-470-72335-7.

[19] **Butcher, John C.** (2008), Numerical Methods for Ordinary Differential Equations, New York: John Wiley & Sons, ISBN 978-0-470-72335-7.

[20] **Chauvet, Erica, Joseph E. Paullet, Joseph P. Previte, and Zac Walls.** "A Lottka-Volterra Three-Species Food Chain." Mathematics Magazine 75.4 (2002): 243-55

[21] **Lalith Devireddy** https://sites.math.washington.edu/~morrow/336_16/2016papers/lalith.pdf

more species and generalized known approaches for three species models. Simultaneously, the author gave the algorithm for the solution of a general model.

2. Definitions and notations

2.1. Definition of quadratic spline

Let us consider two sets of nodes given on the segment $[t_0, T]$:

$t_0 < t_1 < t_2 < ....t_i < t_{i+1} < ... < t_l$

$\bar{t}_1 < \bar{t}_2 < \bar{t}_3 < ....\bar{t}_i < \bar{t}_{i+1} < ... < \bar{t}_{l+1}$, where $l \geq 2$.

$t_{i-1} < \bar{t}_i < t_i, i = 1,2,...,l$

Let $h_i = t_{i+1} - t$, $i = 0,1,..l-1$, $\bar{h}_i = t_{i+1} - \bar{t}_{i+1}, i = 0,1,..l-1$

Throughout what follows we will assume that $h_i = h, i = 1,2,...,n, \bar{h} = h/2$.

For $t \in [t_i, t_{i+1}]$, $i = 0,1,...,n-1$ consider spline (Stechkin, Subbotin[22]), approximating functions $p_k(t), k = 1,2,...,n$, that have the following form:

(i) $$S_{2ki}(t, p_k) = p_k(t_i) + m_{ki}(t - t_i) + c_{ki}(t - t_i)^2 \quad (1)$$

(ii) $$S_{2ki}(t, p_k) \in C^1[a,b], \quad (2)$$

(iii) $$S_{2ki}(t_i, p_k) = p_k(t_i) \quad (3)$$

(iv) $$\frac{dS_{2ki}}{dt}\Big|_{t=t_i} = m_{ik} \quad (4)$$

where $i = 1,2,...,l-1, k = 1,2,...,n.$

Numbers $\bar{t}_i$ are called spline nodes, and numbers t_i *are called interpolation nodes.*

We require that the spline (1) –(4) satisfy the system of the ODE (3.1), (3.2).

2.2. Extending the Lottka-Volterra model

Author [23] proposed an extension of the Lotka-Volterra model

$$\frac{dp_k}{dt} = g_k p_k(t) + \sum_{1 \leq i,k,j \leq n, k \neq j} d_{k,j} p_k(t) p_j(t) \quad (5)$$

Where

i) $p_k(t)$ is the population of $k-th$ species,

ii) $g_k p_k(t)$ represents either the growth or natural death rate of the species and simultaneously it is proportional to the species population,

iii) $d_{k,j}$ is a constant representing $j-th$ contribution to assess the effect of $d_{k,j} p_k(t) p_j(t)$

to the differential $\frac{dp_k(t)}{dt}$ which is the measure of the rate of change of

[22] **Stechkin, Subbotin** Splines in Computational Mathematics. Moscow, Nauka, p.35.,272 p., 1976.

[23] **Lalith Devireddy** Extending the Lotka-Volterra Equations, https://sites.math.washington .edu/~morrow/336_16/2016papers/lalith.pdf

$k-th$ population at time t following after the interaction with $j-th$ specie.

iv) we will assume that two species are interacting to benefit which other. Depending on the sign in front of the coefficient $d_{k,j}$ the rate $d_{i,j}p_k(t)P_j(t)$ could be positive or negative. In the first case the contribution to the $\frac{dp_k(t)}{dt}$ is positive from the interaction between $k-th$ and $j-th$ species. The negative sign in front of the coefficient $d_{k,j}$ proofs that the $j-th$ specie causes the decrease of the population of $k-th$ specie from the interaction with $j-th$ specie.

Everywhere below we will assume that the system of countries is considering as species of the Lottka-Volterra model. Countries amounts of military spending are given similarly to the population of the Lottka -Volterra model denoted as $p_k(t), k=1,2,...,n$.

Let us consider the system of ordinary differential equations as the example of (5) as follows:

$$\frac{dp_1}{dt}=g_1p_1(t)+d_{1,2}p_1(t)p_2(t)+d_{1,3}p_1(t)p_3(t)+d_{1,4}p_1(t)p_4(t)+d_{1,5}p_1(t)p_5(t) \quad (6)$$

$$\frac{dp_2}{dt}=g_2p_2(t)+d_{2,1}p_1(t)p_2(t)+d_{2,3}p_2(t)p_3(t)+d_{2,4}p_2(t)p_4(t)+d_{2,5}p_2(t)p_5(t) \quad (7)$$

$$\frac{dp_3}{dt}=g_3p_3(t)+d_{3,1}p_1(t)p_3(t)+d_{3,2}p_3(t)p_2(t)+d_{3,4}p_3(t)p_4(t)+d_{3,5}p_3(t)p_5(t) \quad (8)$$

$$\frac{dp_4}{dt}=g_4p_4(t)+d_{4,1}p_1(t)p_4(t)+d_{4,2}p_4(t)p_2(t)+d_{4,3}p_3(t)p_4(t)+d_{4,5}p_4(t)p_5(t) \quad (9)$$

$$\frac{dp_5}{dt}=g_5p_5(t)+d_{5,1}p_1(t)p_5(t)+d_{5,2}p_5(t)p_2(t)+d_{5,3}p_3(t)p_5(t)+d_{5,4}p_4(t)p_5(t) \quad (10)$$

Assume that $t\in[0,T],\ \ p_k(0)=p_{k0}, k=1,2,...,5,$ where as we proposed $p_k(t)$ is amount of military spending of $k-th$ country, $k=1,2,...,5$.

3. Solution of five countries model (6)-(10)

Assume that $t\in[t_i,t_{i+1}]$, and require that the spline (1) –(4) satisfies the system of the ODE (6) - (10). We get

$$\frac{dS_{21i}}{dt}=g_1S_{21i}+d_{1,2}S_{21i}S_{22i}+d_{1,3}S_{21i}S_{23i}+d_{1,4}S_{21i}S_{24i}+d_{1,5}S_{21i}S_{25i} \quad (11)$$

$$\frac{dS_{22i}}{dt}=g_2S_{22i}+d_{2,1}S_{21i}S_{22i}+d_{23}S_{22i}S_{23i}+d_{2,4}S_{22i}S_{24i}+d_{2,5}S_{22i}S_{25i} \quad (12)$$

$$\frac{dS_{23i}}{dt} = g_3 S_{23i} + d_{3,1} S_{23i} S_{21i} + d_{32} S_{22i} S_{23i} + d_{3,4} S_{23i} S_{24i} + d_{3,5} S_{23i} S_{25i} \quad (13)$$

$$\frac{dS_{24i}}{dt} = g_4 S_{24i} + d_{4,1} S_{24i} S_{21i} + d_{32} S_{22i} S_{24i} + d_{4,3} S_{23i} S_{24i} + d_{4,5} S_{24i} S_{25i} \quad (14)$$

$$\frac{dS_{25i}}{dt} = g_5 S_{25i} + d_{5,1} S_{25i} S_{21i} + d_{52} S_{22i} S_{25i} + d_{5,3} S_{25i} S_{23i} + d_{5,4} S_{24i} S_{25i} \quad (15)$$

Assume that $t \in [t_i, t_{i+1}]$, then from (4), (11) – (15) follows:

$$m_{i1} = \frac{dS_{21i}}{dt}\Big|_{t=t_i} = p_1(t_i)(g_1 + d_{12}p_2(t_i) + d_{13}p_3(t_i) + d_{14}p_4(t_i) + d_{15}p_5(t_i) \quad (16)$$

$$m_{i2} = \frac{dS_{22i}}{dt}\Big\{_{t=t_i} = p_2(t_i)(g_2 + d_{21}p_1(t_i) + d_{23}p_3(t_i) + d_{24}p_4(t_i) + d_{25}p_5(t_i) \quad (17)$$

$$m_{i3} = \frac{dS_{23i}}{dt}\Big|_{t=t_i} = p_3(t_i)(g_3 + d_{31}p_1(t_i) + d_{32}p_3(t_i) + d_{34}p_4(t_i) + d_{35}p_5(t_i) \quad (18)$$

$$m_{i4} = \frac{dS_{24i}}{dt}\Big|_{t=t_i} = p_4(t_i)(g_4 + d_{41}p_1(t_i) + d_{42}p_3(t_i) + d_{43}p_4(t_i) + d_{45}p_5(t_i) \quad (19)$$

$$m_{i5} = \frac{dS_{25i}}{dt}\Big|_{t=t_i} = p_5(t_i)(g_5 + d_{51}p_1(t_i) + d_{52}p_3(t_i) + d_{53}p_4(t_i) + d_{54}p_5(t_i) \quad (20)$$

Let

$$r_{ik} = p_k(t_i) + m_{ik}(t - t_i) \quad (21)$$

$$w_{i1} = g_1 h^2 + d_{12} r_{i2} h^2 + d_{13} r_{i3} h^2 + d_{14} r_{i4} h^2 + d_{15} r_{i5} h^2 \quad (22)$$

$$w_{i2} = g_2 h^2 + d_{21} r_{i1} h^2 + d_{23} r_{i3} h^2 + d_{24} r_{i4} h^2 + d_{25} r_{i5} h^2 \quad (23)$$

$$w_{i3} = g_3 h^2 + d_{31} r_{i1} h^2 + d_{32} r_{i2} h^2 + d_{34} r_{i4} h^2 + d_{35} r_{i5} h^2 \quad (24)$$

$$w_{i4} = g_4 h^2 + d_{41} r_{i1} h^2 + d_{42} r_{i2} h^2 + d_{43} r_{i3} h^2 + d_{45} r_{i5} h^2 \quad (25)$$

$$w_{i5} = g_5 h^2 + d_{51} r_{i1} h^2 + d_{52} r_{i2} h^2 + d_{53} r_{i3} h^2 + d_{54} r_{i4} h^2 \quad (26)$$

$$q_{i1} = g_1 r_{i1} + d_{12} r_{i2} r_{i1} + d_{13} r_{i1} r_{i3} + d_{14} r_{i4} r_{i1} + d_{15} r_{i5} r_{i1} \quad (27)$$

$$q_{i2} = g_2 r_{i2} + d_{21} r_{i2} r_{i1} + d_{23} r_{i2} r_{i3} + d_{24} r_{i4} r_{i1} + d_{25} r_{i5} r_{i2} \quad (28)$$

$$q_{i3} = g_3 r_{i3} + d_{31} r_{i3} r_{i1} + d_{32} r_{i2} r_{i3} + d_{34} r_{i4} r_{i3} + d_{35} r_{i5} r_{i3} \quad (29)$$

$$q_{i4} = g_4 r_{i4} + d_{41} r_{i4} r_{i1} + d_{42} r_{i2} r_{i4} + d_{43} r_{i4} r_{i3} + d_{45} r_{i5} r_{i4} \quad (30)$$

$$q_{i5} = g_5 r_{i5} + d_{51} r_{i5} r_{i1} + d_{52} r_{i2} r_{i5} + d_{53} r_{i3} r_{i5} + d_{54} r_{i5} r_{i4} \quad (31)$$

where $k = 1,2,\ldots,5,\ i = 1,2,\ldots,l.$

Assume that $t \in [t_i, t_{i+1}]$, then from (1), (3), (4), (11) – (15), (21)-(26) follows:

$$m_{i1}+2c_{i1}h=\frac{dS_{2i1}}{dt}\Big|_{t=t_{i+1}}=c_{i1}w_{i1}+c_{i2}d_{12}r_{i1}h^2+c_{i3}d_{13}r_{i1}h^2+c_{i4}d_{14}r_{i1}h^2+c_{i5}d_{15}r_{i1}h^2+$$

$$d_{12}c_{i1}c_{i2}h^4+d_{13}c_{i1}c_{i3}h^4+d_{14}c_{i1}c_{i4}h^4+d_{15}c_{i1}c_{i5}h^4+q_{i1}, \qquad (32)$$

$$m_{i2}+2c_{i2}h=\frac{dS_{2i2}}{dt}\Big|_{t=t_{i+1}}=c_{i2}w_{i2}+c_{i1}d_{21}r_{i2}h^2+c_{i3}d_{23}r_{i2}h^2+c_{i4}d_{24}r_{i2}h^2+c_{i5}d_{25}r_{i2}h^2+$$

$$d_{21}c_{i1}c_{i2}h^4+d_{23}c_{i2}c_{i3}h^4+d_{24}c_{i2}c_{i4}h^4+d_{25}c_{i2}c_{i5}h^4+q_{i2}, \qquad (33)$$

$$m_{i3}+2c_{i3}h=\frac{dS_{2i3}}{dt}\Big|_{t=t_{i+1}}=c_{i3}w_{i3}+c_{i1}d_{31}r_{i3}h^2+c_{i3}d_{32}r_{i3}h^2+c_{i4}d_{34}r_{i3}h^2+c_{i5}d_{35}r_{i3}h^2+$$

$$d_{31}c_{i1}c_{i3}h^4+d_{32}c_{i2}c_{i3}h^4+d_{34}c_{i3}c_{i4}h^4+d_{35}c_{i3}c_{i5}h^4+q_{i3}, \qquad (34)$$

$$m_{i4}+2c_{i4}h=\frac{dS_{2i4}}{dt}\Big|_{t=t_{i+1}}=c_{i4}w_{i4}+c_{i1}d_{41}r_{i4}h^2+c_{i2}d_{42}r_{i4}h^2+c_{i3}d_{43}r_{i4}h^2+c_{i5}d_{45}r_{i4}h^2+$$

$$d_{41}c_{i1}c_{i4}h^4+d_{42}c_{i2}c_{i4}h^4+d_{43}c_{i3}c_{i4}h^4+d_{45}c_{i4}c_{i5}h^4+q_{i4}, \qquad (35)$$

$$m_{i5}+2c_{i5}h=\frac{dS_{2i5}}{dt}\Big|_{t=t_{i+1}}=c_{i5}w_{i5}+c_{i1}d_{51}r_{i5}h^2+c_{i2}d_{52}r_{i5}h^2+c_{i3}d_{53}r_{i5}h^2+c_{i4}d_{54}r_{i5}h^2+$$

$$d_{51}c_{i1}c_{i5}h^4+d_{52}c_{i2}c_{i5}h^4+d_{53}c_{i3}c_{i5}h^4+d_{54}c_{i4}c_{i5}h^4+q_{i5} \qquad (36)$$

Assume that $t\in[t_0,t_1]$, then from (32) – (36) follows:

$$m_{01}-q_{01}=c_{01}(w_{01}-2h)+c_{02}d_{12}r_{01}h^2+c_{03}d_{13}r_{01}h^2+c_{04}d_{14}r_{01}h^2+c_{05}d_{15}r_{01}h^2$$

$$d_{12}c_{01}c_{02}h^4+d_{13}c_{01}c_{03}h^4+d_{14}c_{01}c_{04}h^4+d_{15}c_{01}c_{05}h^4, \qquad (37)$$

$$m_{02}-q_{02}=c_{02}(w_{02}-2h)+c_{01}d_{21}r_{02}h^2+c_{03}d_{23}r_{02}h^2+c_{04}d_{24}r_{02}h^2+c_{05}d_{25}r_{02}h^2$$

$$d_{21}c_{01}c_{02}h^4+d_{23}c_{02}c_{03}h^4+d_{24}c_{02}c_{04}h^4+d_{25}c_{02}c_{05}h^4, \qquad (38)$$

$$m_{03}-q_{03}=c_{03}(w_{03}-2h)+c_{01}d_{31}r_{03}h^2+c_{02}d_{32}r_{03}h^2+c_{04}d_{34}r_{03}h^2+c_{05}d_{35}r_{03}h^2$$

$$d_{31}c_{01}c_{03}h^4+d_{32}c_{02}c_{03}h^4+d_{34}c_{03}c_{04}h^4+d_{35}c_{03}c_{05}h^4, \qquad (39)$$

$$m_{04}-q_{04}=c_{04}(w_{04}-2h)+c_{01}d_{41}r_{04}h^2+c_{02}d_{42}r_{04}h^2+c_{03}d_{43}r_{04}h^2+c_{05}d_{45}r_{04}h^2$$

$$d_{41}c_{01}c_{04}h^4+d_{42}c_{02}c_{04}h^4+d_{43}c_{03}c_{04}h^4+d_{45}c_{04}c_{05}h^4, \qquad (40)$$

$$m_{05}-q_{05}=c_{05}(w_{05}-2h)+c_{01}d_{51}r_{05}h^2+c_{02}d_{52}r_{05}h^2+c_{03}d_{53}r_{05}h^2+c_{05}d_{54}r_{05}h^2$$

$$d_{51}c_{01}c_{05}h^4+d_{52}c_{02}c_{05}h^4+d_{53}c_{03}c_{05}h^4+d_{54}c_{04}c_{5}h^4. \qquad (41)$$

4. **Simulation model[24].** Let consider the group {1,2,3,4,5} of 5 countries having interests in one economic region.

5. **An assessment of constants** $d_{ij}, i,j=1,2,3,4,5$**,** $d_{ii}=g_i, i=1,2,3,4,5$**.** The assessment of constants $d_{ij}, i,j=1,2,3,4,5$, $d_{ii}=g_i, i=1,2,3,4,5$ will be implemented considering the partition of coun-

[24] **Remark.** In the simulation model, the group of countries supporting Armenia is conditionally selected. However, the model allows to consider other groups of countries instead of the option of countries.

tries on groups each with three country cooperative games and estimating Shapley[25] vectors. As groups of countries we'll consider groups as follows: {1,2,4}, {1,3,5}, {3,4,5}, {235}.

Table.1

Data by country of the region

Country	GDP Bln USA dollars	Military Spending Bln USA dollars	%GDP	Popularion (mln)	Area (thousand square km)
Armenia	13,67	0.6	0.044	2.97	29.74
Azerbaijan	48,05	2.3	0.048	10.22	86.6
Iran	468.15	17.4	0.037	85.09	1.648
Turkey	761.43	21.9	0.029	85.04	783.56
Russia	1,700	61.4	0.036	145.91	17,098.24
Total	2991.3	103.6		329.23	19646.14

Source:https://worldpopulationreview.com/country-rankings/military-spending-by-country

Table 2

Military spending in percentage to total military spending of the region by country

Country	% to total military spending in the region	Military Spending Bln USA dollars
Armenia (c_1)	0.006	0.6
Azerbaijan (c_2)	0.024	2.3
Iran (c_3)	0.17	17.4
Turkey (c_4)	0.21	21.9
Russia (c_5)	0.59	61.4
Total	100	103.6

Source: https://worldpopulationreview.com/country-rankings/military-spending-by-country

Table 3

Numbering of countries

Armenia	Azerbaijan	Iran	Turkey	Russia
1	2	3	4	5

1. For the group {1,2,4} of countries consider the cooperative game $< c(S),(c_1,c_2,c_5) >$ defined as follows:

$c(1) = c_1,\ c(2) = c_1 + c_2 = c(1,2),$

$c(4) = c_1 + c_2 + c_4 = c(1,4), c(2,4) = c(1,2,4)$

Denote Shapley[26] vector of this game as $\Phi(c) = (\Phi_1(c), \Phi_2(c), \Phi_4(c))$, where

[25] **Professor Giacomo Bonanno** Game Theory COOPERATIVE GAMES: the SHAPLEY VALUEShapley.pdf (ucdavis.edu)

[26] **Professor Giacomo Bonanno** Game Theory COOPERATIVE GAMES: the SHAPLEY VALUE Shapley.pdf (ucdavis.edu)

$\Phi_i(c) = \sum_{S:i\in i} \frac{(s-1)!(n-1)!}{n!}(c(S) - c(S \setminus \{i\})$, where n is the number of players, s is the number of players of the S coalition.

Thus, $\Phi_1(c) = \frac{c_1}{3}$, $\Phi_2(c) = \frac{c_1 + c_2}{3} + \frac{c_2}{6} = \frac{2c_1 + 3c_2}{6}$,

$$\Phi_4(c) = \frac{2(c_1 + c_2 + c_4) + c_2 + c_4 + c_4}{6} = \frac{2c_1 + 3c_2 + 4c_4}{6}.$$

Relations between players {2,4} and player {1} are conflicting in the economic - political region. Therefore, players {2} and {4} are pursuing the goal to defeat {1}- st country through the crushing of 1st country manpower, eliminating light and heavy weapons, occupying the enemy's territory, damaging the economic development and GDP, and Military spending growth, preventing military expenditures, preventing the ability of the country {1} to recover recent military and economic power.

An assessment of coalitions joint payoffs gives values as follows:

$$\Phi_1(c) + \Phi_2(c) = \frac{4c_1 + 3c_2}{6},\ \Phi_1(c) + \Phi_4(c) = \frac{4c_1 + 3c_2 + 4c_4}{6},$$

$$\Phi_2(c) + \Phi_4(c) = \frac{4c_1 + 6c_2 + 4c_4}{6}.$$

Considering the opposite goals of country's 2 and 4 against country 1 for the definition of constants d_{ij} we have: $d_{12} = -d_{21}, d_{14} = -d_{41}$. Consequently,

$$d_{12} = -\ (\Phi_1(c) + \Phi_2(c)) = -\frac{4c_1 + 3c_2}{6},$$

$$d_{14} = -\ (\Phi_1(c) + \Phi_4(c)) = -\ \Phi_1(c) + \Phi_4(c) = -\frac{4c_1 + 3c_2 + 6c_4}{6},$$

$$d_{21} = \Phi_1(c) + \Phi_2(c) = \frac{4c_1 + 3c_2}{6},$$

$$d_{41} = \Phi_1(c) + \Phi_4(c) = \Phi_1(c) + \Phi_4(c) = \frac{4c_1 + 6c_2 + 4c_4}{6}.$$

$$d_{24} = d_{42} = \Phi_2(c) + \Phi_4(c) = \frac{4c_1 + 6c_2 + 4c_4}{3}.$$

1. For the group {1,3,5} of countries consider the cooperative game $< c(S), (c_1, c_3, c_5) >$ and take into account the following features. The relations between the countries {3,5} and county {1} are constructive, they pursue the same goals for the given economic-political region. Moreover, the countries {3.5} realize that in order to protect their own interests in a given political-economic region, to reduce or eliminate countries {2,4} attacks on the country {1} on behalf of his statehood, economy, and army it is necessary to provide support as the military aid to the country {1}. Thus, the country {1} will enable them to regain their recent military power in the region as a powerful military unit. As a result

the pair {3,5} of countries having powerful partner country {1} get the opportunity to fulfill their demands in the given economic-political region.

Thus, in order to achieve these goals, countries {3,5} must support the country {1} to receive military assistance, enable him to strengthen his army, increase military spending, and restore lost military power. Consequently, to achieve these goals, it is obvious that the coalitions formed by the country {1} between the countries {3,5} will receive more than the expected payo ff due to the increase of the country's {1} military capabilities through the military investments of county's {3,5}. Payoff functions of the game of the group of countries {1,3,5} based on requirements given above are forming as follows:

$c(1) = c_1$
$c(3) = c_1+c_3$
$c(1,3) = 3c_1+c_3$
$c(5) = c_1+c_3+c_5$
$c(1,5) = 3c_1+c_3+c_5$
$c(3,5) = c_1+c_3+c_5$
$c(1,3,5) = 3c_1+c_3+c_5$

From the forming of coalitions (1,3), (1,5) and (1,3,5) follows that the payoffs increase due to the increase of the payoff of the player {1} through the military investments of countries {3,5}. Shapley vectors assessment is as follows:

$$\Phi_1(c) = \frac{8c_1}{6},\ \Phi_3(c) = \frac{4c_1+4c_3}{6},\ \Phi_5(c) = \frac{2c_1+c_3+3c_5}{3}.$$

An assessment of coalitions joint payoffs gives values as follows:

$$\Phi_1(c)+\Phi_3(c) = \frac{16c_1+5c_3}{6},\ \Phi_1(c)+\Phi_5(c) = \frac{7c_1+2c_3+2c_5}{6},$$

$$\Phi_3(c)+\Phi_5(c) = \frac{6c_1+5c_3+4c_5}{6}.$$

Considering the similar goals between country's 3, 5, and country 1 for the definition of constants d_{ij} we have:

$$d_{13} = d_{31} = \Phi_1(c)+\Phi_3(c) = \frac{6c_1+2c_3}{3},$$

$$d_{15} = d_{51} = \Phi_1(c)+\Phi_5(c) = \frac{6c_1+c_3+3c_5}{3},$$

$$d_{35} = d_{53} = \Phi_3(c)+\Phi_5(c) = \frac{4c_1+3c_3+3c_5}{6}.$$

2. For the group {3,4,5} of countries consider the cooperative game $< c(S),(c_3,c_4,c_5) >$

with payoff functions defined as follows:

$c(3) = c_3,\ c(4) = c_3+c_4 = c(3,4),$

$c(5) = c_3 + c_4 + c_5 = c(3,5) = c(4,5) = c(3,4,5)$

$\Phi_3(c) = \frac{c_3}{3}$, $\Phi_4(c) == \frac{3c_4 + 2c_3}{6}$, $\Phi_5(c) = \frac{2c_3 + 3c_4 + 6c_5}{6}$.

Thus, the constants for countries group {3,4,5} is as follows:

$$d_{34} = d_{43} = \Phi_3(c) + \Phi_4(c) = \frac{4c_3 + 3c_4}{6},$$

$$d_{35} = d_{53} = \Phi_3(c) + \Phi_5(c) = \frac{4c_3 + 3c_4 + 6c_5}{6},$$

$$d_{45} = d_{54} = \Phi_4(c) + \Phi_5(c) = \frac{2c_3 + 3c_4 + 3c_5}{3},$$

4. For the group {2,3, 5} of countries consider the cooperative game $< c(S), (c_2, c_3, c_5) >$

with payoff functions defined as follows:

$c(2) = c_2$,

$c(3) = c_3 + c_2 = c(2,3)$, $c(5) = c_2 + c_3 + c_5 = c(2,5) = c(3,5) = c(2,3,5)$

$\Phi_2(c) = \frac{c_2}{3}$, $\Phi_3(c) == \frac{3c_3 + 2c_2}{6}$, $\Phi_5(c) = \frac{2c_2 + 3c_3 + 6c_5}{6}$.

Thus, the constants for countries group {2,3, 5} is as follows:

$$d_{23} = d_{32} = \Phi_2(c) + \Phi_3(c) = \frac{3c_3 + 4c_2}{6},$$

$$d_{25} = d_{52} = \Phi_2(c) + \Phi_5(c) = \frac{4c_2 + 3c_3 + 6c_5}{6},$$

Substituting values of $d_{ij}, i, j = 1,2,3,4,5$, $d_{ii} = g_i, i = 1,2,3,4,5$ in the model (37)-(41) and using expressions (21)-(31) we are getting equations representing military spending for the end of one year as follows:

Table 4

Constants $d_{ij}, i, j = 1,2,3,4,5$, $d_{ii} = g_i, i = 1,2,3,4,5$ of the model

	1	2	3	4	5
1	0.006	-0.016	0.125333	-0.121	0.658666667
2	0.016	0.024	0.101	0.133	0.691
3	0.12533333	0.101	0.17	0.218333333	0.808333333
4	0.121	0.133	0.218333	0.21	0.913333333
5	0.65866667	0.691	0.808333	0.913333333	0.59

Source: Authors estimation

-54.33=0.44 c_{01} -0.00023 c_{02} +0.0014 c_{03} -0.0032 c_{04} +0.0037 c_{05} -0.0000016 $c_{01}c_{02}$ +1.57667E-05 $c_{01}c_{03}$ -0.0000226 $c_{01}c_{04}$ +2.60333E-05 $c_{01}c_{05}$ (42)

5025.79=3.017 c_{02} +0.00016 c_{01} +0.00097 c_{03} +0.00714 c_{04} +0.00691 c_{05} +0.0000016 $c_{01}c_{02}$ +9.7E-06 $c_{02}c_{03}$ +0.0000714 $c_{02}c_{04}$ +0.0000691 $c_{02}c_{05}$ (43)

-15135.1=1.673 c_{03} +0.13302 c_{01} +0.0818 c_{02} +0.184 c_{04} +0.456 c_{05} + +1.58E-05 $c_{03}c_{01}$ ++9.7E-06 $c_{03}c_{02}$ +2.18E-05 $c_{03}c_{04}$ +0.00005 $c_{03}c_{05}$ (44)

54218.4=2.96 c_{04} +0.398 c_{01} +1.258 c_{02} +0.385 c_{03} +1.856 c_{05} +0.00002 $c_{04}c_{01}$ + +0.00007 $c_{04}c_{02}$ +2.18E-05 $c_{04}c_{03}$ + $c_{04}c_{05}$ (45)

63968.1=2.233 c_{05} +0.707c c_{01} +1.876 c_{02} +1.469 c_{03} +2.86 c_{04} +2.6 $c_{01}c_{05}$ +0.000069 $c_{02}c_{05}$ +0.00005 $c_{03}c_{05}$ +0.0001 $c_{04}c_{05}$ (46)

Table 5

Solutions of the system of equations (42) – (46)

Coefficients	c_{01}	c_{02}	c_{03}	c_{04}	c_{05}
Values	-2.0	-2.0	-9.227	-1.7625	0.1
Errors *	0.0014	0.0909	0.1819	0.7276	0.0

*) Errors must be multiplied by 1.0E-11. Author's estimations based on MatLab

Using the equation (1), values of coefficients c_{ik} m_{ik}, $i = 0,1,2, k = 1,2,3,4,5.$ we are getting assessments for the first, second and third years as follows:

Table 6

Military spending by country

	Bln (starting year	Bln (First year)	Bln (second year)	Bln(third year)
Armenia	0.6	3.008534	4.264367031	5.520200062
Azerbaijan	2.3	3.3840363	5.040126077	6.696026853
Iran	17.4	18.3550976	19.40828578	20.46149495
Turkey	21.9	23.21518075	24.59052029	25.75399001
Russia	61.4	63.6136161	66.21914804	66.32340172
Total	103.6	111.5764648	119.5224472	124.7551136

Source: Author's estimations

Armenia forms the coalition with Iran and Russia as countries that have similar interests like Armenia. They have common steps to defense interests. Thanks to this, estimating the expected military spending for Armenia grew up to 3.008534 and continued to grow during the second and third years. During the integration, Iran and Russia Armenia adapted such that the estimated military spending stays higher than in the situation when Armenia acted out of the coalition. This result is almost optimal behavior for Armenia to defense for own interests. Moreover, using this approach we could find an appropriate behavior for Armenia to adjust military and economic behavior.

Conclusion

We proposed an approach for the modeling of territorial conflict adopted to the Lottka – Volterra model. We assumed that territorial conflict involves different states which are interesting in one economic-political region. The feature of the conflict is such that between conflicting countries we distinguish two countries that are directly opposed to each other, and the other countries join conflicting countries depending on their own interests. Taking into account that the problem studied in the paper consists of multiple tasks we joint the solution of these tasks in one common framework. Through the modeling of territorial conflict using the Lottka – Volterra model we presented the conflict as the system of differential equations and consequently assessed the degree of the interactions between countries using a game theoretical approach. The set of countries has been divided to groups of countries and each group of countries was admitted as the set of players of the game. Through the estimation of the Shapley vector we assessed the degree of interactions between countries.

We proposed an approach to model multi-operational extension of the Lottka-Volterra model considering an example of five countries having interests in one geo-political-economic region. Among these countries we distinguished two countries having interests and conflicting caused by the disorder of territorial integrity and as a consequence interacting for the exhausting of economic resources of one another. The model considers also the group of countries interacting with these two distinguished countries as well as having mutual interactions and interests in the region separately from the distinguished two countries.

The solution of the general model is given using the spline approximation method. We argue that spline approximation method allows getting a procedure for the solution based on the system of nonlinear equations. The solution of the model implemented for five states on general and the model allows to change considering states by other groups of states depending on newly states economic-political interests in considering the region.

We argue also, that the coalitions considered in the model allow providing military support providing an increase of military spending of one of the conflicting states with weak abilities to further enhance the amount of own military budget.

The numerical solution of the model for the group of countries allowed us substantiate practical usage of the model and as the outcome, we defined country's military spending amount for the period of three years.

In addition, we argue that Lottka –Volterra model allows the enhancement as means to adjust the economic-military policy of the cooperation between conflicting countries and define appropriate coalitions between conflicting countries.

ԱՐԱՄ ԱՌԱՔԵԼՅԱՆ, ԼԵՈՆ ՄԱԿԱՐՅԱՆ – *Տարածքային կոնֆլիկտի և միջազգային ռազմական համագործակցության մոդել* - Հոդվածում քննարկվում է Լոտտկա – Վոլտերրայի մոդելի ընդլայնումը՝ որպես տնտեսական շրջանի նկատմամբ տարածքային կոնֆլիկտային հարաբերություններ ունեցող երկրների ռազմական ծախսերի մոդելավորման միջոց։

Ուսումնասիրվող հիմնախնդիրը ներառում է այնպիսի հարցեր, ինչպիսիք են ռազմական ծախսերի կարգավորումը, համագործակցությունը, պետությունների միջև փոխգործակցությունը, բանակցությունների կամ պատերազմի հանգեցնող գործողություններ:

Որպես հիմնական ապրանք ուսումնասիրվում է Լոտտկա-Վոլտերրայի մոդելի ընդլայնումը: Լուծման համար դիտարկվում է սպլայն մոտարկման մեթոդը, որի հիման վրա ստացվել է ոչ գծային հավասարումների համակարգ:

Հետազոտության գործնական նշանակությունը հիմնավորելու նպատակով ուսումնասիրվել է տարածաշրջանային կոնֆլիկտի իմիտացման մոդելը, և այդ մոդելի լուծումը ներկայացվել է ոչ գծային հավասարումների միջոցով:

Որպես արդյունք՝ տրվել են տարածքային կոնֆլիկտային հարաբերություններ ունեցող երկրների ռազմական ծախսերի հաշվարկները երեք տարվա կտրվածքով:

Բանալի բառեր – *Լոտտկա-Վոլտերրայի մոդել, տարածքային կոնֆլիկտ, ռազմական ծախսեր, կարգավորում, կոոպերատիվ խաղ, Շեպլիի վեկտոր*

АРАМ АРАКЕЛЯН, ЛЕОН МАКАРЯН – ***Модель территориального конфликта и международного военного сотрудничества.*** **–** Статья посвящена применению модели Лоттка-Вольтерра как средства моделирования военных расходов стран, вовлеченных в конфликтые отношения касательно экономического региона. Рассматриваются такие вопросы, как регулирование военных расходов, сотрудничество, межгосударственная кооперация, решение конфликтов путем переговоров или военных действий.

Модель Лоттка-Вольтерра рассматриваетсяк как основной продукт с точки зрения конфликтов. Для решения модели рассматривается сплайн аппроксимация, на основе которой получается система нелинейных уравнений. Для обоснования прктической значимости задачи исследования была изучена имитационная модель регионального конфликта, решение которой было представлено в виде системы нелинейных уравнений. В результате даны оценки военных расходов стран, вовлеченных в территориальный конфликт на трехлетний период.

Ключевые слова: *модель Лотка-Вольтерра, территориальный конфликт, военные расходы, регулирование, кооперативная игра, вектор Шепли*