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Mechanics

TASK TO POSSIBLE REDUCTION OF THE MAXIMUM DEFLECTION OF CIRCULAR SECTION ROD UNDER THE INFLUENCE OF CENTRALIZED FORCE

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A round rod of variable cross-section is considered, to which a centralized force is applied. An optimization problem is stated for obtaining such a regularity of transverse section variability, for which the value of maximum deflection of the rod would be the least. The case of pin-jointing of the rod was considered and the problem was solved by means of variational method. An analytical solution to this problem was obtained, according to which the corresponding plots of unknown functions were constructed.

***Keywords*:** round rod, optimization problem, deflection, variable section.

Let $2l$ long circular section rod be pin-jointed in points $x=\pm l$ and a centralized force $q=p\delta(x)$ be applied in $x=0$ point, where $\delta(x)$ is the Dirac function.

Consider the following optimization problem for a circular section rod of variable radius: to obtain such a function $h(x)=r^2(x)$ describing the variability of transverse section of round rod under the influence of applied force that would provide the least possible value of maximum deflection, when the rod volume has a preassigned value ($r(x)$ is the radius of transverse section).

The surface area of transverse section $S(x)$ and the hardness of the material of round rod $D=EI(x)$ are determined by means of $h(x)$ function [1, 2].

$$S(x)=B_2 h\ (x),\qquad EI(x)=A_2 h^2(x), \tag{1}$$

where $B_2=\pi$, $A_2=\dfrac{E\pi}{4}$, $I(x)$ is the moment of inertia of the section, E is the Young modulus of the material of rod.

The main relationships of the formulated problem are as follows:

$$\begin{gathered}(DW_{xx})_{xx}=q,\qquad W|_{x=\pm l}=DW_{xx}|_{x=\pm l}=0,\\ \int_{-l}^{l} S(x)dx=V_0,\ J=\int_{-l}^{l} W(x)\delta(x)dx=W(0)\xrightarrow[h(x)]{}\min,\end{gathered} \tag{2}$$

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where $W(x)$ is the function of rod deflection and J is the purpose of the functional.

In the dual problem of the formulated task it is required to reduce the mass of round rod M as much as possible, so that the maximum deflection of the circular section rod adopted the prespecified value.

Now let us obtain the required optimization conditions by making use of the variational method [1, 3, 4].

If we write down the variational equation for rod bending as

$$(h^2 \delta W_{xx})_{xx} + 2(h\delta W_{xx}\delta h)_{xx} = 0\,, \tag{3}$$

multiply that by some conjugate function $V(x)$ and integrate by parts, then demanding a vanishing of growth factor δW in the integrand and of $V(x)$ function together with the boundary values of those summands that include the derivatives of $V(x)$ in the integrated term, which do not become zero due to the presence of $W(x)$ function and of boundary values of its derivatives, we obtain from $\delta J = 0$ condition [1]

$$\begin{cases} (h^2 V_{xx})_{xx} + \lambda q = 0, \\ 2hW_{xx}V_{xx} + 1 = 0, \end{cases} \tag{4}$$

where λ is Lagrange multiplier.

It is easy to see after elimination of $V(x)$ function from this system that $V = -\lambda W$. After substitution of this relation into the second equation of system (4) we have

$$hW_{xx}^2 = C^2, \tag{5}$$

where C is an unknown constant that may be determined based on the equality of the round rod, and the energy of elastic strain [1]

$$P = \frac{A_2}{B_2 W(0)} \int_{-l}^{l} h^2 W_{xx}^2 dx = \frac{A_2 C^2 V_0}{B_2 W(0)}. \tag{6}$$

$h(x)$ parameter is determined from the optimality equation

$$h(x) = C^2 / W_{xx}^2. \tag{7}$$

Hence, after removal of $h(x)$ function from the boundary value problem of bending equation for round rod we shall have the following boundary value problem for the deflection function $W(x)$:

$$\begin{cases} (W_{xx}^{-3}) = 0 \qquad (0 < x < l,\ -l < x < 0), \\ W(-l) = W(l) = hW_{xx}(-l) = hW_{xx}(l) = 0. \end{cases} \tag{8}$$

On integrating this boundary value problem under symmetry condition $W_x(0) = 0$ we have for $W(x)$ function

$$\begin{cases} W(x) = \dfrac{36P\pi^2 l^5}{250 A_2 V_0^2}\left(1 - \dfrac{x}{l}\right)\left(5 - 3\sqrt[3]{\left(1 - \dfrac{x}{l}\right)^2}\right), & 0 \le x \le l, \\ W(x) = \dfrac{36P\pi^2 l^5}{250 A_2 V_0^2}\left(1 + \dfrac{x}{l}\right)\left(5 - 3\sqrt[3]{\left(1 + \dfrac{x}{l}\right)^2}\right), & -l \le x \le 0. \end{cases} \tag{9}$$

Then the optimal thickness $h(x)$ follows from conditions (5) and (7):

$$\begin{cases} h(x) = \dfrac{5V_0}{6\pi l}\left(1 - \dfrac{x}{l}\right)^{2/3}, & 0 \le x \le l, \\ h(x) = \dfrac{5V_0}{6\pi l}\left(1 + \dfrac{x}{l}\right)^{2/3}, & -l \le x \le 0. \end{cases} \quad (10)$$

The Figure for deflection function $W(x)$ and $h(x)$ function will be plotted according to (9) and (10) formulae (*M* is bending moment).

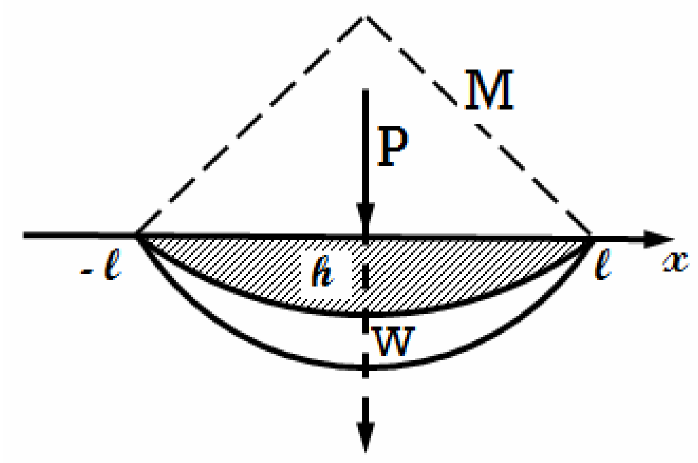

Figure.

In these calculations the constant C is determined from condition (6) taking into account the isopermic limitation.

The greatest deflection for such a round rod will be

$$J_* = W(0) = \frac{9(12\pi)^{1/2} l^5 P}{2 \cdot 5^{1/2} V_0^{3/2} E}.$$

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