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Mathematics

COMPLETELY INVARIANT SUBSPACES OF FREE ALGEBRAS

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A structural theorem is proved for pMqBM completely invariant subspaces of free associative algebras with a unit, having a countable number of free variables over the field of characteristic zero. In particular, it is shown that such spaces contain a Lie nilpotent polynomial.

Keywords: free algebra, *T*-ideals, linear space, endomorphism, invariant space, module, Lie nilpotent polynomial.

Let F be a free associative algebra with a unit, containing a countable number of free variables $T=\{t_1,t_2,...\}$ over the field K of characteristic zero. Its elements (polynomials) are formal K-linear combinations of different associative words (monomials) in the alphabet T with the natural operations of multiplication that turn it into a linear K-space [1].

Definition 1 [2]. The K-linear subspace $L\subset F$ is called a completely invariant K-space (F-module, F-bimodule), if L is invariant under all endomorphisms of the free algebra F, i.e. for any $\varphi\in\mathrm{End}(F)$, $\varphi(L)\subset L$.

Note, that T-ideals [1], linear T-spaces [4] of the free algebra F are examples of these spaces.

Definition 2 [2]. The completely invariant subspace $L\subset F$ is called a k-quasi finitely generated K-space (F-module, F-bimodule), if there exists a finite set of polynomials $\{f_1,...,f_k\}\subset L$, such that L as the K-space (as F-module, as F-bimodule) is generated by the set $\{\sigma f_i \,|\, i=1,2,...,k,\ \sigma\in S,\ S$ being the countable symmetric group$\}$.

Definition 3. The K-linear subspace $L\subset F$ is called a pMqBM space, if $L=L_1+L_2$, where L_1 is a p-quasi finitely generated F-module, L_2 is a q-quasi finitely generated F-bimodule.

Let $h=\sum_i \lambda_i U_i$ be an associative polynomial, where $\sum_i \lambda_i\neq 0, g=[V_1,V_2,...,V_k]$ is the Lie polynomial (U_i, V_j are arbitrary associative monomials in T); $L\subset F$ is a linear space.

Condition 1. There exists a Lie polynomial $g\in L$, and a completely invariant bimodule $L'\subset L$, such that $[g,h]\in L'$ for some associative polynomial h as above.

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Condition 2. If $g \in L$ is a Lie polynomial, then there exists a completely invariant bimodule $L' \subset L$ and an associative polynomial h as above such that $[g,h] \in L'$.

Employing the methods from Refs. [1–3, 5], we prove using some combinatorial considerations:

Theorem. If L is a completely invariant subspace in F over the field K of zero characteristic, satisfying the Condition 1, then L is a pM1BM space. Conversely, if L is a completely invariant pMqBM subspace in F with Condition 2, then L contains a polynomial $[t_1, t_2, ..., t_m]$ for some integer $m \geq 2$.

This Theorem is a generalization of some early results [2, 4, 5].

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