

## Fractal Differential Equations in Short-Term River Flow Forecasting

Ekaterina Gaidukova<sup>1</sup>, Varduhi Margaryan<sup>2\*</sup>, Sargis Qelyan<sup>2</sup>

<sup>1</sup> Russian State Hydrometeorological University, Saint Petersburg, Russia

<sup>2</sup> Yerevan State University, Yerevan, Armenia

### ABSTRACT

The article proposes a method for short-term forecasting of water discharge using models in the form of differential equations of fractional dimension. The dimension of the model is determined at the calibration stage by the fractal diagnostics method, and this certain dimension is preserved for the forecast lead time. The method based on the calculation of the correlation integral is chosen as the main method of fractal diagnostics. The resulting fractional differential equation is solved by numerical methods, and the efficiency of the approach is estimated by a generally accepted criterion. The algorithm of the proposed short-term forecasting method is implemented in the MatLab software application. The proposed approach was tested on river catchments with different conditions for the formation of river runoff during the spring flood and autumn floods. A hydrometeorological database with a daily resolution was formed: water discharge, precipitation amount, air temperature and snow cover height for the spring flood period. It was found that the efficiency of forecasting water consumption using fractional differential equations during autumn floods and spring floods increases by 75–100 %, respectively. This approach is suitable for use in operational practice when forecasting hydrological characteristics for the needs of water-dependent sectors of the economy. This approach is recommended for use in the presence of computer applications that allow solving differential equations of any order, including fractional ones, in real time.

**Keywords:** river runoff, differential equations, order of equation, fractional dimension, forecasting

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### \*Corresponding Author:

Varduhi Margaryan, Department of General Geography, Faculty of Geography and Geology, Armenia, Yerevan, A. Manoukian Street 1, 0025.

Email: [vmargaryan@ysu.am](mailto:vmargaryan@ysu.am)