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Physics

LIGHT PROPAGATION IN METAMATERIAL BASED ANISOTROPIC LAYER

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Light propagation (reflection, refraction and transmission) through homogeneous uniaxial medium layer with a finite thickness having double anisotropy (i.e. with anisotropy of both dielectric and magnetic permittivities) and an arbitrary orientation of its optical axis in the plane of incidence are investigated. Conditions when layer works as an omnidirectional reflector (independent from incident light polarization and angle) and also conditions for total transmission (independent from incident light polarization and for fixed incident angle) are considered. In purpose of getting laser cavity applications we have discussed certain case of anisotropic layer in which the light accumulation reaches two hundred times more than the intensity of incident light.

***Keywords*:** dielectric permittivity, resonant frequency, piezoelectric crystals, capacitance, inductance.

Introduction. Metamaterial is a system of artificial structural elements constructed for achieving useful and/or unusual electromagnetic properties. They exhibit linear and nonlinear optical properties such as negative refraction, inverse Doppler effect, propagation of energy of electromagnetic wave in the direction opposite to the wave vector etc. [1–5]. The concept of such media, with simultaneously negative dielectric and magnetic permittivities in certain frequency range, was theoretically proposed by Veselago in 1967 [1]. Natural isotropic crystals with simultaneously negative dielectric and magnetic susceptibilities are not yet revealed, but artificial media were created recently with use of periodically placed frames and fringes of a conducting material [2, 3] exhibiting negative refraction. These structures find also such astounding applications as perfect lenses [6], invisible masking [7, 8], perfect absorbers [9] etc.

Studies of such anisotropic metamaterials are currently of great interest (see, in particular, works [10–23]). We note that, as distinct from isotropic crystals (with negative refraction), there exist natural anisotropic crystals with different signs of components of dielectric permittivity tensor [24, 25]. In [20, 26] it is investigated the possibilities of total reflection at the boundary, isotropic medium-anisotropic medium, and a condition for total reflection is obtained. The dispersion equations, dispersion surfaces and their classifications for anisotropic metamaterials are studied in [15, 23, 27]. The most general, for uniaxial media, case of orientation of optical axis in space is considered in [28], where the influence of changes of

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orientation of optical axis in space on the wave vectors of modes excited in medium is studied. Non-reciprocity of waves in such media also was studied and possible applications of considered effects are proposed.

The photonic crystals (PCs) and metamaterials have been widely used as laser cavities. Dowling et al. predicted [29] lasing at the band edges of photonic band gaps (PBGs) materials based on the argument that light slows down near the band edge and so spontaneous emission would be enhanced.

In the present paper we have discussed all analogical studies of [26], but instead of half-space here a layer of finite thickness is observed. Based on the continuity of the field tangent components on the system borders is investigated light propagation (reflection, refraction and transmission) through homogeneous uniaxial medium layer having double anisotropy (i.e. with anisotropy of both dielectric and magnetic permittivities) and an arbitrary orientation of its optical axis in the plane of incidence. Conditions when the layer works as an omnidirectional reflector (independent from incident light polarization and angle) and conditions for total transmission (independent from incident light polarization and for fixed incident angle) also are considered. Certain case of anisotropic layer in which the light accumulation exceeds the initial intensity of incident light more than two hundred times, is discussed. This can be very important in construction of chiral metamaterials and their use in laser applications [30, 31].

Boundary Problem of Anisotropic Layer. Let consider light reflection, refraction and transmission on the anisotropic uniaxial metamaterial layer surrounded by isotropic media. The geometry of the problem is following: the uniaxial medium occupies layer $0 \le z \le d$ (d is the layer thickness), i.e. the interfaces between the media are parallel to the xy-plane and the plane of incidence coincides with the plane xz (xyz is the lab system). Electromagnetic wave at the wavelength is incident from medium 1 onto the considered layer (medium 2) at the angle and it is transmitted in medium 3. Medium 1 and medium 3 are homogeneous and isotropic with parameters and (dielectric and magnetic permittivities of the medium).

The problem of reflected and transmitted field amplitudes finding is reduced to the eight linear inhomogeneous equation system (obtained from the continuity of the field tangent components on the system borders) can be presented as follows:

$$\vec{E}_r = \hat{R}\vec{E}_i, \ \vec{T}_r = \hat{R}\vec{T}_i, \tag{1}$$

indices i, r and t denote the incident, reflected and transmitted fields respectively, $\hat{R} = \begin{pmatrix} r_p & 0 \\ 0 & r_s \end{pmatrix}$ and $\hat{T} = \begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix}$ are the reflection and transmission matrices $\vec{E}_{i,r,t} = E^p_{i,r,t}\vec{n}_p + E^s_{i,r,t}\vec{n}_s = \begin{pmatrix} E^p_{i,r,t} \\ E^s_{i,r,t} \end{pmatrix}$, $E^p_{i,r,t}$ and $E^s_{i,r,t}$ are field components parallel (p) and perpendicular (s) to the incident plane, $\vec{n}_p$ and $\vec{n}_s$ are p- and s-polarization orts. Solving the boundary problem, we get for the reflection matrix elements:

$$r_p = \frac{\nu_0 - \eta^2\cos^2\alpha}{\nu_0 + \eta^2\cos^2\alpha - 2\eta\cos\alpha\sqrt{\nu_0}\,\dfrac{d_1 + d_2}{d_1 - d_2}}, \quad r_s = \frac{\nu_0\cos^2\alpha - \gamma^2}{\nu_0\cos^2\alpha + \gamma^2 + 2\gamma\cos\alpha\sqrt{\nu_0}\,\dfrac{d_3 + d_4}{d_3 - d_4}},$$

$$t_p = \frac{-4d_1 d_2/(d_1-d_2)\eta\cos\alpha\sqrt{\nu_0}}{\nu_0+\eta^2\cos^2\alpha-2\eta\cos\alpha\sqrt{\nu_0}\dfrac{d_1+d_2}{d_1-d_2}},\ t_s = \frac{4d_3 d_4/(d_3-d_4)\gamma\cos\alpha\sqrt{\nu_0}}{\nu_0\cos^2\alpha+\gamma^2+2\gamma\cos\alpha\sqrt{\nu_0}\dfrac{d_3+d_4}{d_3-d_4}},$$

where $$\nu_0 = \frac{\varepsilon_0}{\mu_0},\quad \eta = \frac{\varepsilon_m\sqrt{(\delta_\varepsilon^2-1)\left(n_x^2+\varepsilon_m\mu_m(\delta_\mu-1)(1-\delta_\varepsilon\cos 2\phi)\right)}}{n_x^2+\varepsilon_m\mu_m(\delta_\mu-1)(1-\delta_\varepsilon\cos 2\phi)},$$

$$d_i = e^{ik_{zi}d},\quad \gamma = \frac{\sqrt{(\delta_\mu^2-1)\left(n_x^2+\varepsilon_m\mu_m(\delta_\varepsilon-1)(1-\delta_\mu\cos 2\phi)\right)}}{\mu_m(1-\delta_\mu^2)}.$$

The obtained reflection and transmission matrices allow calculating the reflected and transmitted field amplitudes, and reflection and transmission coefficients, $R=|E_r|^2/|E_i|^2$, $T=|E_t|^2/|E_i|^2$, and some other optical characteristics can be expressed through the reflection and transmission matrices, too.

According to the law of conservation of energy, if there is not absorbance or amplification, we have $R+T=1$.

Using above mentioned boundary conditions the refracted four fields get the following forms: $E_{1,2} = t_{1,2}\, E_x^i$, $E_{3,4} = t_{3,4}\, E_y^i$, where

$$t_{1,2} = \frac{2d_{1,2}/(d_1-d_2)\sqrt{\nu_0}\left(\pm\sqrt{\nu_0}-\eta\cos\alpha\right)}{\nu_0+\eta^2\cos^2\alpha-2\eta\cos\alpha\sqrt{\nu_0}\,(d_1+d_2)/(d_1-d_2)},$$
$$t_{3,4} = \frac{2d_{3,4}/(d_3-d_4)\sqrt{\nu_0}\,\cos\alpha\left(\pm\sqrt{\nu_0}\,\cos\alpha+\gamma\right)}{\nu_0\,\cos^2\alpha+\gamma^2+2\gamma\cos\alpha\sqrt{\nu_0}\,(d_3+d_4)/(d_3-d_4)}. \qquad (2)$$

Now let us consider the possibilities of total reflection (for arbitrary incident angle and light polarizations) and total transmission (independent on incident light polarization but for fixed incident angle, which is defined from the characteristic parameters of anisotropic layer). If the layer is thick enough to satisfy $(d_1+d_2)/(d_1-d_2)\approx 1$ condition, the r_p and r_s get the forms corresponding to half space [26], and t_p and t_s become zeros, which is also the case of the half space. So, if the layer is thick enough the conditions of total reflection and total transmission (for fixed incident angle) have the same form as in [26]. To confirm the above mentioned claims, in Fig.1 it is presented *s*- and *p*-reflection coefficients and $(d_1+d_2)/(d_1-d_2)$ value dependences on the d/λ relation. The layer is defined by following parameters: $\varepsilon_1 = 1.7$, $\varepsilon_2 = 2.8$, $\mu_1 = -1.0$, $\mu_2 = -1.4$, $\phi = \pi/4$. One can see that *s*- and *p*-reflection coefficients are converging jointly with $(d_1+d_2)/(d_1-d_2)$ coefficient to the value one (Fig. 1). It is worth noting that here total transmission for arbitrary incident angle and polarization is only possible, if $\delta_\varepsilon=\delta_\mu=0$, $\varepsilon_m = \pm\varepsilon_0$, $\mu_m = \pm\mu_0$, $\varepsilon_m\mu_m = \varepsilon_0\mu_0$ and consequently $n_{z1} = n_{z2} = -n_{z3} = -n_{z4} = -\sqrt{\varepsilon_0\mu_0}\cos\alpha$, which means that layer becomes an isotropic.

Now let us pass to detailed analysis of light reflection from anisotropic layer in the cases of all possible dispersion curves (see [28]). In the Fig. 2 it is presented the dependence of $R = (R_p+R_s)/2$ on the incident angle α, and the angle ϕ

(characteristic angle of optical axis) for all possible types of dispersion curves. In Fig. 2, a, b and c the cases are shown, when both (dielectric and magnetic) dispersion curves are ellipses, when one is hyperbole and other is ellipse and when both are hyperboles correspondingly.

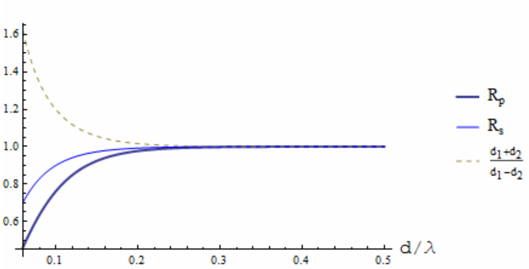
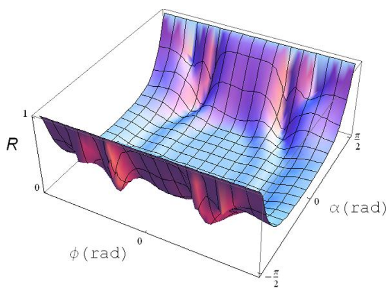
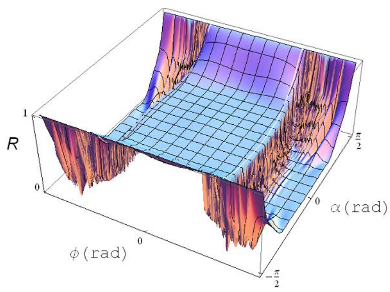


Fig. 1. R_p, R_s and $(d_1+d_2)/(d_1-d_2)$ dependences on the relation of layer thickness and wavelength, d/λ. Here $\varepsilon_1=1.7$, $\varepsilon_2=2.8$, $\mu_1=-1.0$, $\mu_2=-1.4$, $\phi=\pi/4$, $\alpha=\pi/3$.

From those dependences one can see that the slight changes of optical axes orientation for fixed incident angle cause to the sharp variation of reflection coefficient, and opposite, for the fixed orientation of optical axis a small change of incident angle can lead to the big increase or decrease of reflection. All the characteristic parameters which bring diferent types of dispersion surfaces are taken from the analogical reflection coefficient studies of article [26].





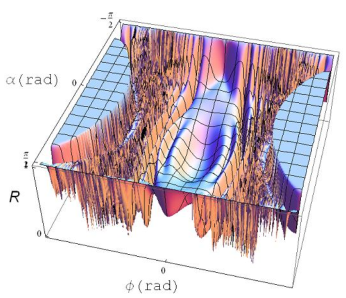


Fig. 2. Dependence of $R=(R_p+R_s)/2$ on the incident angle α and the angle ϕ (characteristic angle of optical axis) for the cases of all possible types of dispersion curves. Here

a) $\varepsilon_1=2.5$, $\varepsilon_2=1.5$, $\mu_1=1{,}7$, $\mu_2=2.9$, $\varepsilon_l=1$, $\mu_l=1$, $d=1\ \mu m$;
b) $\varepsilon_1=3.1$, $\varepsilon_2=2.5$, $\mu_1=-1.3$, $\mu_2=2.2$, $\varepsilon_l=1$, $\mu_l=1$, $d=1\ \mu m$;
c) $\varepsilon_1=-2.2$, $\varepsilon_2=3$, $\mu_1=0.3$, $\mu_2=-0.8$, $\varepsilon_l=1$, $\mu_l=1$, $d=1\ \mu m$.

The peculiarities of reflection and transmission from anisotropic layer in case, when the dispersion curve becomes the straight line, i.e. when (and only when) $\delta=\pm1$, are also analogical with the results of half-space, if $(d_1+d_2)/(d_1-d_2)\approx 1$.

Finally we will discuss also the refracted field intensity distribution in anisotropic layer calculated from (2):

$$I(z)=E_x(z)(E_x(z))^*+E_y(z)(E_y(z))^*, \tag{3}$$

where $E_x(z)=t_1E_x^ie^{i\,k_{z1}z}+t_2E_x^ie^{i\,k_{z2}z}$, $E_y(z)=t_3E_y^ie^{i\,k_{z3}z}+t_4E_y^ie^{i\,k_{z4}z}$.

In Fig. 3 for different wavelengths it is presented refracted light intensity distribution in the layer. One can see from graph that the intensity of refracted light in certain regions reaches almost two hundred times more than initial incident light intensity and in some regions that relation gets almost zero.

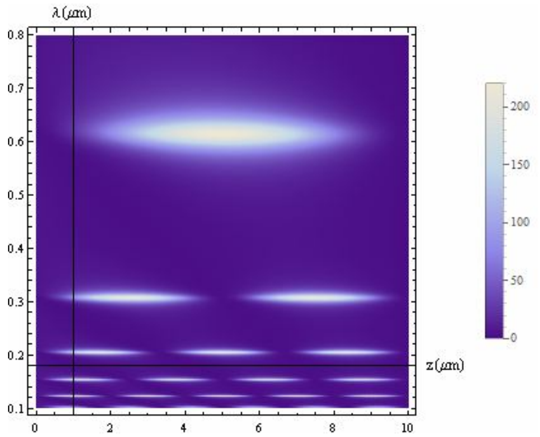


Fig. 3. Refracted light intensity in the anisotropic layer. Here $\varepsilon_1 = 0.0014$, $\varepsilon_2 = 2.2$, $\mu_1 = -1.5$, $\mu_2 = 0.2$, $\varepsilon_l = 1$, $\mu_l = 1$, $\phi = 0.7$, $\alpha = \pi/6$.

Conclusion: We considered light reflection, refraction and transmission from layer filled with medium having dielectric and magnetic anisotropies, at arbitrary orientation of its optical axis in the plane of incidence. We obtained analytical expressions for reflected, refracted and transmitted waves. The conditions of total reflection and transmission are also considered. Peculiarities of reflection of light from the half-space are studied for all types of dispersion curves. For certain values of characteristic parameters of layer, we have showed that the light accumulation in layer can be more than two hundred times bigger than initial intensity of incident light.

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