ON NON-MONOTONOUS PROPERTIES OF SOME CLASSICAL AND NONCLASSICAL PROPOSITIONAL PROOF SYSTEMS

A. A. CHUBARYAN *, A. A. HAMBARDZUMYAN **

Chair of Discrete Mathematics and Theoretical Informatics, YSU, Armenia

We investigate the relations between the proof lines of non-minimal tautologies and its minimal tautologies for the Frege systems, the sequent systems with cut rule and the systems of natural deductions of classical and nonclassical logics. We show that for these systems there are sequences of tautologies $\psi_n$, every one of which has unique minimal tautologies $\phi_n$ such that for each $n$ the minimal proof lines of $\phi_n$ are an order more than the minimal proof lines of $\psi_n$.

https://doi.org/10.46991/PYSU:A/2020.54.3.127

MSC2010: 03F20; 03F07.

Keywords: minimal tautology, Frege system, sequent system, natural deduction system, proof lines, proof sizes, monotonous and strongly monotonous system.

Introduction. The minimal tautologies play a main role in the proof complexity area. Namely, the well-known propositional formulas (Pigeon Hole Principle, Clique Colouring pair, Topsy-Turvy Matrix, Hool’s theorem, Ramsey theorem and some others), the proof complexities of which are investigated in many papers, are minimal tautologies. There is a traditional assumption that a non-minimal tautology shouldn’t be less complicated than its minimal tautology, that means it must be some monotonicity of proofs. This idea was first revised by Anikeev in [1]. He introduced the notion of a monotonous proof system and gave examples of monotonous and non-monotonous systems, but both of them are not complete systems. In [2–5], the notion of strongly monotonous systems for propositional proof systems is additionally introduced and the properties of monotonous and strongly monotonous for many propositional proof systems of classical and nonclassical logics are investigated. Some of the investigated systems (resolution systems, cut-free sequent systems) are monotonous systems, in each of which the proof lines of non-minimal tautologies are not less than the proof lines of their minimal tautologies. Some others are not monotonous (systems based on the splitting method, elimination systems), the proof lines of some formulas can

* E-mail: achubaryan@ysu.am
** E-mail: arsen.hambardzumyan2@ysumail.am
A. A. CHUBARYAN, A. A. HAMBARDZUMYAN

be less than the proof lines of some of their minimal tautologies. It was proved that many known systems are not strongly monotonous, but the question of monotonicity for some strong propositional systems still remains open. In [6] it is proved that the Frege systems of classical logic are not monotonous. In this paper we generalize this result for some other systems of classical logic (the sequent systems with cut rule and the systems of natural deductions), as well as for analogous systems of intuitionistic and Johansson’s logics. It is shown that for each mentioned systems there are the sequences of tautologies \( \psi_n \) (respectively sequents \( \rightarrow \psi_n \)), each of which has unique minimal tautologies \( \phi_n \) (respectively sequents \( \rightarrow \phi_n \)) such that for every \( n \) the minimal proof lines of \( \phi_n \) (\( \rightarrow \phi_n \)) are an order more than the minimal proof lines of \( \psi_n \) (\( \rightarrow \psi_n \)).

**Preliminaries.** We will use the current concepts of propositional formula, subformula, elementary subformula, sequent, tautology in different logics, Frege proof systems, sequent systems, systems of natural deductions, proof and proof complexity [2–9]. Let us recall some of them.

Taking into account that in nonclassical logics none of logical connectives can be presented through the others, we assume that the language for the presented propositional formulas contains logical connectives \( \neg, \&, \lor, \supset \) and may be constant \( \bot \) (false) as well. Further we use also \( A \Leftrightarrow B \) for the presentation of formula \( (A \supset B) \& (A \supset B) \).

**The Complexity Properties of Formulas Proofs and Proofs Systems.** We denote by \( |\phi| \) the size of the formula \( \phi \), defined as the number of all logical signs entries in it. It is obvious that the full size of the formula, which is understood to be the number of all symbols, is bounded by some linear function in \( |\phi| \).

**Definition 1.** A tautology of some logic is called minimal, if the replacement result of all occurrences of each its non-elementary subformulas by some new variable is not a tautology of the same logic.

**Definition 2.** A minimal tautology \( \phi \) of some logic is minimal of some formula, \( \psi \) if \( \phi \) is \( \psi \), or \( \phi \) is the replacement result of all occurrences of some non-elementary subformulas of \( \psi \) by some new variable.

We denote by \( M(\psi) \) the set of all minimal tautologies of the tautology \( \psi \).

In the theory of proof complexity one main characteristics of the proof is \( t \)-complexity (length), defined as the number of proof steps. We denote by \( t^\phi(\psi) \) the minimal possible value of \( t \)-complexity for all proofs of the tautology \( \phi \) in the system \( \phi \).

**Definition 3.** The proof system \( \phi \) is called \( t \)-monotonous, if for every tautology \( \psi \) there is a minimal tautology \( \phi \), such that \( \phi \in M(\psi) \) and \( t^\phi(\psi) = t^\phi(\phi) \).

**Definition 4.** The proof system \( \phi \) is called \( t \)-strongly monotonous, if for every tautology \( \psi \) there is no minimal tautology \( \phi \), such that \( \phi \in M(\psi) \) and \( t^\phi(\phi) > t^\phi(\psi) \).
Definitions of Investigated Systems. Here we give the descriptions of the Frege systems, the sequent systems with cut rule and the systems of natural deductions for classical (CL), intuitionistic (IL) and Johansson’s (JL) logics.

Definition 5. A Frege system $\mathcal{FC}$ for $\text{CL}$ uses a denumerable set of propositional variables, a finite, complete set of propositional connectives; $\mathcal{FC}$ has a finite set of inference rules defined by a figure of the form $\frac{A_1,A_2,\ldots,A_n}{B}$ (the rules of inference with zero hypotheses are the axioms schemes); $\mathcal{FC}$ must be sound and complete, i.e. for each rule of inference $\frac{A_1,A_2,\ldots,A_n}{B}$ every truth-value assignment, satisfying $A_1,A_2,\ldots,A_n$ also satisfies $B$, and $\mathcal{FC}$ must prove every tautology.

Definition 6. Two proof systems are called $t$-linearly equivalent, if any proof in one system can be modified to a proof of the same tautology in another system, so that the $t$-complexity of the proof is increased not more than linearly.

In [8], it is proved that all Frege systems of $\text{CL}$ are $t$-linearly equivalent.

For the definition of Frege systems for nonclassical logic, we must give some additional notions [9–13].

Definition 7. The inference rule $\frac{A_1,A_2,\ldots,A_n}{B}$ is called derivable in the system $\phi$, if the formula $(A_1 \supset (A_2 \ldots (A_n \supset B) \ldots))$ is provable in $\phi$.

Definition 8. The inference rule $\frac{A_1,A_2,\ldots,A_n}{B}$ is called admissible in the system $\phi$, if the formula $B$ is provable in $\phi$ from premises $A_1,A_2,\ldots,A_n$.

Note that for the systems of $\text{CL}$ the derivability property of some rule implies the admissibility properties and vice versa, but for the nonclassical logics we have another situation: in [13] it is proved that there are some inference rules for the systems of $\text{IL}$ ($\text{JL}$), which are admissible, but not derivable and it is proved in [11] ([12]) that the verification of the rule admissibility for the systems of $\text{IL}$ ($\text{JL}$) can be done with the linear $t$-complexity in some specific system for $\text{IL}$ ($\text{JL}$).

To give the definitions of Frege systems for $\text{IL}$ ($\text{JL}$), we must fix the following main proof systems for classical, intuitionistic and Johansson’s logics respectively by Cm, Im and Jm [9].

For each propositional formulas $A,B,C$ the axiom schemas of the classical system Cm are:

1) $A \supset (B \supset A)$
2) $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$
3) $A \supset (B \supset A \land B)$
4) $A \land B \supset A; A \land B \supset B$
5) $A \supset A \lor B; B \supset A \lor B$
6) $(A \supset C) \supset ((B \supset C) \supset (A \lor B \supset C))$
7) $(A \supset B) \supset ((A \supset \neg B) \supset \neg A)$
8) $\neg \neg A \supset A$
Inference rule is modus ponens \( \frac{A, A \supset B}{B} \) (m.p.).

In the system Im instead of axiom schema \( \neg A \supset A \) is taken \( \neg A \supset (A \supset B) \), which is omitted in the system Jm, where each formula \( \neg A \) is replaced by \( A \supset \bot \). Therefore, the axiom Schema 7 can be represented by the axiom Schema 2 and it must be omitted from Jm as well.

It is known that the deduction theorem holds in all three systems.

For each formula \( F \) we denote by J-image the formula, which is obtained from \( F \) by replacing each of its \( \neg A \)-type subformulas with formula \( A \supset \bot \). Note, that the size of J-image of formula \( F \) can not be more than \( 2|F| \).

It is known that if some formula \( F \) is provable in Cm, then the formula \( \neg \neg F \) is provable in Im, and if some formula \( F \) is provable in Im, then its J-image is provable in Jm.

**Definition 9.** The Frege system \( \forall I (\forall J) \) for \( IL (JL) \) is finite set of schematic axioms derivable in \( IL (JL) \) and schematic inference rules admissible in \( IL (JL) \) provided \( \forall I (\forall J) \) contains up to a linear translation by lines to main system Im (Jm).

Note that in [11, 12] the analogous definition is given with the last phrase “contains up to a polynomial translation by size to main system Im (Jm)” explaining the background for such definition, but it is the background for our Definition 9 as well.

As consequence we obtain

**Proposition 1.** Any two Frege systems over \( \neg, \& , \vee, \supset \) for \( IL \) are t-linearly equivalent [11]. Any two Frege systems over \( \& , \vee, \supset, \bot \) for \( JL \) are t-linearly equivalent [12].

Since the systems Cm, Im and Jm are Frege systems for \( CL, IL \) and \( JL \) respectively and it is proved in [9, 14] that the corresponding sequent systems with cut rule, the corresponding systems of natural deductions and mentioned main systems are t-linearly equivalent, from Proposition 1 it follows that

**Proposition 2.**
- The Frege systems, the sequent systems with cut rule and the systems of natural deductions of \( CL \) are t-linearly equivalent.
- The Frege systems, the sequent systems with cut rule and the systems of natural deductions over \( \neg, \& , \vee, \supset \) of \( IL \) are t-linearly equivalent.
- The Frege systems, the sequent systems with cut rule and the systems of natural deductions over \( \& , \vee, \supset, \bot \) of \( JL \) are t-linearly equivalent.

**Important Formulas.** In some papers on propositional proof complexity for 2-valued classical logic the following tautologies (Topsy-Turvy Matrix) play a key role

\[
TTM_{n,m} = \bigvee_{(\sigma_1,\sigma_2,\ldots,\sigma_n) \in E^n} \bigwedge_{j=1}^{m} \bigvee_{i=1}^{n} p_{ij}^{\sigma_j}(n \geq 1, 1 \leq m \leq 2^n - 1, E = 0, 1).
\]
For all fixed $n \geq 1$ and $m$ in the above indicated intervals every formula of this kind expresses the following true statement: given a $0,1$-matrix of order $n \times m$ we can “topsy-turvy” some strings (writing 0 instead of 1 and 1 instead of 0) such that each column will contain at least one 1.

Let $A_n = TTM_{n,2^n - 1}$. In [15], it is proved that in each Frege system of classical logic it holds the relation $t(A_n) = \Omega \left( \frac{|A_n|^2}{\log^2(|A_n|)} \right)$.

Note that the formula $\neg \neg A_n$ is provable in $IL$, its J-image is provable in $JL$ and it is obvious that the minimal numbers of their proof lines in corresponding systems $Im$ and $Jm$ can not have less order than the proof lines of $A_n$ in $Cm$.

**Main Results.** Here we prove that the mentioned systems of $CL$, $IL$, and $JL$ are non $t$-monotonous, consequently non $t$-strongly monotonous, but first we should give the following auxiliary statements relatively to the proof systems $Cm$, $Im$ and $Jm$.

**Lemma 1.** Let $P(A)$ be a tautology in $Cm$ ($Im$, $Jm$), $A$ be some of its subformulas of size $n$, and the variable $q$ is not presented in $P(A)$. There is a modification of the subformula $A$ to $A'(q)$, and there are not more than $2n$ tautologies $T_i(q)$, which can be proved in $Cm$ ($Im$, $Jm$) in a constant number of steps, so that no minimal tautology of $Cm$ ($Im$, $Jm$), can be obtained from the tautology $P'(A',q) = (q \supset P(A'(q))) \& T_1(q) \& \ldots \& T_i(q) \& \ldots$ by replacing with new variables neither any of the formulas $A'(q), T_1(q), T_2(q), \ldots$, nor their non-elementary subformulas.

**Proof.** Let’s describe the modification of each subformula $U$ of $A$, while we will add with the conjunctions the formulas $T_i(q)$.

1. If $U$ is a variable, then its modification coincides with itself, that is $U' = U$. No new formula will be added.

2. If $U = (U_1 \supset U_2)$, and $q$ is true in (1), then $U$ is equivalent to $(U_1 \supset (q \supset U_2))$. The modification of $U$ will be $U' = (U'_1 \supset (q \supset U'_2))$, where $U'_1$ and $U'_2$ are modifications of $U_1$ and $U_2$, respectively. We will add with conjunctions the formulas $\left( ( \neg q ) \supset ( U'_1 \supset ( q \supset U'_2 )) \right)$ and $\left( (\neg q) \supset ( q \supset U'_2) \right)$ for the systems $Cm$, $Im$ and its J-image for $Jm$. If we replace $(U'1 \supset (q \supset U'2))$ in $P'$, then $((\neg q) \supset (U'_1 \supset (q \supset U'_2)))$ will not remain a tautology. If we replace $(q \supset U'_2)$, then $((\neg q) \supset (q \supset U'_2))$ will not remain a tautology of corresponding system.

3. If $U = (\neg U_1)$, then we should modify $U_1$ to $U'_1$. The modified representation of $U$ will be $U' = (\neg U'_1)$. In case $U'_1$ isn’t a contradiction, we add with conjunctions the formula $(U'_1 \supset (\neg U'_2))$ for the systems $Cm$, $Im$ and its J-image for $Jm$, and otherwise the formula $((\neg U'_1) \supset U'_2)$ for $Cm$, $((\neg(\neg U'_1)) \supset U'_2)$ for $Im$ and its J-image for $Jm$. If we replace $(\neg U'_1)$ or $((\neg U'_1))$ in $P'$, then the added formulas won’t remain a tautology of the corresponding system.

4. If $U = (U_1 \& U_2)$, and $q$ is true in (1), then $U$ is equivalent to $(U_1 \& (q \& U_2))$. The modification of $U$ will be $U' = (U'_1 \& (q \& U'_2))$, where $U'_1$ and $U'_2$ are the modifications of $U_1$ and $U_2$, respectively. Then we will add with the conjunctions the formulas $((U'_1 \& (q \& U'_2)) \supset q)$ and $((q \& U'_2) \supset q)$ for the systems $Cm$, $Im$ and its J-image for
A. A. CHUBARY AN, A. A. HAMBARDZUMYAN

We can obtain by repeating all the steps for the \( J \)-images of mentioned formulas. The modification of \( q \) formulas (tautology), we can prove in \( C_m \) and \( I_m \) in a constant number of steps as well. Thus, constructing \( A \) following formulas will not remain a tautology of the corresponding system.

Finally, we will add \( q \) to \( P(A'(q)) \), because we assumed \( q \) is true in \( 1 \) when modifying \( \lor, \land, \lor \).

Further, if the variable \( q \) is not represented in a formula \( A \), then we call its modification \( A'(q) \) for the systems \( C_m \), \( I_m \) and \( J_m \) by the method described in Lemma 1. \( q \)-discharged presentation of \( A \).

Lemma 2. Let \( A \) be a formula of size \( n \) for one of the systems \( C_m \), \( I_m \) or \( J_m \) and let \( A'(q) \) be corresponding \( q \)-discharged representation. Then there exists a proof of \( q \vdash A'(q) \supset A \) in \( O(n) \) steps.

Proof. For any formulas \( C \) and \( D \) each of the following formulas
\[
(q \supset ((C \supset (q \supset D)) \supset (C \supset D))), \quad (q \supset ((C \supset D) \supset (C \supset (q \supset D))))
\]
\[
(q \supset ((C \land (q \land D)) \supset (C \land D))), \quad (q \supset ((C \land D) \supset (C \land (q \land D))))
\]
\[
(q \supset ((C \lor (q \lor D)) \supset (C \lor D))), \quad (q \supset ((C \lor D) \supset (C \lor (q \lor D))))
\]
is provable in \( C_m \) and \( I_m \) in a constant number of steps. Therefore from premise \( q \) the following formulas
\[
((C \supset (q \supset D)) \supset (C \supset D)) \text{ and } ((C \supset D) \supset (C \supset (q \supset D)))
\]
\[
((C \land (q \land D)) \supset (C \land D)) \text{ and } ((C \land D) \supset (C \land (q \land D)))
\]
\[
((C \lor (q \lor D)) \supset (C \lor D)) \text{ and } ((C \lor D) \supset (C \lor (q \lor D)))
\]
are provable in \( C_m \) and \( I_m \) in a constant number of steps as well.

We have, that for any formulas \( F, F', E \) and \( E' \) the following formulas
\[
(F \equiv F') \supset ((E \equiv E') \supset ((F \supset E) \supset (F' \supset E'))),
\]
\[
(F \equiv F') \supset ((E \equiv E') \supset ((F \land E) \equiv (F' \land E'))),
\]
\[
(F \equiv F') \supset ((E \equiv E') \supset ((F \lor E) \equiv (F' \lor E'))),
\]
\[
(F \equiv F') \supset ((F \equiv F') \equiv (F' \equiv F'))
\]
also are provable in \( C_m \) and \( I_m \) in a constant number of steps. Thus, constructing \( A \) step by step connecting to two equivalent formulas, not the old ones, but equivalent formulas (see Replacement Theorem 6 from [9]), we can prove in \( C_m \) and \( I_m \) from premise \( q \) the formula \( A'(q) \supset A \) with \( O(n) \) steps. Analogous result for the system \( J_m \) we can obtain by repeating all the steps for the \( J \)-images of mentioned formulas.

Lemma 3.

a) If the tautology \( E \) is a minimal for the tautology \( C \) in the system \( I_m \) (\( J_m \)), then tautology \( \neg E ((E \supset \bot) \supset \bot) \) is a minimal for tautology \( \neg C ((C \supset \bot) \supset \bot) \);

b) for any tautology \( A \) of the system \( I_m \) (\( J_m \)) the minimal steps of proof for the
formula \( \neg A((A \supset \bot) \supset \bot) \) in the system Im (Jm) can be more, than the minimal steps of proof for the formula A only with some constant.

\[ \text{Proof.} \text{ Is obvious.} \]

\[ \text{Theorem 1.} \text{ The systems Cm, Im and Jm are not t-monotonous and, consequently, are not t-strongly monotonous.} \]

\[ \text{Proof.} \text{ Consider the formula } P = A \lor (p \supset p), \text{ where } p, q, t, r \text{ are not presented} \]

\[ \text{in } A \text{ and } |A| = n. \text{ Obviously, } t \lor (p \supset p) \text{ is minimal for } P. \text{ If } A \text{ is a minimal tautology,} \]

\[ \text{then } A \lor r \text{ is also minimal to } P. \text{ We can construct } P' \text{ applying Lemma 12 to } P: \]

\[ P_n(A', q) = (q \lor ((p \supset p) \lor A'(q))) \& T_1(q) \& \ldots \& T_i(q) \& \ldots \]

\[ \text{It’s obvious that } P_n(A', q) \text{ can be proved in } O(n) \text{ steps and its unique minimal} \]

\[ \text{is the following:} \]

\[ B_n(q, r, A') = (q \lor (r \lor A'(q))) \& T_1(q) \& \ldots \& T_i(q) \& \ldots \]

\[ \text{Suppose it can be proved in Cm in } f(n) \text{ steps. Then the following three} \]

\[ \text{tautologies } B_n(q, \neg r, A'), B_n(\neg q, r, A'), B_n(\neg q, \neg r, A') \text{ can be proved in at most } f(n) \]

\[ \text{steps as well.} \]

\[ \text{Now we can prove that any minimal tautology } A \text{ is proved in Cm with} \]

\[ O(f(n) + |A|) \text{ steps.} \]

\[ \text{Let the formula } B_n(q, r, A') \text{ is proved already. Then in } O(n) \text{ steps by } \&\text{-elimination} \]

\[ \text{rule we can prove the formula } (q \lor (r \lor A'(q))). \text{ Now we add two premises } q, \neg r \text{ and} \]

\[ \text{continue the Proof:} \]

\[ q, \neg r \vdash (q \lor (r \lor A'(q))) \]

\[ \vdash q \]

\[ \vdash r \lor A'(q) \]

\[ \vdash \neg r \]

\[ \vdash \neg r \lor ((r \lor A'(q)) \lor A'(q)) \text{ tautology(*)} \]

\[ \vdash ((r \lor A'(q)) \lor A'(q)) \]

\[ \vdash A'(q) \]

\[ \ldots \ldots \ldots \ldots \]

\[ \vdash A'(q) \lor A \text{ by Lemma 2} \]

\[ \vdash A \]

\[ q \vdash \neg r \lor A \]

\[ \text{By analogy, after proving the formulas } B_n(q, \neg r, A'), B_n(\neg q, r, A'), B_n(\neg q, \neg r, A'), \]

\[ \text{we obtain: } q, \neg r \vdash (q \lor (r \lor A'(q))) \ldots q \vdash \neg r \lor A \text{ consequently, using the proof} \]

\[ \text{of tautology } (\neg r \lor A) \lor ((\neg r \lor A) \lor A) \text{ (**), we obtain } q \vdash A. \]

\[ \text{Further by analogy } \neg q, \neg r \vdash (\neg q \lor (r \lor A'(q))) \ldots \neg q \vdash \neg r \lor A \text{ and } \neg q, \neg r \vdash \]

\[ (\neg q \lor (r \lor A'(q))) \ldots \neg q \vdash \neg r \lor A \ldots, \neg q \vdash A \text{ and finally, using the proof of} \]

\[ \text{tautology } (q \lor A) \lor ((q \lor A) \lor A) \text{ (***) we prove } A. \]

\[ \text{Thus, we got a proof of any minimal tautology of size } n \text{ with } O(f(n) + n) \]

\[ \text{steps. On the other hand, as it was noted in “Important Formulas’, there exists a} \]

\[ \text{sequence of minimal tautologies } A_n, \text{ such that in each Frege system of classical logic} \]
\[ t(A_n) = \Omega \left( \frac{|A_n|^2}{\log_2(|A_n|)} \right) \] and consequently, \( f(n) = \Omega \left( \frac{n^2}{\log_2(n)} \right) \). So, for the system Cm we constructed the tautologies \( P_n'(A', q) \), which are proved in \( O(n) \) steps, while its unique minimal tautologies \( B_n(q, r, A') \) in \( \Omega \left( \frac{n^2}{\log_2(n)} \right) \) steps.

Note that we use the classical tautology (*) and results of the substitutions in it, which are tautology in Im and its J-images are tautologies in Jm, but the classical tautologies (**) and (***) are not provable in Im. Therefore, to prove the theorem for the system Im and Jm, we need to use the tautologies \( (\neg r \supset A) \supset ((\neg\neg r \supset A) \supset \neg\neg A) \), \( (q \supset A) \supset ((\neg q \supset A) \supset \neg\neg A) \) for Im and its J-images for Jm, which give us the proof of tautology \( \neg\neg A \) for any minimal tautology \( A \) for Im and J-image of \( A \) for Jm, as well as Lemma 3.

**Theorem 2.**

a) The Frege systems, the sequent systems with cut rule and the systems of natural deductions of CL are not t-monotonous, and consequently, are not t-strongly monotonous.

b) The Frege systems, the sequent systems with cut rule and the systems of natural deductions over \( \neg, \&, \lor, \supset \) of IL are not t-monotonous and, consequently, are not t-strongly monotonous.

c) The Frege systems, the sequent systems with cut rule and the systems of natural deductions over \( \&, \lor, \supset, \perp \) of JL are not t-monotonous and, consequently, are not t-strongly monotonous.

**Proof.** Follows from Theorem 1 and Proposition 2. Note, that for the sequent systems with cut rule and the systems of natural deductions for any minimal tautology \( A \) it is proved the correspondent sequents.

This work was supported by the Science Committee of the Ministry of Education, Science, Culture and Sport of RA, in the frames of the research project Grant No 18T-1B034.

Received 31.07.2020  
Reviewed 25.08.2020  
Accepted 18.11.2020

**REFERENCES**

[https://science.rau.am/rus/50/701](https://science.rau.am/rus/50/701)

http://www.flib.sci.am/eng/Reports/Frame.html


https://european-science.org/mainpage/sample-page/


http://www.flib.sci.am/eng/Reports/Frame.html


https://european-science.org/mainpage/sample-page/
В настоящей работе для систем Фреге, секвенциальных систем с правилом сечения и систем натуральных выводов классической и неклас- сических логик исследовано соотношение количества шагов выводов неми- нимальных тавтологий и их минимальных тавтологий. Доказано, что для каждой из рассмотренных систем существуют такие последовательности неминимальных тавтологий $\psi_n$, каждая из которых имеет единственную минимальную $\phi_n$ и для каждого $n$ наименьшее количество шагов выводов формулы $\phi_n$ по порядку больше наименьшего количества шагов выводов формулы $\psi_n$. 

А. А. ЧУБАРЯН, А. А. АМБАРЦУМЯН

О СВОЙСТВЕ НЕМОНОТОННОСТИ НЕКОТОРЫХ КЛАССИЧЕСКИХ И НЕКЛАССИЧЕСКИХ ПРОПОЗИЦИОНАЛЬНЫХ СИСТЕМ ВЫВОДОВ