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ON RESOLUTION STRATEGIES IN FUZZY LOGIC

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In this paper resolution strategies in fuzzy logic are investigated. Some new deduction strategies are introduced in fuzzy logic, analogous to those of classic logic, aimed to narrow deduction search space. In particular, semantic resolution is redefined for fuzzy logic, and some special cases are considered. The completeness of these strategies is proved.

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Introduction. For the principles of resolution method we refer the reader to [1]; the basic notions of fuzzy logic are given in [2] and [3]. The problem of existence and completeness of resolution methods in fuzzy logic deserves attention because the theory behind it is scarcely developed. In this paper semantic resolution for classical two-valued logic is studied, an analogous deduction system is defined for fuzzy logic, and its completeness is proved. The deduction system uses the general resolution rule for fuzzy logic, defined in [3], and its special case – the standard resolution.

1. Preliminary Facts. We consider the standard resolution rule, defined as follows:
Definition 1.1 [3]. The standard resolution rule (r'_R) is the following

$$r'_R := \frac{a/P(x) \bigtriangledown Q(x),\ b/\neg Q(y) \bigtriangledown R(z)}{a \otimes b/P(x)_\sigma \bigtriangledown R(z)_\sigma}, \tag{1}$$

where σ is the most general unifier of formulas $Q(x)$ and $Q(y)$; or, in the case of propositional logic:

$$r'_R := \frac{a/P \bigtriangledown Q,\ b/\neg Q \bigtriangledown R}{a \otimes b/P \bigtriangledown R}. \tag{2}$$

Theorem 1.1 [3]. Fuzzy logic is complete with the rule r'_R.

Bellow we give the definitions of "satisfiability" and "unsatisfiability", as well as we state some fundamental Theorems from [2].

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Definition 1.2 [2]. An interpretation D is said to satisfy a formula A, if $D(A) > 0.5$. An interpretation D is said to falsify a formula A, if $D(A) < 0.5$. If $D(A) = 0.5$, then D both satisfies and falsifies A. A formula is said to be unsatisfiable, if it is falsified by every interpretation of it.

Theorem 1.2 [2]. A set S of clauses in fuzzy logic is unsatisfiable, if and only if there is a finite unsatisfiable (in fuzzy logic) set S' of ground instances of S over the Herbrand universe of S.

Theorem 1.3 [2]. A set S of clauses is unsatisfiable in fuzzy logic, if and only if it is unsatisfiable in two-valued logic.

Lemma 1.1. Let S be a set of ground clauses in fuzzy logic. If there is a unit ground clause L in S, let S_1 be the result obtained from S by deleting those ground clauses in S containing L. If S_1 is empty, then S is satisfiable. Otherwise, if we denote by S_2 the set obtained from S_1 by deleting $\neg L$ from S_1, then S_2 is unsatisfiable, if and only if $\neg S$ is.

Proof. The proof solely relies on Theorem 1.3 and the Davis-Putnam method from [1], where similar result holds for two-valued logic. □

2. Semantic Resolution. The semantic resolution is described by the use of an interpretation to divide clauses into two groups and of an ordering to reduce the number by possible resolutions.

Definition 2.1. Let I be an interpretation, and P be an ordering of predicate symbols. A finite set of clauses $\{E_1,\dots,E_q,N\}, q \geq 1$, is called a semantic clash with respect to P and I (or PI-clash for short), if and only if $E_1,\dots,E_q$ (called electrons) and N (called the nucleus) satisfy the following conditions:

1. $E_1,\dots,E_q$ are falsified by I.
2. Let $R_1 = N$. For each $i = 1,\dots,q$ there is a resolvent R_{i+1} of R_i and E_i (under the r'_R resolution rule).
3. The literal in E_i, which is resolved upon, contains the largest predicate symbol in E_i, $i = 1,\dots,q$.
4. R_{q+1} is falsified by I.

R_{q+1} is called a PI-resolvent of the PI-clash $\{E_1,\dots,E_q,N\}$.

Definition 2.2. Let I be an interpretation for a set S of clauses in fuzzy logic, and P be an ordering of predicate symbols appearing in S. A deduction from S is called a PI-deduction, if and only if each clause in the deduction is either a clause in S or a PI-resolvent.

We will show here that the semantic resolution strategy is complete, i.e. for every finite unsatisfiable set S of clauses in fuzzy logic there is a PI-deduction of $\perp$ from S.

Theorem 2.1. If P is an ordering of predicate symbols in a finite unsatisfiable set S of ground clauses in fuzzy logic, and I is an interpretation of S, then there is a PI-deduction of $\perp$ from S.

Proof. This Theorem can be proved by induction. Let A be the atom set of S. If A consists of a single element, say Q, then the clauses Q and $\neg Q$ will be among the elements of S. Clearly, the resolvent of Q and $\neg Q$ is $\perp$. Since one of Q and $\neg Q$ must be falsified by I and $\perp$ is falsified by I, then $\perp$ is a PI-resolvent. Therefore, Theorem 2.1 holds for this case.

Assume Theorem 2.1 holds, when A consists of i elements, $1 \leq i \leq n$. To complete the induction we consider A, such that A consists of exactly $n+1$ elements. There are two possible cases:

Case 1. S contains a unit clause L that is falsified by I (L is a literal). Let S' be a set obtained from S by deleting those clauses containing the literal L and by deleting $\neg L$ from the remaining clauses. From Lemma 1.1 we conclude that S' is unsatisfiable. Since S' contains n or fewer atoms, by the induction hypothesis there is a *PI*-deduction D' of $\perp$ from S'. From the deduction D' we can obtain a *PI*-deduction of $\perp$ from S. This is done as follows:

- First, for each *PI*-clash $\{E'_1,\dots,E'_q,N'\}$, where $E'_1,\dots,E'_q,N'$ are clauses attached to initial nodes of D', if N' is obtained from a clause N in S by deleting $\neg L$ from N, replace the clash $\{E'_1,\dots,E'_q,N'\}$ by the *PI*-clash $\{E'_1,\dots,E'_q,L,N\}$ (where $E'_1,\dots,E'_q$, L are electrons and N is the nucleus).
- Second, if E'_i is obtained from a clause E_i in S by deleting $\neg L$ from E_i, attach the *PI*-clash $\{L,E_i\}$ above the node of E'_i. It is clear, after performing the above process, that we will obtaine a *PI*-deduction of $\perp$ from S.

Case 2. S doesn't contain a unit clause that is falsified by I. In this case, choose an element B in the atom set A of S such that B contains the smallest predicate symbol. Either B or $\neg B$ must be falsified by I. Let L be the element in $\{B,\neg B\}$ that is falsified by I. Let S' be the set obtained from S by deleting those clauses containing the literal $\neg L$ and by deleting L from the remaining clauses. Then S' is unsatisfiable. Since S' contains n or fewer atoms, by the induction hypothesis there is a *PI*-deduction D' of $\perp$ from S. Let D_1 be the deduction obtained from D' by putting the literal L back to those clauses, from which it was deleted. D_1 is still a *PI*-deduction, since L contains the smallest predicate symbol and L is falsified by I. D_1 is either a *PI*-deduction of $\perp$ or L. In the former case, we are done. In the second case, consider the set $(S\cup\{L\})$. Since $(S\cup\{L\})$ contains a unit clause L that is falsified by I, by the proof of Case 1 given above, there is a *PI*-deduction D_2 of $\perp$ from $(S\cup\{L\})$. Combining D_1 and D_2, we can obtain a *PI*-deduction of $\perp$ from S. This completes the proof of Theorem 2.1. □

Theorem 2.2 (Completeness of semantic resolution). If P is an ordering of predicate symbols in a finite and unsatisfiable set S of clauses in fuzzy logic, and I is an interpretation of S, then there is a *PI*-deduction of $\perp$ from S.

Proof. Since S is unsatisfiable, then by Theorem 1.2 there is a finite unsatisfiable set S' of ground instances of clauses in S. By Theorem 2.1, there is a *PI*-deduction D' of $\perp$ from S'. Using the *PI*-deduction D', we now show that we can produce a *PI*-deduction of $\perp$ from S. To see this, we merely attach to each node of D' a clause over or above the ground clause already there as follows: to each initial node, attach a clause in S, its ground clause is already in this node; then, for each non-initial node, if clauses have been attached in this way to each of its immediate predecessor nodes and constitute a *PI*-clash, attach to it the *PI*-resolvent whose ground clause is already in this node (existence of such resolvent is proved in [2]). In this fashion, a clause is attached to each node of which the ground clause already at the node is an instance. The clause attached to the terminal node must be $\perp$, since the clause is already $\perp$. It's easy to see that the deduction tree, together with the attached clauses, is a *PI*-deduction of $\perp$ from S. This completes the proof. □

3. Special cases of semantic resolution. In this section two special kinds of interpretations for semantic resolution are introduced.

Definition 3.1. A clause is called positive, if it doesn't contain any negation sign. A clause is called negative, if its every literal contains the negation sign. A clause is called mixed, if it's neither positive nor negative.

Definition 3.2. A positive hyperresolution is a special case of *PI*-resolution, in which every literal is falsified by the interpretation *I*.

Definition 3.3. A negative hyperresolution is a special case of *PI*-resolution, in which every literal is satisfied by the interpretation *I*.

Corollary 3.1. From Theorem 2.2 both positive and negative hyperresolutions are complete in fuzzy logic.

A Theorem consists of axioms $A_1 \ldots A_n$ and a conclusion B. To prove the theorem we are proving that $A_1 \& \ldots \& A_n \& \neg B$ is unsatisfiable. Since $A_1 \& \ldots \& A_n$ is usually satisfiable, it might be wise to avoid resolving clauses in $A_1 \& \ldots \& A_n$. To accomplish that, the set-of-support strategy is used.

Definition 3.4. A subset T of a set S of clauses is called a set of support of S if $S-T$ is satisfiable. A set-of-support resolution is a resolution of two clauses that are not both from $S-T$. A set-of-support deduction is a deduction, in which every resolution is a set-of-support resolution.

Using the Theorem 2.2, we can prove the following theorem.

Theorem 3.1. If S is a finite unsatisfiable set of clauses and T is a set of support of S, then there is a set-of-support deduction of $\perp$ from S.

Proof. Since $S-T$ is satisfiable, there is an interpretation I that satisfies $S-T$. Choose any ordering P of predicate symbols in S. By Theorem 2.2 there is a *PI*-deduction D of $\perp$ from S. Consider any *PI*-clash $\{E_1, \ldots, E_q, N\}$ in D. The *PI*-resolvent of this clash is obtained by first resolving E_1 and N, then resolving E_2 with this resolvent, etc. Every resolvent involves an electron E_i. Every electron is falsified by I, therefore, for every resolution the two clauses cannot both belong to $S-T$. Thus, the deduction D can be transformed into a set-of-support deduction of $\perp$ from S. □

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