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CONTACT PROBLEM FOR A PIECEWISE-HOMOGENEOUS INFINITE PLATE WITH STACKED ELASTIC PIECEWISE-HOMOGENEOUS INFINITE STRINGER

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The contact problem has been considered for elastic composite (piecewise-homogeneous) infinite plate consisting of two semi-infinite plates interlinked along the common straight border. Parallel to this line of heterogeneity of these semi-infinite plates with different elastic properties, an elastic piecewise-homogeneous infinite stringer is continuously glued over its full length and width on the upper semi-infinite plate, the layer of glue during the deformation being in the state of pure shear. The contacting triple (plate–glue–stringer) is simultaneously deformed by codirectional concentrated forces applied to the stringer and uniformly distributed horizontal tensile stresses of piecewise–constant intensity acting at infinity on the plate. According to the generalized Fourier integral transform, under certain conditions the solution of contact problem under consideration reduces to a solution of functional equation in the Fourier transforms of an unknown function on the real axis. A closed form solution of the contact problem in question is given in an integral form. As a result of investigations it was shown that due to the presence of the layer of glue the tangential contact forces have no singularities in the points of application of forces and in sections of semi-infinite stringer attachment.

Keywords: composite plate, contact, stringer, layer of glue, pure shear, Fourier transform, functional equation, asymptotic representation of function.

1. Let an elastic solid isotropic plate in the form of thin composite (piecewise-homogeneous) infinite plate of small constant thickness h, consisting of two semi-infinite plates showing different elastic properties, that are linked along the common straight border, be strengthened along $y = a(a > 0)$ line on the upper semi-infinite plate by an elastic piecewise-homogeneous infinite stringer of sufficiently small constant rectangular cross-section. Being continuously glued over its full length and width to the upper semi-infinite plate, the elastic piecewise-homogeneous infinite stringer is directed along and parallel to the dividing heterogeneity line of the mentioned semi-infinite plates as axis of abscissa, has different elastic characteristics and the contact with them is realized via sufficiently thin layer of glue having constant thickness h_k and width d_k.

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The problem consists in determination of the intensity distribution of tangential contact forces acting along the line of contact between the stringer and the upper semi-infinite plate when the contacted triple (plate–glue–stringer) is simultaneously deformed by codirectional and concentrated forces $P\delta(x-b)\delta(y-a)(b>0)$ and $Q\delta(x+c)\delta(y-a)(c>0)$ applied to the stringer, and uniformly distributed horizontal tensile stresses of piecewise constant intensity $\sigma_0(y)$ acting at infinity of elastic composite (piecewise-homogeneous) infinite plate.

The contact problem under investigation is solved based on the following three basic assumptions [1–8]:

• for elastic piecewise-homogeneous infinite stringer a model of uniaxial stress state in combination with the model of contact along the line was adopted, i.e. it was assumed that the tangential contact forces are concentrated along the medial line of contact area;

• during the deformation the elastic composite infinite plate is assumed to be in a generalized plane state, owing to which it is deformed as a plane;

• during the deformation the layer of glue is assumed to be in the state of pure shear.

Under the terms of contact problem under consideration the modulus of elasticity, the cross-sectional area of stringer and the piecewise-constant function $\sigma_0(y)$ of distribution intensity are determined by formulae

$$\left\{E_S(x);F_S(x)\right\}=\left\{E_S^{(1)};F_S^{(1)}\right\}\theta(x)+\left\{E_S^{(2)};F_S^{(2)}\right\}\theta(-x) \quad (-\infty<x<\infty), \quad (1.1)$$

$$\sigma_0(y)=\frac{\sigma_0}{2E}[E+E_1+(E-E_1)\operatorname{sgn} y] \quad (-\infty<y<\infty;\ |x|\to\infty), \quad (1.2)$$

where $\theta(t)$ is the Heaviside unit step function, $\operatorname{sgn} t$ is the well-known sign function and $E_S^{(n)},\ F_S^{(n)},\ E,\ E_1,\ \sigma_0=\text{const}\ (n=1,2)$.

Turning now to the derivation of resolving functional equation for the contact problem in question it is easy to see that the stringer is stretched or shrunken in the horizontal direction while being in a uniaxial stress state. Then, based on the aforesaid the equilibrium differential equations of separate parts of the piecewise-homogeneous stringer making allowance for the Hook's law will assume the following forms:

$$\frac{d^2u_S^{(1)}(x;a)}{dx^2}=\frac{\tau^{(1)}(x)}{E_S^{(1)}F_S^{(1)}}-\frac{P\delta(x-b)}{E_S^{(1)}F_S^{(1)}} \quad (0<x<\infty), \quad (1.3)$$

$$\frac{d^2u_S^{(2)}(x;a)}{dx^2}=\frac{\tau^{(2)}(x)}{E_S^{(2)}F_S^{(2)}}-\frac{Q\delta(x+c)}{E_S^{(2)}F_S^{(2)}} \quad (-\infty<x<0), \quad (1.4)$$

that respectively satisfy the following boundary conditions and conditions at infinity:

$$\left.\frac{du_S^{(1)}(x;a)}{dx}\right|_{x=+0}-\left.\frac{du_S^{(2)}(x;a)}{dx}\right|_{x=-0}=\frac{X_0}{E_S^{(1)}F_S^{(1)}}-\frac{X_0}{E_S^{(2)}F_S^{(2)}},$$
$$\left.\frac{du_S^{(1)}(x;a)}{dx}\right|_{x\to+\infty}=\frac{\sigma_0}{E};\quad \left.\frac{du_S^{(2)}(x;a)}{dx}\right|_{x\to-\infty}=\frac{\sigma_0}{E}, \quad (1.5)$$

as well as the conditions of the equilibrium of elastic semi-infinite stringers:

$$\int_0^\infty \tau^{(1)}(s)ds = P + P_0 - X_0 \quad (a); \qquad \int_{-\infty}^0 \tau^{(2)}(s)ds = Q - Q_0 + X_0 \quad (b), \quad (1.6)$$

where $P_0 = \dfrac{E_S^{(1)}F_S^{(1)}}{E}\sigma_0$ and $Q_0 = \dfrac{E_S^{(2)}F_S^{(2)}}{E}\sigma_0$ are the forces that arise in the elastic infinite stringer when $x \to +\infty$ and $x \to -\infty$ respectively.

In (1.1)–(1.6) $u_S^{(1)}(x;a)$ and $u_S^{(2)}(x;a)$ are horizontal displacements of stringer points on $y = a$ line when $0 < x < \infty$ and $-\infty < x < 0$, respectively; $\tau^{(1)}(x) = d_S^{(1)}\tau^{(1)}(x;a)$, where $\tau^{(1)}(x;a)$ are tangential contact stresses on $y = a$ line when $0 < x < \infty$, $d_S^{(1)}$ is the width of that stringer; $\tau^{(2)}(x) = d_S^{(2)}\tau^{(2)}(x;a)$, where $\tau^{(2)}(x;a)$ are tangential contact stresses on $y = a$ line when $-\infty < x < 0$, $d_S^{(2)}$ is the width of that stringer; $E_S^{(1)}$ and $E_S^{(2)}$ are the elastic moduli, $F_S^{(1)} = d_S^{(1)}h_S^{(1)}$ and $F_S^{(2)} = d_S^{(2)}h_S^{(2)}$ are the cross sectional areas of the semi-infinite stringers when $0 < x < \infty$ and $-\infty < x < 0$, $h_S^{(1)}$ and $h_S^{(2)}$ are their heights; $\delta(x)$ is the Dirac delta function; P and Q are the concentrated forces; X_0 is an unknown longitudinal force acting in the $x = 0$ section of the stringer; σ_0 is the intensity of uniformly distributed horizontal tensile stresses acting at infinity of the upper semi-infinite plate [6, 8].

To express the differential equations (1.3) and (1.4) with due regard for conditions (1.5) as a single equation for all $x(-\infty < x < \infty)$ the following function is introduced [2, 4, 11]:

$$U_S(x;a) \overset{def}{=} \theta(x)\frac{du_S^{(1)}(x;a)}{dx} + \theta(-x)\frac{du_S^{(2)}(x;a)}{dx} \quad (-\infty < x < \infty). \quad (1.7)$$

After differentiation of (1.7) in the sense of the theory of generalized functions, one arrives at expression

$$\frac{dU_S(x;a)}{dx} = \frac{\tau_+^{(1)}(x)}{E_S^{(1)}F_S^{(1)}} + \frac{\tau_-^{(2)}(x)}{E_S^{(2)}F_S^{(2)}} - \frac{P\delta(x-b)}{E_S^{(1)}F_S^{(1)}} - \frac{Q\delta(x+c)}{E_S^{(2)}F_S^{(2)}} + \frac{X_0\delta(x)}{E_S^{(1)}F_S^{(1)}} - \frac{X_0\delta(x)}{E_S^{(2)}F_S^{(2)}}. \quad (1.8)$$

The solution of differential equation (1.8) is represented in the following form:

$$\begin{aligned} U_S(x;a) = &-\frac{1}{E_S^{(1)}F_S^{(1)}}\int_{-\infty}^{\infty}\theta(s-x)\tau_+^{(1)}(s)ds - \frac{1}{E_S^{(2)}F_S^{(2)}}\int_{-\infty}^{\infty}\theta(s-x)\tau_-^{(2)}(s)ds + \\ &+ \frac{P\theta(b-x)}{E_S^{(1)}F_S^{(1)}} + \frac{Q\theta(-c-x)}{E_S^{(2)}F_S^{(2)}} - \frac{X_0\theta(-x)}{E_S^{(1)}F_S^{(1)}} + \frac{X_0\theta(-x)}{E_S^{(2)}F_S^{(2)}} + \frac{\sigma_0}{E} \quad (-\infty < x < \infty). \end{aligned} \quad (1.9)$$

Note, that as in formulas (1.8) and (1.9)

$$\tau_+^{(1)}(x) = \theta(x)\tau^{(1)}(x); \quad \tau_-^{(2)}(x) = \theta(-x)\tau^{(2)}(x); \quad \tau_+^{(1)}(x) + \tau_-^{(2)}(x) = \tau(x), \quad (1.10)$$

stringer equilibrium the conditions of (1.6) will be equivalent to condition

$$\int_{-\infty}^{\infty}\tau(s)ds = P + Q + P_0 - Q_0. \quad (1.11)$$

On the other hand, when tangential contact stresses of $\tau(x)$ intensity ($-\infty < x < \infty$) act on $y = a$ line and simultaneously uniformly distributed horizontal tensile stresses of constant σ_0 intensity act at infinity, we have for the horizontal deformation of the upper semi-infinite plate [6, 8]:

$$hlU(x;a) \overset{def}{=} hl\frac{du(x;a)}{dx} = \frac{1}{\pi}\int_{-\infty}^{\infty}\left[\frac{1}{s-x} + K(s-x)\right]\tau(s)ds + \frac{hl}{E}\sigma_0 \quad (-\infty < x < \infty). \quad (1.12)$$

Here the following notations are introduced:

$$U(x;a) = \theta(x)\frac{du^{(1)}(x;a)}{dx} + \theta(-x)\frac{du^{(2)}(x;a)}{dx}, \qquad l = \frac{8\mu}{3-\nu} = \frac{4E}{(1+\nu)(3-\nu)},$$

$$K(x) = -\frac{d_1 x}{x^2+4a^2} + \frac{8d_2a^2x}{(x^2+4a^2)^2} + \frac{2d_3a^2x(x^2-12a^2)}{(x^2+4a^2)^3}, \quad (1.13)$$

$$d_1 = \frac{k(3-\nu)[k(3-\nu)(1+\nu_1)+2(1-\nu)(1-\nu_1)] - (3-\nu_1)[8-(1+\nu)(3-\nu)]}{(3-\nu)[k(3-\nu)+1+\nu]\,[3-\nu_1+k(1+\nu_1)]},$$

$$d_2 = \frac{(k-1)(1+\nu)}{k(3-\nu)+1+\nu}, \quad d_3 = \frac{2(k-1)(1+\nu)^2}{(3-\nu)\,[k(3-\nu)+1+\nu]}, \quad k = \frac{\mu_1}{\mu} = \frac{E_1(1+\nu)}{E(1+\nu_1)},$$

where $u^{(1)}(x;a)$ and $u^{(2)}(x;a)$ are horizontal deflections of the upper semi-infinite plate on the $y=a$ line when $0<x<\infty$ and $-\infty<x<0$ respectively; $(E;\mu;\nu)$ and $(E_1;\mu_1;\nu_1)$ are the elastic characteristics of the upper and lower semi-infinite plates; E and E_1 are the elastic moduli, μ and μ_1 are the shear moduli, ν and ν_1 are the Poisson ratios of the semi-infinite plate materials.

Now, keeping in mind that each differential element of the glue layer during deformation is in the pure shear state, we obtain [3, 5–8]:

$$U_S(x;a) - U(x;a) = h_k\frac{d\gamma_k(x;a)}{dx};\ \tau(x) = d_k\,\tau(x;a) = d_kG_k\gamma_k(x;a) \quad (-\infty < x < \infty), \quad (1.14)$$

where $\gamma_k(x;a)$ is the shear deformation and G_k is the shear modulus of the glue layer, $d_k = \min\{d_k^{(1)}; d_k^{(2)}\}$, $d_k^{(1)} \equiv d_S^{(1)}$ and $d_k^{(2)} \equiv d_S^{(2)}$ are the widths of glue layers.

To obtain the resolving functional equation of the contact problem under consideration, we use the generalized Fourier integral transform. For this purpose, applying the generalized Fourier integral transform to formulas (1.9), (1.12) and (1.14), we respectively obtain [8, 9]:

$$\bar{U}_S(\sigma;a) = \left[-\frac{\bar{\tau}_+^{(1)}(\sigma)}{E_S^{(1)}F_S^{(1)}} - \frac{\bar{\tau}_-^{(2)}(\sigma)}{E_S^{(2)}F_S^{(2)}} + \frac{Pe^{i\sigma b}}{E_S^{(1)}F_S^{(1)}} + \frac{Qe^{-i\sigma c}}{E_S^{(2)}F_S^{(2)}} - \right.$$
$$\left. -\frac{X_0}{E_S^{(1)}F_S^{(1)}} + \frac{X_0}{E_S^{(2)}F_S^{(2)}}\right]\left[\pi\delta(\sigma) - \frac{i}{\sigma}\right] + \frac{\sigma_0}{E}2\pi\delta(\sigma) \quad (-\infty < \sigma < \infty), \quad (1.15)$$

$$hl\bar{U}(\sigma;a) = \frac{hl}{E}\sigma_0 2\pi\delta(\sigma) - i\,\mathrm{sgn}\,\sigma\left[1 + (-d_1 + 2d_2a|\sigma| - d_3a^2\sigma^2)e^{-2a|\sigma|}\right]\bar{\tau}(\sigma), \quad (1.16)$$

$$\bar{U}_S(\sigma;a) - \bar{U}(\sigma;a) = \frac{h_k}{G_kd_k}(-i\sigma)\bar{\tau}(\sigma) \qquad (-\infty < \sigma < \infty), \quad (1.17)$$

where σ is the spectral parameter of Fourier transform; $\bar{A}(\sigma) = F[A(x)]$ is the Fourier transformant of $A(x)$ function and $F[\bullet]$ is the Fourier operator. Note that for obtaining of (1.15)–(1.17) the following formulas were used [8, 9]:

$$F[\operatorname{sgn} t]=\frac{2i}{\sigma};\quad F[\theta(\pm t)]=\pi\delta(\sigma)\pm\frac{i}{\sigma};\quad F\left[\frac{t}{t^2+y^2}\right]=\frac{i\pi\operatorname{sgn}\sigma}{e^{|\sigma y|}},$$

$$F\left[\frac{t}{(t^2+y^2)^2}\right]=\frac{i\pi\sigma}{2|y|e^{|\sigma y|}},\quad F\left[\frac{t}{(t^2+y^2)^3}\right]=\frac{i\pi\sigma}{8|y|^3}\cdot\frac{1+|\sigma y|}{e^{|\sigma y|}}\quad(-\infty<\sigma,t,y<\infty).$$

After simple transformations we obtain based on comparison of formulas (1.15), (1.16) and (1.17) the following functional equation with respect to Fourier transforms of distribution functions of unknown tangential contact forces of $\tau_+^{(1)}(x)$ and $\tau_-^{(2)}(x)$ intensities $(-\infty<x<\infty)$, which are the basic unknown functions of the contact problem under consideration:

$$[\lambda_1+|\sigma|+\alpha\sigma^2+\bar{B}(|\sigma|)]\bar{\tau}_+^{(1)}(\sigma)+[\lambda_2+|\sigma|+\alpha\sigma^2+\bar{B}(|\sigma|)]\bar{\tau}_-^{(2)}(\sigma)=\bar{f}(\sigma)\quad(-\infty<\sigma<\infty),\tag{1.18}$$

the following notations being introduced here:

$$\bar{B}(|\sigma|)=(-d_1|\sigma|+2d_2a\sigma^2-d_3a^2|\sigma|^3)e^{-2a|\sigma|},\ \lambda_1=\frac{hl}{E_S^{(1)}F_S^{(1)}},\ \lambda_2=\frac{hl}{E_S^{(2)}F_S^{(2)}},$$
$$\bar{f}(\sigma)=\lambda_1P\cdot e^{i\sigma b}+\lambda_2Q\cdot e^{-i\sigma c}+(\lambda_2-\lambda_1)X_0,\quad \alpha=\frac{2(1+\nu_k)}{E_kd_k}h_khl,\tag{1.19}$$

where E_k and ν_k are the Young modulus and Poisson ratio of the material of glue layer. Functional equation (1.18) may be transformed to the form

$$\bar{G}(\sigma)\bar{\tau}_+^{(1)}(\sigma)+\bar{\tau}_-^{(2)}(\sigma)=\bar{g}(\sigma)\qquad(-\infty<\sigma<\infty),\tag{1.20}$$

the kernel $\bar{G}(\sigma)$ and the free term $\bar{g}(\sigma)$ in which are defined as

$$\bar{G}(\sigma)=\frac{\lambda_1+|\sigma|+\alpha\sigma^2+\bar{B}(|\sigma|)}{\lambda_2+|\sigma|+\alpha\sigma^2+\bar{B}(|\sigma|)},\quad \bar{g}(\sigma)=\frac{\bar{f}(\sigma)}{\lambda_2+|\sigma|+\alpha\sigma^2+\bar{B}(|\sigma|)}.\tag{1.21}$$

It is easy to see that condition (1.11) is equivalent to condition

$$\bar{\tau}(0)=P+Q+P_0-Q_0.\tag{1.22}$$

Thus, under the assumptions made the solution of stated contact problem was reduced to solution on the whole real axis of functional equation (1.20) with respect to $\bar{\tau}_+^{(1)}(\sigma)$ and $\bar{\tau}_-^{(2)}(\sigma)$ subject to condition (1.22). It should be noted that since functions $\bar{\tau}_+^{(1)}(\sigma)$ and $\bar{\tau}_-^{(2)}(\sigma)$ are the boundary values of analytical functions $\bar{\tau}_+^{(1)}(\alpha)$ and $\bar{\tau}_-^{(2)}(\alpha)$ $(\alpha=\sigma+i\tau)$ that are regular in the upper and lower semi-planes respectively, the functional equation can be solved either as a Riemann boundary value problem in analytical function theory or by means of the Wiener-Hopf method. However, the solution of functional equation (1.20) under condition (1.22) was constructed using the method set out in [3, 4, 10].

2. Now note with the view of solving the functional equation (1.20) subject to condition (1.22) that as $\tau(x)$ $(-\infty<x<\infty)$ is a summable function on the whole real axis, functions $\bar{\tau}_+^{(1)}(\sigma)$ and $\bar{\tau}_-^{(2)}(\sigma)$ tend to zero when $|\sigma|\to\infty$. On the other hand, it is easy to see that $\bar{G}(\sigma)\to1$ when $|\sigma|\to\infty$. Thus, the kernel $\bar{G}(\sigma)$ of functional equation (1.20) is represented as [2–4]

$$\bar{G}(\sigma)=\frac{1+\bar{K}_+(\sigma)}{1+\bar{K}_-(\sigma)} \qquad (-\infty<\sigma<\infty). \tag{2.1}$$

Here $\bar{K}_+(\sigma)$ and $\bar{K}_-(\sigma)$ are boundary values of analytical functions $\bar{K}_+(\alpha)$ and $\bar{K}_-(\alpha)$, which are regular and have no zeros in the upper $\operatorname{Im}\alpha>0$ and the lower $\operatorname{Im}\alpha<0$ semi-planes respectively with

$$\bar{K}_+(\sigma)=\exp\{\bar{F}_+(\sigma)\}-1, \qquad \bar{K}_-(\sigma)=\exp\{-\bar{F}_-(\sigma)\}-1,$$

$$\ln\bar{G}(\sigma)=\bar{F}(\sigma)=\bar{F}_+(\sigma)+\bar{F}_-(\sigma), \quad F(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\ln\bar{G}(\sigma)e^{-i\sigma x}d\sigma,$$

$$\bar{F}_+(\sigma)=\int_0^{\infty}F(x)e^{i(\sigma+i0)x}dx=\ln\sqrt{\bar{G}(\sigma)}+\frac{1}{\pi i}\int_{-\infty}^{\infty}\frac{\ln\sqrt{\bar{G}(s)}ds}{s-\sigma}, \tag{2.2}$$

$$\bar{F}_-(\sigma)=\int_{-\infty}^{0}F(x)e^{i(\sigma-i0)x}dx=\ln\sqrt{\bar{G}(\sigma)}-\frac{1}{\pi i}\int_{-\infty}^{\infty}\frac{\ln\sqrt{\bar{G}(s)}ds}{s-\sigma},$$

and the integrals with Cauchy kernel are conceived in the sense of their main value.

It is obvious, that when $|\sigma|\to\infty$ $\bar{F}_{\pm}(\sigma)\to 0$. Then, in view of representation (2.1), the functional equation (1.20) is written in the following form:

$$[1+\bar{K}_+(\sigma)]\bar{\tau}_+^{(1)}(\sigma)+[1+\bar{K}_-(\sigma)]\bar{\tau}_-^{(2)}(\sigma)=\bar{g}(\sigma)[1+\bar{K}_-(\sigma)] \quad (-\infty<\sigma<\infty). \tag{2.3}$$

On the other side, since

$$\bar{g}(\sigma)[1+\bar{K}_-(\sigma)]=\bar{\varphi}(\sigma)=\bar{\varphi}_+(\sigma)+\bar{\varphi}_-(\sigma) \qquad (-\infty<\sigma<\infty), \tag{2.4}$$

where

$$\bar{\varphi}_+(\sigma)=\int_0^{\infty}\varphi(x)e^{i(\sigma+i0)x}dx; \ \bar{\varphi}_-(\sigma)=\int_{-\infty}^{0}\varphi(x)e^{i(\sigma-i0)x}dx,$$

$$\varphi(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{g}(\sigma)[1+\bar{K}_-(\sigma)]e^{-i\sigma x}d\sigma, \tag{2.5}$$

one can represent equality (2.3) as follows:

$$\bar{L}_+^{(1)}(\sigma)\overset{def}{=}\left[1+\bar{K}_+(\sigma)\right]\bar{\tau}_+^{(1)}(\sigma)-\bar{\varphi}_+(\sigma)=\bar{\varphi}_-(\sigma)-\left[1+\bar{K}_-(\sigma)\right]\bar{\tau}_-^{(2)}(\sigma)\overset{def}{=}\bar{L}_-^{(2)}(\sigma)$$
$$(-\infty<\sigma<\infty). \tag{2.6}$$

Now, applying the generalized inverse Fourier integral transform to equality (2.6), we eventually arrive at the following equality:

$$L_+^{(1)}(x)=L_-^{(2)}(x) \qquad (-\infty<x<\infty), \tag{2.7}$$

which means that $L_+^{(1)}(x)$ and $L_-^{(2)}(x)$ are generalized functions concentrated in the zero point. Therefore, one can represent them in the following form [3, 4]:

$$L_+^{(1)}(x)=L_-^{(2)}(x)=\sum_{k=0}^{n}a_k\delta^{(k)}(x) \qquad (-\infty<x<\infty), \tag{2.8}$$

where $\delta^{(n)}(x)$ is the n-th derivative of Dirac's function $\delta(x)$. Next, applying to (2.8) the generalized integral Fourier transform, we obtain as a result:

$$\bar{L}_+^{(1)}(\sigma)=\bar{L}_-^{(2)}(\sigma)=\sum_{k=0}^{n}a_k(-i\sigma)^k \qquad (-\infty<\sigma<\infty). \tag{2.9}$$

Based on the aforesaid it is easy to see that when $|\sigma|\to\infty$ $\overline{L}_{+}^{(1)}(\sigma)\to 0$ and $\overline{L}_{-}^{(2)}(\sigma)\to 0$, and we obtain from (2.9) that $a_k=0$ $(k=\overline{0;n})$. This means that for all $-\infty<\sigma<\infty$ $\overline{L}_{+}^{(1)}(\sigma)\equiv\overline{L}_{-}^{(2)}(\sigma)\equiv 0$. Then the solution of functional equation (1.20) is obtained from (2.6) and is written in the following form:

$$\overline{\tau}_{+}^{(1)}(\sigma)=\frac{\overline{\varphi}_{+}(\sigma)}{1+\overline{K}_{+}(\sigma)},\quad \overline{\tau}_{-}^{(2)}(\sigma)=\frac{\overline{\varphi}_{-}(\sigma)}{1+\overline{K}_{-}(\sigma)}\qquad(-\infty<\sigma<\infty).\tag{2.10}$$

If we now apply the generalized inverse Fourier integral transform to (2.10) with due regard for representations (1.10), we respectively obtain for intensities $\tau^{(1)}(x)$ and $\tau^{(2)}(x)$ of unknown tangential contact stresses:

$$\tau^{(1)}(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\overline{\varphi}_{+}(\sigma)}{1+\overline{K}_{+}(\sigma)}e^{-i\sigma x}d\sigma\qquad(0<x<\infty),\tag{2.11}$$

$$\tau^{(2)}(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\overline{\varphi}_{-}(\sigma)}{1+\overline{K}_{-}(\sigma)}e^{-i\sigma x}d\sigma\qquad(-\infty<x<0)\tag{2.12}$$

Thus, the closed-form solution of contact problem under consideration was obtained in integral form the unknown functions $\tau^{(1)}(x)$ and $\tau^{(2)}(x)$ being (2.11) and (2.12) respectively.

3. It should be noted that the solution of the contact problem under investigation is also reducible to the solution of Fredholm integral equation of the second kind assuming the solution by means of the method of iterative approximations. Even though equations (2.10) contain functions of complex structure the numerical values of functions $\tau^{(1)}(x)$ and $\tau^{(2)}(x)$ are readily determined by means of this method. Indeed, solving functional equation (1.20) with respect to $\overline{\tau}(\sigma)$ under condition (1.22), we obtain:

$$\overline{\tau}(\sigma)=\frac{(\lambda_2-\lambda_1)\overline{\tau}_{+}^{(1)}(\sigma)}{\lambda_2+|\sigma|+\alpha\sigma^2+\overline{B}(|\sigma|)}+\frac{\overline{f}(\sigma)}{\lambda_2+|\sigma|+\alpha\sigma^2+\overline{B}(|\sigma|)}\ (-\infty<\sigma<\infty).\tag{3.1}$$

If we now apply the generalized inverse Fourier integral transform to (3.1), after some simplifications we obtain the following integral equation:

$$\tau(x)=(\lambda_2-\lambda_1)\int_{-\infty}^{\infty}K_1(|x-s|)\tau_{+}^{(1)}(s)ds+p(x)\qquad(-\infty<x<\infty).\tag{3.2}$$

Here the following notations are introduced:

$$K_1(|x|)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-i\sigma x}d\sigma}{\lambda_2+|\sigma|+\alpha\sigma^2+\overline{B}(|\sigma|)}=\frac{1}{\pi}\int_{0}^{\infty}\overline{\Lambda}(\sigma)\cos(\sigma x)d\sigma\ \ (-\infty<x<\infty),\tag{3.3}$$

$$p(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\overline{f}(\sigma)e^{-i\sigma x}d\sigma}{\lambda_2+|\sigma|+\alpha\sigma^2+\overline{B}(|\sigma|)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{\Lambda}(|\sigma|)\overline{f}(\sigma)e^{-i\sigma x}d\sigma\ \ (-\infty<x<\infty),\tag{3.4}$$

$$[\overline{\Lambda}(||\sigma)]^{-1}=\lambda_2+|\sigma|+\alpha\sigma^2+\overline{B}(||\sigma)\qquad(-\infty<\sigma<\infty).\tag{3.5}$$

The sought for Fredholm integral equation of the second kind for determination of tangential contact stresses $\tau^{(1)}(x)$ $(0<x<\infty)$ is obtained from (3.2) based on representations (1.10) as well as taking into account that $\tau_{+}^{(1)}(x)=0$ when $-\infty<x<0$ and $\tau_{-}^{(2)}(x)=0$ when $0<x<\infty$:

$$\tau^{(1)}(x) = (\lambda_2 - \lambda_1)\int_0^\infty K_1(|x-s|)\tau^{(1)}(s)ds + p(x) \qquad (0 < x < \infty). \qquad (3.6)$$

It is obvious, that the function $p(x)$ can be represented also as follows:

$$p(x) = \lambda_1 PK_1(|x-b|) + \lambda_2 QK_1(|x+c|) + (\lambda_2 - \lambda_1)X_0 K_1(|x|) \quad (-\infty < x < \infty). \quad (3.7)$$

Then, determining from (3.6) $\tau^{(1)}(x)$ function $(0 < x < \infty)$, one can determine from (3.2) $\tau^{(2)}(x)$ function $(-\infty < x < 0)$ in the following form:

$$\tau^{(2)}(x) = (\lambda_2 - \lambda_1)\int_0^\infty K_1(|x-s|)\tau^{(1)}(s)ds + p(x) \qquad (-\infty < x < 0). \qquad (3.8)$$

Note, that the unknown constant X_0 is determined from conditions (1.6).

Thus, in this case the solution of the contact problem under study was reduced to solution of Fredholm integral equation of the second kind (3.6). It is easy to see that the condition $N|\lambda_2 - \lambda_1| < 1$, where

$$N = \sup_{s\in(0;\infty)} \int_0^\infty \left|K_1(|x-s|)\right| dx, \qquad (3.9)$$

is sufficient condition for solvability of integral equation (3.6) in the space $L_1(0,\infty)$ of summable functions that allows to solve equation (3.6) by means of the method of iterative approximations.

4. Now investigate the behavior of $\tau(x)$ function $(-\infty < x < \infty)$ of the distribution intensity of unknown tangential contact stresses both close to and far from the application points of concentrated forces. First, let us investigate the behavior of $\tau(x)$ function when $|x| \to \infty$. It is easy to see that in case of $|\sigma| \to 0$ the following asymptotic representation for $\overline{\Lambda}(|\sigma|)$ is obtained from (3.5):

$$\overline{\Lambda}(|\sigma|) = a_0 - a_1|\sigma| + a_2\sigma^2 - a_3|\sigma|^3 + a_4\sigma^4 - a_5|\sigma|^5 + O(\sigma^6), \qquad (4.1)$$

the following notations being introduced here:

$$a_0 = \frac{1}{\lambda_2};\ a_1 = \frac{1-d_1}{\lambda_2^2};\ a_2 = \frac{1}{\lambda_2^3}\left[(1-d_1)^2 - \lambda_2(\alpha + 2ad_2 + 2ad_1)\right],$$

$$a_3 = \frac{1}{\lambda_2^4}\left[(1-d_1)^3 - 2\lambda_2(1-d_1)(\alpha + 2ad_2 + 2ad_1) - \lambda_2^2 a^2(d_3 + 4d_2 + 2d_1)\right],$$

$$a_4 = \frac{1}{\lambda_2^5}\Big[(1-d_1)^4 - 3\lambda_2(1-d_1)^2(\alpha + 2ad_2 + 2ad_1) - 2\lambda_2^2 a^2(1-d_1)(d_3 + 4d_2 + 2d_1) +$$
$$+ \lambda_2^2(\alpha + 2ad_2 + 2ad_1)^2 - \frac{2}{3}\lambda_2^3 a^3(3d_3 + 6d_2 + 2d_1)\Big], \qquad (4.2)$$

$$a_5 = \frac{1}{\lambda_2^6}\Big[(1-d_1)^5 - 4\lambda_2(1-d_1)^3(\alpha + 2ad_2 + 2ad_1) - 3\lambda_2^2 a^2(1-d_1)^2(d_3 + 4d_2 + 2d_1) +$$
$$+ 3\lambda_2^2(1-d_1)(\alpha + 2ad_2 + 2ad_1)^2 - \frac{4}{3}\lambda_2^3 a^3(1-d_1)(3d_3 + 6d_2 + 2d_1) +$$
$$+ 2\lambda_2^3 a^2(\alpha + 2ad_2 + 2ad_1)(d_3 + 4d_2 + 2d_1) - \frac{2}{3}\lambda_2^4 a^4(3d_3 + 4d_2 + d_1)\Big].$$

Applying the generalized inverse Fourier integral transform to (4.1), the following asymptotic representation is obtained for the kernel $K_1(|x|)$ of Fredholm integral equation of the second kind (3.6) when $|x|\to\infty$:

$$K_1(|x|)=\frac{1}{\pi}\left(\frac{a_1}{x^2}-\frac{6a_3}{x^4}+\frac{120a_5}{x^6}\right)+O\left(\frac{1}{x^8}\right). \quad (4.3)$$

This means that when $|x|\to\infty$ $K_1(|x|)$, therefore, both $p(x)$ and $\tau(x)$ have the order of $O(x^{-2})$. Now, let us investigate the behavior of function $\tau(x)$ when $|x|\to 0$. It is easy to see that $\bar{\Lambda}(|\sigma|)$ function can be represented as follows [8]:

$$\bar{\Lambda}(|\sigma|)=\bar{\Lambda}_1(|\sigma|)-\bar{\Lambda}_2(|\sigma|) \qquad (-\infty<\sigma<\infty), \quad (4.4)$$

where the functions $\bar{\Lambda}_1(|\sigma|)$ and $\bar{\Lambda}_2(|\sigma|)$ are respectively

$$\bar{\Lambda}_1(|\sigma|)=\frac{1}{\lambda_2+|\sigma|+\alpha\sigma^2}=\frac{1}{\alpha(b_2-b_1)}\left(\frac{1}{|\sigma|+b_1}-\frac{1}{|\sigma|+b_2}\right),\quad b_1=\frac{2\lambda_2}{\sqrt{1-4\alpha\lambda_2}+1},$$

$$\bar{\Lambda}_2(|\sigma|)=\frac{\bar{B}(|\sigma|)}{(\lambda_2+|\sigma|+\alpha\sigma^2)(\lambda_2+|\sigma|+\alpha\sigma^2+\bar{B}(|\sigma|))},\quad b_2=\frac{2\lambda_2}{1-\sqrt{1-4\alpha\lambda_2}}. \quad (4.5)$$

As in case of $|\sigma|\to\infty$ the following asymptotic representation takes place

$$\frac{k}{k+|\sigma|}=\sum_{n=0}^{\infty}(-1)^n\left(\frac{k}{|\sigma|}\right)^{n+1}, \quad (4.6)$$

then applying the generalized inverse Fourier transform to representation (4.4) with due regard for (4.5) and (4.6), we obtain in case of $|x|\to 0$ the following asymptotic formulae for kernel $K_1(|x|)$ of Fredholm integral equation of the second kind (3.6) [6, 8]:

$$K_1(|x|)=\frac{1}{\alpha(b_2-b_1)}\sum_{n=0}^{\infty}(-1)^{n+1}\left[\frac{|b_2x|^{2n+1}}{2(2n+1)!}-\frac{(b_2x)^{2n+2}}{\pi(2n+2)!}\left(\psi(2n+3)+\ln\frac{1}{|b_2x|}\right)-\right.$$
$$\left.-\frac{|b_1x|^{2n+1}}{2(2n+1)!}+\frac{(b_1x)^{2n+2}}{\pi(2n+2)!}\left(\psi(2n+3)+\ln\frac{1}{|b_1x|}\right)+\alpha(b_2-b_1)\bar{B}_n\frac{x^{2n}}{(2n)!}\right], \quad (4.7)$$

here $\bar{B}_n=\frac{1}{\pi}\int_0^{\infty}\frac{\bar{B}(\sigma)\sigma^{2n}d\sigma}{(\lambda_2+\sigma+\alpha\sigma^2)(\lambda_2+\sigma+\alpha\sigma^2+\bar{B}(\sigma))}$, $\psi(x)$ is well known psi-function.

It is noteworthy here that asymptotic formulae (4.3) and (4.7) were obtained using the values of following integrals [8, 9]:

$$F^{-1}[\sigma^{2n}]=(-1)^n\delta^{(2n)}(x),\quad F^{-1}[|\sigma|^{2n+1}]=\frac{(2n+1)!}{\pi}\cdot\frac{(-1)^{n+1}}{x^{2n+2}} \quad (n=0,1,2,...), \quad (4.8)$$

$$F^{-1}[|\sigma|^{-2n-1}]=\frac{(-1)^n\cdot x^{2n}}{\pi(2n)!}\left[\psi(2n+1)+\ln\frac{1}{|x|}\right],\quad F^{-1}[\sigma^{-2n-2}]=\frac{(-1)^{n+1}\cdot|x|^{2n+1}}{2(2n+1)!},$$

where $F^{-1}[\bullet]$ is Fourier inverse operator.

As the kernel $K_1(|x|)$ is seen from representation (4.7) to be finite when $|x|\to 0$, therefore, the function $p(x)$ is finite in points $x=0$, $x=-c$ and $x=b$.

Hence follows that the tangential contact forces in these points $x=0$, $x=-c$ and $x=b$ also have finite values due to the presence of glue layer. It should be observed that the series (4.7) are convergent for any value of x, $-\infty < x < \infty$.

Finally note that in the absence of glue layer the tangential contact forces have a logarithmic singularity [12] in $x=0$, $x=-c$ and $x=b$ points, the singularity in $x=-c$ and $x=b$ points being caused by concentrated forces P and Q, and that in $x=0$ by the non-homogeneity of stringer.

Thus, two approaches were adopted for solution of contact problem under study, in the first approach (Section 2) a closed solution of the problem in integral form was constructed, and in the second one (Section 3) the problem is reduced to solution of Fredholm integral equation of the second kind (3.6) without singularities, for solution of which the method of iterative approximations is used.

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REFERENCES

1. **Muki R., Sternberg E.** On the Diffusion of Load from a Transverse Tension Bar into a Semi-Infinite Sheet. // PM. Proc. Amer. Eng. Mech. Ser. E, 1968, № 4, p. 124–135.
2. **Grigoryan E.Kh.** Load Transmission from the Piece–Homogeneous Infinite Stringer to Elastic Semi-Plane. // Uchenie Zapiski EGU, 1979, № 3, p. 29–34 (in Russian).
3. **Grigoryan E.Kh.** Contact Problem for Elastic Semi-Plane with an Elastic Infinite Piecewise Homogeneous Plate Glued to its Boundary. // Izv. NAN RA. Mekhanika, 1990, v. 43, № 4, p. 24–34 (in Russian).
4. **Aghayan K.L., Grigoryan E.Kh.** Deformable Body Mechanics. Yerevan: AN of Arm. SSR, 1990, p. 31–38 (in Russian).
5. **Grigoryan E.Kh., Kerobyan A.V., Sarkisyan V.S.** Contact Problem for Elastic Halfplane with Forced Boundary by Glued Elastic Semi-Infinite Plates. // Izv. RAN MTT, 1992, № 3, p. 180–184 (in Russian).
6. **Grigoryan E.Kh., Hovhannisyan H.V.** A Contact Problem for a Piecewise Homogeneous Infinite Plate Strengthened with Two Parallel Different Infinite Elastic Stringers // Izv. NAN RA. Mekhanika, 2009, v. 62, № 3, p. 29–43 (in Russian).
7. **Benthem J.P.** On the Dffusion of a Load from a Semi-Infinite Stringer Bonded to Sheet. // Contribution to the Theory of Aircraft Structures. Delft Univer. Press, 1973, p. 117–134.
8. **Hovhannisyan H.V.** Contact Problem for an Infinite Composite Elastic (Piecewise-Homogeneous) Plate with an Infinite Elastic Stringer Glued to the Plate Surface. // Proceedings of the YSU. Physical and Mathematical Sciences, 2012, № 1, p. 27–32.
9. Handbook of Special Functions. M.: Nauka, 1979, 832 p. (in Russian).
10. **Gakhov F.D.** Boundary Value Problems. M.: Nauka, 1977, 640 p. (in Russian).
11. **Shilov G.E.** Mathematical Analysis. Second Special Course. M.: Nauka, 1965, 327 p. (in Russian).
12. **Hovhannisyan H.V.** Proceedings of Int. Conf. “Topical Problems of Continuum Mechanics”, v. 2, p. 59–63 (in Russian).