PROCEEDINGS OF THE YEREVAN STATE UNIVERSITY

Physical and Mathematical Sciences 2012, № 1, p. 27–32

Mechanics

CONTACT PROBLEM FOR AN INFINITE COMPOSITE ELASTIC (PIECEWISE-HOMOGENEOUS) PLATE WITH AN INFINITE ELASTIC STRINGER GLUED TO THE PLATE SURFACE

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Contact problem has been considered for an infinite composite elastic (piecewise homogeneous) plate consisting of two semi-infinite plates having different elastic characteristics, that are linked to each other along the common straight border. It was assumed that an infinite elastic stringer (IES) is glued over its full length and width to the upper surface of infinite composite elastic plate parallel to the line of heterogeneity of the mentioned two semi-infinite plates and have different elastic properties.The layer of glue being in the state of pure shear. The contacting triple (plate–glue–stringer) is simultaneously deformed by a concentrated force applied to IES and by uniformly distributed horizontal tension stresses of constant intensity acting at infinity on the infinite composite elastic plate. The solution of contact problem under consideration is reduced under certain condition to the solution of functional equation according to Fourier transformant of required function. The closed-form solution of the contact problem at issue is constructed in the integral form. The tangential contact and normal stresses arising in IES have been determined. Asymptotic formulae describing the behavior of stresses both near and far from the origin of force have been obtained.

***Keywords*:** composite plate, contact, stringer, glue layer, pure shear, generalized integral Fourier transform, asymptotics, singularity.

1. Let an isotropic elastic solid sheet in the form of thin composite elastic (piecewise homogeneous) infinite plate of small constant thickness h consisting of two semi-infinite plates having different elastic characteristics and interlinked along the common rectilinear boundary, be strengthened along $y = a\ (a > 0)$ line on its upper surface by sufficiently thin h_s and small width d_s infinite elastic stringer (IES) of rectangular cross-section. It is assumed that IES is parallel to the boundary line between the mentioned semi-infinite elastic plates, is continuously glued lengthwise and widthwise to the upper plate and the contact between them is realized through thin layer of glue of constant thickness h_k and width d_k $(d_k = d_s)$, and the abscissa axis coincides with the dividing line of the mentioned semi-infinite plates.

The aim of the work consists in the determination of distribution intensity of tangential contact forces acting along the binding line of the infinite stringer with the upper semi-infinite elastic plate, and normal stresses arising in IES, when the contacting triple (plate–glue–stringer) is deformed simultaneously by a concentrated force $P\delta(x)\delta(y-a)$, applied to IES and by uniformly distributed horizontal

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tensile stresses of constant intensity $\sigma_0(y)$ that act at the infinity of elastic infinite composite (piecewise homogeneous) plate.

As concerns IES in the problem under study, we accept the model of uniaxial stress state in combination with the model of contact along line [1–3], i.e. it is assumed that the intensity of distribution of the unknown tangential contact forces is concentrated along the medial line of the contact area, and accept that for the plate the model of generalized stress state is valid, due to which it is deformed as a plane. It also should be noted, that here it is assumed that during the deformation the glue layer is in the state of pure shear. Here assumed, that the function $\sigma_0(y)$ has the following form:

$$\sigma_0(y)=\frac{\sigma_0}{E}\left[E\theta(y)+E_1\theta(-y)\right] \quad \left(E,E_1,\sigma_0=\text{const};\ -\infty<y<\infty;\ |x|\to\infty\right). \quad (1.1)$$

Now obtain the resolving equation of the contact problem in question. Since in the uniaxial stress state the infinite IES is stretched or compressed in the horizontal direction, the differential equation of equilibrium of IES element as written in generalized functions will be

$$\frac{du_s(x;a)}{dx}=-\frac{1}{E_sF_s}\int_{-\infty}^{\infty}\theta(s-x)\tau(s)ds+\frac{P\theta(-x)}{E_sF_s}+\frac{\sigma_0}{E} \qquad (-\infty<x<\infty), \quad (1.2)$$

here boundary conditions and the condition of equilibrium stringer have the form

$$\left.\frac{du_s(x;a)}{dx}\right|_{|x|\to\infty}=\frac{\sigma_0}{E} \ (a); \qquad \int_{-\infty}^{\infty}\tau(s)ds=P \ (b). \quad (1.3)$$

Note that according to (1.2) the normal stresses arising in IES along $y=a$ line are:

$$\sigma_x(x;a)=-\frac{1}{F_s}\int_{-\infty}^{\infty}\theta(s-x)\tau(s)ds+\frac{P\theta(-x)}{F_s}+\frac{E_s}{E}\sigma_0 \qquad (-\infty<x<\infty). \quad (1.4)$$

In (1.1) – (1.4) $u_s(x;a)$ are the displacements of horizontal points of IES on $y=a$ line; $\tau(x)=d_s\tau(x;a)$, where $\tau(x;a)$ are tangential contact stresses on $y=a$ line; E_s is the elasticity modulus and $F_s=d_sh_s$ is the cross-sectional area of IES; P is the intensity of concentrated force applied to IES at $(0;a)$ point; E and E_1 are elasticity moduli of the upper and lower semi-infinite elastic plates; $\theta(u)$ is the Heaviside unit step function; σ_0 is the intensity of uniformly distributed horizontal tensile stresses acting at the infinity of the semi-infinite elastic plate.

Now, having in view the aforesaid when on $y=a$ line the tangential contact forces with intensity $\tau(x)$ $(-\infty<x<\infty)$ and uniformly distributed horizontal tensile stresses of the constant intensity σ_0 at the infinity of the upper semi-infinite elastic plate are in action, we have for the horizontal deformation of the plate

$$hl\frac{du(x;a)}{dx}=\frac{1}{\pi}\int_{-\infty}^{\infty}K(s-x)\tau(s)ds+\frac{hl}{E}\sigma_0 \qquad (-\infty<x<\infty). \quad (1.5)$$

Let us introduce the notation [3]:

$$K(t)=\frac{1}{t}-\frac{d_1t}{t^2+4a^2}+\frac{8d_2a^2t}{(t^2+4a^2)^2}+\frac{2d_3a^2t(t^2-12a^2)}{(t^2+4a^2)^3} \qquad (-\infty<t<\infty), \quad (1.6)$$

$$d_1=\frac{k(3-\nu)\left[k(3-\nu)(1+\nu_1)+2(1-\nu)(1-\nu_1)\right]-(3-\nu_1)\left[8-(3-\nu)(1+\nu)\right]}{(3-\nu)\left[k(3-\nu)+1+\nu\right]\left[3-\nu_1+k(1+\nu_1)\right]},$$

$$d_2=\frac{(k-1)(1+\nu)}{k(3-\nu)+1+\nu};\ d_3=\frac{2(k-1)(1+\nu)^2}{(3-\nu)\left[k(3-\nu)+1+\nu\right]};\ l=\frac{4E}{(3-\nu)(1+\nu)};\ k=\frac{\mu_1}{\mu},$$

where $u(x;a)$ – are the horizontal displacements of the points of the upper semi-infinite plate on $y=a$ line; $(E;\mu;\nu)$ and $(E_1;\mu_1;\nu_1)$ – are the elastic characteristics of the upper and the lower semi-infinite plates respectively; μ and μ_1 – are shear moduli; ν and ν_1 – are the Poisson coefficients of the material of elastic semi-infinite plates.

As during the deformation each differential element of the glue layer is in the state of pure shear, then we have [2, 4, 5]:

$$u_s(x;a)-u(x;a)=h_k\gamma_k(x;a);\ \ \tau(x)=d_s\,\tau(x;a)=d_sG_k\gamma_k(x;a)\quad(-\infty<x<\infty),\quad(1.7)$$

where $\gamma_k(x;a)$ – is the shear deformation and G_k -is the shear modulus of glue layer.

To obtain the resolving functional equation of the contact problem under consideration, we use the generalized integral Fourier transform. Applying the generalized integral Fourier transform to formulas (1.2), (1.5) and (1.7), we obtain respectively [3, 6]:

$$-i\sigma\bar{u}_s(\sigma;a)=\frac{P-\bar{\tau}(\sigma)}{E_sF_s}\left(\pi\delta(\sigma)-\frac{i}{\sigma}\right)+\frac{\sigma_0}{E}2\pi\delta(\sigma)\qquad(-\infty<\sigma<\infty),\qquad(1.8)$$

$$-i\sigma hl\bar{u}(\sigma;a)=\frac{hl}{E}\sigma_0 2\pi\delta(\sigma)+\bar{K}(\sigma;|\sigma|)\bar{\tau}(\sigma)\qquad(-\infty<\sigma<\infty),\qquad(1.9)$$

$$\bar{u}_s(\sigma;a)-\bar{u}(\sigma;a)=\frac{h_k}{G_kd_s}\bar{\tau}(\sigma)\qquad(-\infty<\sigma<\infty),\qquad(1.10)$$

Where $\bar{K}(\sigma;|\sigma|)=-i\operatorname{sgn}\sigma+\left(d_1-2d_2a|\sigma|+d_3a^2\sigma^2\right)i\operatorname{sgn}\sigma\cdot e^{-2a|\sigma|}$; $\delta(\sigma)$ is well-known Dirac delta-function; σ is the parameter of Fourier transform; $\bar{u}_s(\sigma;a)=F[u_s(x;a)]$, $\bar{u}(\sigma;a)=F[u(x;a)]$, $\bar{\tau}(\sigma)=F[\tau(x)]$ are the Fourier transforms of functions $u_s(x;a)$, $u(x;a)$, $\tau(x)$ respectively; and $F[\bullet]$ is the Fourier operator. Now, by comparing formulas (1.8), (1.9) and (1.10) after some transformations with respect to the Fourier transform of the distribution function of unknown tangential contact forces of $\tau(x)$ $(-\infty<x<\infty)$ intensity, which is the basic unknown function of the contact problem under study, we obtain the following functional equation:

$$\left[\lambda+|\sigma|+\alpha\sigma^2+B(|\sigma|)\right]\bar{\tau}(\sigma)=\lambda P\qquad(-\infty<\sigma<\infty),\qquad(1.11)$$

The following notations are introduced:

$$B(|\sigma|)=\left(-d_1|\sigma|+2d_2a\sigma^2-d_3a^2|\sigma|^3\right)e^{-2a|\sigma|},\quad\lambda=\frac{hl}{E_sF_s},\quad\alpha=\frac{2(1+\nu_k)}{E_kd_s}h_khl,\qquad(1.12)$$

where E_k and ν_k – are the modulus of elasticity and the Poisson coefficient of the material of glue layer. It is easy to see that in this case the condition (1.3(b)) of IES equilibrium is equivalent to the following condition:

$$\bar{\tau}(0)=P.\qquad(1.13)$$

Solving (1.11) with respect to $\bar{\tau}(\sigma)$ $(-\infty<\sigma<\infty)$ under the condition (1.13), we obtain

$$\bar{\tau}(\sigma)=\frac{\lambda P}{\lambda+|\sigma|+\alpha\sigma^2+B(|\sigma|)}\qquad(-\infty<\sigma<\infty).\qquad(1.14)$$

Here we must notice that the function $\overline{\tau}(\sigma)$ $(-\infty<\sigma<\infty)$ defined by the formulae (1.14) uniformly satisfies the condition (1.13) and is an even function, therefore, $\tau(x)$ $(-\infty<x<\infty)$ is also an even function.

To determine the Fourier transformant of the function of normal stresses $\sigma_x(x;a)$ $(-\infty<x<\infty)$ arising in IES, we obtain by applying the generalized integral Fourier transform to (1.4) with due regard for (1.14):

$$\overline{\sigma}_x(\sigma;a)=\frac{P}{iF_s}\cdot\frac{\operatorname{sgn}\sigma+\alpha\sigma+B_1(\sigma;|\sigma|)}{\lambda+|\sigma|+\alpha\sigma^2+B(|\sigma|)}+\frac{E_s}{E}2\pi\sigma_0\delta(\sigma)\qquad(-\infty<\sigma<\infty),\tag{1.15}$$

where $B_1(\sigma;|\sigma|)=\left(-d_1\operatorname{sgn}\sigma+2d_2a\sigma-d_3a^2\sigma|\sigma|\right)e^{-2a|\sigma|}$; $\overline{\sigma}_x(\sigma;a)=F\left[\sigma_x(x;a)\right]$.

If we apply the generalized integral inverse Fourier transform to (1.14) and (1.15), then the unknown functions take on the form

$$\tau(x)=\frac{\lambda P}{\pi}\int_0^\infty\frac{\cos(\sigma x)\,d\sigma}{\lambda+\sigma+\alpha\sigma^2+B(\sigma)}\qquad(-\infty<x<\infty),\tag{1.16}$$

$$\sigma_x(x;a)=-\frac{P}{\pi F_s}\int_0^\infty\frac{[1+\alpha\sigma+B_1(\sigma)]\sin(\sigma x)}{\lambda+\sigma+\alpha\sigma^2+B(\sigma)}d\sigma+\frac{E_s}{E}\sigma_0\qquad(-\infty<x<\infty).\tag{1.17}$$

Thus, the considered contact problem under consideration is closely solved in the integral form, and the unknown functions are given by (1.16) and (1.17).

Now consider some particular cases:

- An elastic composite (piecewise-homogeneous) infinite plate regardless of the glue layer. In this case, from (1.14) and (1.15) we have [3] for the Fourier transformants of unknown functions on the basis of (1.12):

$$\overline{\tau}(\sigma)=\frac{\lambda P}{\lambda+|\sigma|+B(|\sigma|)};\quad\overline{\sigma}_x(\sigma;a)=\frac{P}{iF_s}\cdot\frac{\operatorname{sgn}\sigma+B_1(\sigma;|\sigma|)}{\lambda+|\sigma|+B(|\sigma|)}+\frac{E_s}{E}2\pi\sigma_0\delta(\sigma).\tag{1.18}$$

- Elastic homogeneous infinite plate $(E=E_1,\ \nu=\nu_1)$. In this case, on proceeding from (1.6) the formulas (1.14) and (1.15) accordingly take on the following form:

$$\overline{\tau}(\sigma)=\frac{\lambda P}{\lambda+|\sigma|+\alpha\sigma^2};\quad\overline{\sigma}_x(\sigma;a)=\frac{P}{iF_s}\cdot\frac{\operatorname{sgn}\sigma+\alpha\sigma}{\lambda+|\sigma|+\alpha\sigma^2}+\frac{E_s}{E}2\pi\sigma_0\delta(\sigma).\tag{1.19}$$

- Elastic homogeneous infinite plate, absence of glue layer $(E=E_1,\ \nu=\nu_1$ and $h_k=0)$. In this case, we obtain from (1.19) on the bases of (1.12) [3, 7]:

$$\overline{\tau}(\sigma)=\frac{\lambda P}{\lambda+|\sigma|};\quad\overline{\sigma}_x(\sigma;a)=\frac{P}{iF_s}\cdot\frac{\operatorname{sgn}\sigma}{\lambda+|\sigma|}+\frac{E_s}{E}2\pi\sigma_0\delta(\sigma)\qquad(-\infty<\sigma<\infty).\tag{1.20}$$

- In the extreme case of $a\to0$, we obtain from (1.14) and (1.15):

$$\overline{\tau}(\sigma)=\frac{\lambda P}{\lambda+(1-d_1)|\sigma|+\alpha\sigma^2};\overline{\sigma}_x(\sigma;a)=\frac{P}{iF_s}\cdot\frac{(1-d_1)\operatorname{sgn}\sigma+\alpha\sigma}{\lambda+(1-d_1)|\sigma|+\alpha\sigma^2}+\frac{E_s}{E}2\pi\sigma_0\delta(\sigma),\tag{1.21}$$

- In the case of $P=0$ we accordingly obtain from (1.14) and (1.15):

$$\overline{\tau}(\sigma)=0;\ \overline{\sigma}_x(\sigma;a)=\frac{E_s}{E}2\pi\sigma_0\delta(\sigma)\qquad(-\infty<\sigma<\infty).\tag{1.22}$$

From (1.22) it follows that $\tau(x)=0;\ \sigma_x(x;a)=\frac{E_s}{E}\sigma_0\ (-\infty<x<\infty)$, i.e. there is no load transfer from elastic composite (piecewise-homogeneous) infinite plate to the IES, despite of the base is deformed by tensions $\sigma_0(y)$.

Thus, the intensity of the distribution of unknown tangential contact forces and normal tensions arising in IES have been determined, when the contact triple was deformed by a concentrated force applied on IES and at the same time by uniformly distributed stretching horizontal tensions of constant intensity, acting at the infinity of elastic composite plate.

2. Let us investigate the behavior of the intensity function $\tau(x)$ $(-\infty < x < \infty)$ of the distribution of unknown tangential contact forces and normal stresses $\sigma_x(x;a)$ $(-\infty < x < \infty)$ arising in the infinite stringer, which characterizes their behavior near and far from the application point of concentrated force. First obtain the asymptotic formulae for $\tau(x)$ when $|x| \to \infty$. Notice that from (1.14) we can obtain the following asymptotic representation for $\overline{\tau}(\sigma)$ when $|\sigma| \to 0$:

$$\overline{\tau}(\sigma) = \left(1 - a_1|\sigma| + a_2\sigma^2 - a_3|\sigma|^3 + a_4\sigma^4 - a_5|\sigma|^5\right)P + O\left(\sigma^6\right) \quad \left(|\sigma| \to 0\right), \tag{2.1}$$

where the following notations are introduced:

$$a_1 = \frac{1-d_1}{\lambda};\ a_2 = \frac{1}{\lambda^2}\Big[\left(1-d_1\right)^2 - \lambda\left(\alpha + 2ad_2 + 2ad_1\right)\Big],$$

$$a_3 = \frac{1}{\lambda^3}\Big[\left(1-d_1\right)^3 - 2\lambda\left(1-d_1\right)\left(\alpha + 2ad_2 + 2ad_1\right) - \lambda^2 a^2\left(d_3 + 4d_2 + 2d_1\right)\Big],$$

$$\begin{aligned} a_4 = \frac{1}{\lambda^4}\Big[&\left(1-d_1\right)^4 - 3\lambda\left(1-d_1\right)^2\left(\alpha + 2ad_2 + 2ad_1\right) - 2\lambda^2 a^2\left(1-d_1\right)\left(d_3 + 4d_2 + 2d_1\right) + \\ &+\lambda^2\left(\alpha + 2ad_2 + 2ad_1\right)^2 - \frac{2}{3}\lambda^3 a^3\left(3d_3 + 6d_2 + 2d_1\right)\Big], \end{aligned} \tag{2.2}$$

$$\begin{aligned} a_5 = \frac{1}{\lambda^5}\Big[&\left(1-d_1\right)^5 - 4\lambda\left(1-d_1\right)^3\left(\alpha + 2ad_2 + 2ad_1\right) - 3\lambda^2 a^2\left(1-d_1\right)^2\left(d_3 + 4d_2 + 2d_1\right) + \\ &+3\lambda^2\left(1-d_1\right)\left(\alpha + 2ad_2 + 2ad_1\right)^2 - \frac{4}{3}\lambda^3 a^3\left(1-d_1\right)\left(3d_3 + 6d_2 + 2d_1\right) + \\ &+2\lambda^3 a^2\left(\alpha + 2ad_2 + 2ad_1\right)\left(d_3 + 4d_2 + 2d_1\right) - \frac{2}{3}\lambda^4 a^4\left(3d_3 + 4d_2 + d_1\right)\Big]. \end{aligned}$$

If now we apply the generalized integral inverse Fourier transform to (2.1), we obtain the following asymptotic representation for $\tau(x)$ function when $|x| \to \infty$:

$$\tau(x) = \frac{P}{\pi}\left(\frac{a_1}{x^2} - \frac{6a_3}{x^4} + \frac{120a_5}{x^6}\right) + O\left(\frac{1}{x^8}\right) \qquad \left(|x| \to \infty\right). \tag{2.3}$$

Then based on (2.3) for normal stresses $\sigma_x(x;a)$ when $|x| \to \infty$, we obtain from (1.4) the following asymptotic formula:

$$\sigma_x(x;a) = -\frac{P}{\pi F_s}\left(\frac{a_1}{x} - \frac{2a_3}{x^3} + \frac{24a_5}{x^5}\right) + \frac{E_s}{E}\sigma_0 + O\left(\frac{1}{x^7}\right) \qquad \left(|x| \to \infty\right). \tag{2.4}$$

Now, to obtain the asymptotic formula for $\tau(x)$ when $|x| \to 0$, note that one can represent $\overline{\tau}(\sigma)$ from (1.14) in the following form:

$$\overline{\tau}(\sigma) = \lambda P[\overline{K}_1(|\sigma|) - \overline{K}_2(|\sigma|)] \qquad \left(-\infty < \sigma < \infty\right), \tag{2.5}$$

where the functions $\overline{K}_1\left(|\sigma|\right)$ and $\overline{K}_2\left(|\sigma|\right)$ are respectively

$$\bar{K}_1(|\sigma|)=\frac{1}{\lambda+|\sigma|+\alpha\sigma^2}=\frac{1}{\alpha(|\sigma|+b_1)(|\sigma|+b_2)}=\frac{1}{\alpha(b_2-b_1)}\left(\frac{1}{|\sigma|+b_1}-\frac{1}{|\sigma|+b_2}\right), \tag{2.6}$$

$$\bar{K}_2(|\sigma|)=\frac{B(|\sigma|)}{(\lambda+|\sigma|+\alpha\sigma^2)\left[\lambda+|\sigma|+\alpha\sigma^2+B(|\sigma|)\right]};\ b_1=\frac{2\lambda}{\sqrt{1-4\alpha\lambda}+1};\ b_2=\frac{2\lambda}{1-\sqrt{1-4\alpha\lambda}}. \tag{2.7}$$

As in case of $|\sigma|\to\infty$ the following asymptotic representation holds:

$$\frac{k}{k+|\sigma|}=\sum_{n=0}^{\infty}(-1)^n\left(\frac{k}{|\sigma|}\right)^{n+1}, \tag{2.8}$$

then on the bases (2.8) after application of the generalized integral inverse Fourier transform to (2.5) – (2.7), we obtain the following asymptotic formula for $\tau(x)$ when $|x|\to 0$:

$$\tau(x)=\frac{\lambda P}{\alpha(b_2-b_1)}\sum_{n=0}^{\infty}(-1)^{n+1}\left[\frac{|b_2x|^{2n+1}}{2(2n+1)!}-\frac{(b_2x)^{2n+2}}{\pi(2n+2)!}\left(\psi(2n+3)+\ln\frac{1}{|b_2x|}\right)-\right. \tag{2.9}$$
$$\left.-\frac{|b_1x|^{2n+1}}{2(2n+1)!}+\frac{(b_1x)^{2n+2}}{\pi(2n+2)!}\left(\psi(2n+3)+\ln\frac{1}{|b_1x|}\right)+\alpha(b_2-b_1)R_n\frac{x^{2n}}{(2n)!}\right]\quad(|x|\to 0),$$

where $R_n=\frac{1}{\pi}\int_0^{\infty}\frac{B(\sigma)\sigma^{2n}d\sigma}{(\lambda+\sigma+\alpha\sigma^2)(\lambda+\sigma+\alpha\sigma^2+B(\sigma))}$ and $\psi(x)$ is the psi-function.

It is easy to see from representation (2.9), that $\tau(x)$ when $|x|\to 0$ has a finite value, i.e. at the concentrated force application point the tangential contact tensions have finite values due to the presence of a glue layer. Now, on the basis of (2.9), we obtain from (1.4) the following asymptotic formula for $\sigma_x(x;a)$ when $|x|\to 0$:

$$\sigma_x(x;a)=\frac{\lambda P}{\alpha(b_2-b_1)F_s}\sum_{n=0}^{\infty}(-1)^{n+1}\left[\alpha(b_2-b_1)R_n\frac{x^{2n+1}}{(2n+1)!}+\right.$$
$$+\frac{1}{b_2}\cdot\frac{(b_2x)^{2n+1}|b_2x|}{2(2n+2)!}-\frac{1}{b_2}\cdot\frac{(b_2x)^{2n+3}}{\pi(2n+3)!}\left(\psi(2n+4)+\ln\frac{1}{|b_2x|}\right)- \tag{2.10}$$
$$\left.-\frac{1}{b_1}\cdot\frac{(b_1x)^{2n+1}|b_1x|}{2(2n+2)!}+\frac{1}{b_1}\cdot\frac{(b_1x)^{2n+3}}{\pi(2n+3)!}\left(\psi(2n+4)+\ln\frac{1}{|b_1x|}\right)\right]-\frac{P}{F_s}\theta(x)\quad(|x|\to 0).$$

Notice, that the series (2.9) and (2.10) are convergent for any values of x.

Received 28.07.2011

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