

## INFLUENCE OF THE RASHBA AND DRESSELHAUS SPIN-ORBIT INTERACTIONS ON THE POLARON PROPERTIES OF A TWO-DIMENSIONAL ELECTRONS IN SEMICONDUCTOR HETEROSTRUCTURES

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The interplay of the Rashba and Dresselhaus spin-orbit as well as the Fröhlich type electron-phonon interactions on the energy dispersion relation of the spin subbands in a two-dimensional electron gas in semiconductor polar heterostructures is studied theoretically. The Rayleigh-Schrodinger perturbation theory has been used to obtain in closed form the basic parameters of the polaron state (self-energy and effective mass) as a function of the Rashba and Dresselhaus coupling strengths.

**Keywords:** quantum well, spin-orbit, polaron.

There is currently interest in manipulating electron spins in quantum heterostructures due to potential applications in spintronics [1] and quantum computing [2–4]. In future spin electronic devices, the spin degree of freedom of electrons together with their charge, is used for realizing new device concepts. These devices promise increased speed and lower power consumption, and may offer new functionalities having no counterpart in conventional electronic devices. This has generated an intense interest in effects of spin-orbit interaction (SOI) in low-dimensional semiconductor heterostructures. The SOI is responsible for many noble effects like spin-FET [5, 6], metal-insulator transition in a two-dimensional hole gas [7], spin-resolved ballistic transport [8], spin-galvanic effect [9] and spin Hall effect [10, 11]. As a relativistic effect of dynamic electrons moving in an electric field, the intrinsic SOI exists in bulk semiconductors with structural inversion asymmetry [12], while controllable SOI can be introduced by asymmetrically confining a two-dimensional electron gas in semiconductor heterostructures to create an average electric field across the electron system [13]. The Rashba and Dresselhaus couplings, allow effectively control both magnitude and direction of non-equilibrium spin accumulation in a two dimensional electron gas system [14]. Several schemes have been proposed to generate spin-orbit coupling [15, 16] with tunable spin-orbit (SO) coupling strength.

In systems with SOI a significant role plays the electron-phonon interaction. The Fröhlich interaction is one of the main electron-phonon intrinsic interactions in polar materials originating from the coupling of one itinerant electron with the macroscopic electric field generated by any longitudinal optical (LO) phonon. In

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particular, an important problem is whether the polaron, that is the quasiparticle composed by the electron and its phonon cloud, is strengthened or weakened by the SOI. In previous works [17–19] it has been obtained that the polaronic effects increase for a two-dimensional electron gas with linear Rashba coupling for both short range and long-range electron-phonon interactions.

The aim of the present paper is to investigate the influence of spin-orbit interaction on the energy-momentum relation for the two-dimensional Fröhlich polaron. This allows us, in particular, to obtain in closed form the basic parameters of the polaron state (self-energy and effective mass) as a function of the Rashba and Dresselhaus coupling strengths.

We consider a single electron moving with a parabolic dispersion in the  $x$ - $y$  plane in zinc-blende semiconductor nanostructures with SO coupling, which also interacts with optical phonons of energy  $\hbar\omega_{LO}$ . The total system is then described by the two-dimensional Fröhlich-SO Hamiltonian

$$H = H_{el} + H_{ph} + H_{el-ph}, \quad (1)$$

where

$$H_{el} = \frac{\hbar^2}{2m^*} k_x^2 + k_y^2 + H_R + H_D. \quad (2)$$

The SO coupling is taken into account by adding the linear Rashba and Dresselhaus terms for conduction band electrons in a two-dimensional electron gas:

$$H_R = \alpha(\sigma_x k_y - \sigma_y k_x), \quad H_D = \beta(\sigma_x k_x - \sigma_y k_y), \quad (3)$$

where  $\sigma_x$  and  $\sigma_y$  are Pauli spin matrices,  $\mathbf{k}$  is the in plane electron momentum with its magnitude  $k$  and polar angle  $\phi_k$ . The axes  $x, y$  and  $z$  are aligned along the principal crystallographic axes of zinc-blende semiconductor.  $\alpha$  and  $\beta$  are the Rashba and Dresselhaus SO coupling parameters respectively. The eigenstates  $\psi_{k\lambda}(\mathbf{r})$  of the Hamiltonian  $H_{el}$

$$\psi_{k\lambda}(\mathbf{r}) = \frac{1}{\sqrt{2S}} \begin{pmatrix} ie^{-i\varphi} \\ \lambda \end{pmatrix} \exp(i\mathbf{k}\mathbf{r}) \quad (4)$$

are specified by the chirality  $\lambda = \pm 1$  of the spin branch. The double sign corresponds to the upper (+) and lower (−) parts of the energy spectrum given by

$$E_{\lambda}^0(\mathbf{k}) = (\hbar^2 / 2m^*) [(k + \lambda\zeta(\rho, \theta, \phi_k))^2 - \zeta(\rho, \theta, \phi_k)^2]. \quad (5)$$

The characteristic SO momentum is  $\zeta(\rho, \theta, \phi_k) = \rho\sqrt{1 + \sin(2\theta)\sin(2\phi_k)}$

with  $\rho = (m^* / \hbar^2)\sqrt{\alpha^2 + \beta^2}$  describes the angular anisotropy of the spin splitting.

The angle parameter  $\theta$ , defined as  $\tan\theta = \beta/\alpha$ , describes the relative strength of the Rashba and Dresselhaus SO coupling. The phase  $\varphi$  of the spinor is defined as

$$\varphi(\alpha, \beta, \phi_k) = \text{Arg}[\alpha \exp(i\phi_k) + i\beta \exp(-i\phi_k)]. \quad (6)$$

The polar coupling of a two-dimensional electron to the LO-phonons can be written as

$$H_{el-ph} = \int \Gamma(\mathbf{r})\Phi(\mathbf{r})d\mathbf{r}, \quad (7)$$

where  $\Gamma(\mathbf{r})$  is the generalized electron density operator and  $\Phi(\mathbf{r})$  is the electrostatic potential created by the LO-phonons. The electron density has the standard expansion in second quantized operators  $c_{k\lambda}^+$  and  $c_{k\lambda}$ :

$$\Gamma(\mathbf{r}) = \sum_{k\lambda, k'\lambda'} \psi_{k'\lambda'}^*(\mathbf{r}) \psi_{k\lambda}(\mathbf{r}) c_{k'\lambda'}^+ c_{k\lambda} = (1/2S) \sum_{k\lambda, k'\lambda'} \left( e^{i(\varphi' - \varphi)} + \lambda\lambda' \right) e^{i(k' - k)r} c_{k'\lambda'}^+ c_{k\lambda}. \quad (8)$$

The electrostatic potential  $\Phi(\mathbf{r})$  due to the polarization of the lattice is given by [20]

$$\Phi(\mathbf{r}) = 1/\sqrt{S} \sum_q (1/\sqrt{q}) \left( M_0 \exp(i\mathbf{q}\mathbf{r}) a_q + M_0^* \exp(-i\mathbf{q}\mathbf{r}) a_q^+ \right), \quad (9)$$

where  $M_0 = -i\sqrt{2\pi e^2 \hbar \omega_{LO} (\varepsilon_0^{-1} - \varepsilon_\infty^{-1})}$ ,  $\mathbf{q}$  is the in plane phonon momentum. Here  $\varepsilon_0$  and  $\varepsilon_\infty$  are the static and high frequency dielectric constants of the semiconductor. Substituting the expressions (8) and (9) into (7), we get

$$H_{el-ph} = 1/\sqrt{S} \sum_{k\lambda, q\lambda'} \frac{1}{q} \left( M_0 / \sqrt{q} \right) G_{k-q, k}^{\lambda', \lambda} (a_q + a_q^+) c_{k'\lambda'}^+ c_{k\lambda}, \quad (10)$$

where

$$G_{k+q, k}^{\lambda', \lambda} = \frac{1}{2} \left( \exp \left[ i \left( \text{Arg} \left[ \alpha e^{i\phi_{k+q}} + i\beta e^{-i\phi_{k+q}} \right] - \text{Arg} \left[ \alpha e^{i\phi_k} + i\beta e^{-i\phi_k} \right] \right) \right] + \lambda\lambda' \right). \quad (11)$$

Since the electron-phonon interaction is weak in Ga<sub>1-x</sub>Al<sub>x</sub>As/GaAs or similar systems, Fröhlich coupling constant  $\alpha_{el-ph} = (m^* e^4 / 2\hbar^3 \omega_{LO})^{1/2} (\varepsilon_0^{-1} - \varepsilon_\infty^{-1}) \ll 1$ , the second-order Rayleigh–Schrodinger perturbation theory can be used to obtain the electron energy with electron–LO–phonon coupling:

$$E_\lambda(\mathbf{k}) = E_\lambda^0(\mathbf{k}) - \frac{1}{S} \sum_{q\lambda'} \frac{|M_0|^2}{q} \frac{|G_{k-q, k}^{\lambda', \lambda}|^2}{E_\lambda^0(\mathbf{k} - \mathbf{q}) - E_\lambda^0(\mathbf{k}) + \hbar\omega_{LO}}. \quad (12)$$

After some tedious algebra for the polaron energy in unites of  $\hbar\omega_{LO}$  the following expression is obtained:

$$E_\lambda(\mathbf{k}) = (k + \lambda\zeta(\rho, \theta, \phi_k))^2 - \zeta(\rho, \theta, \phi_k)^2 - \frac{\alpha_{el-ph}}{\pi} \int_0^\pi d\Omega \int_0^\infty dq \frac{p_1 - p_2}{p_1^2 - p_3}, \quad (13)$$

where

$$\begin{aligned} p_1 &= q^2 - 2qk \cos \Omega - \lambda k \rho \sqrt{1 + \sin(2\theta) \sin(2\phi_k)} + 1, \\ p_2 &= 2\lambda \rho \frac{(k - q \cos \Omega - \sin(2\theta)(-k \sin(2\phi_k) + q \sin(\Omega + 2\phi_k)))}{\sqrt{1 + \sin(2\theta) \sin(2\phi_k)}}, \\ p_3 &= 4\rho^2 \left[ k^2 + q^2 - 2qk \cos \Omega + \sin(2\theta) (k^2 \sin(2\phi_k) + \right. \\ &\quad \left. + q(q \sin(2\Omega + 2\phi_k) - 2k \sin(2\Omega + 2\phi_k))) \right]. \end{aligned}$$

Here  $k$ ,  $q$  and  $\rho$  are presented in unites  $\sqrt{2m^* \omega_{LO} / \hbar}$ . To identify the interplay the Rashba and Dresselhaus spin-orbit as well as the Fröhlich type electron-phonon interactions on the energy dispersion relation of the spin subbands in a two-dimensional electron gas in semiconductor polar heterostructures, we present in Fig.1 the contours of constant energy  $0.1\hbar\omega_{LO}$  in the momentum plane ( $k_x, k_y$ ) for electron without SOI (1) and for polaron with SOI (2) for the values.  $k_\alpha = 0.4 k_0$ ,  $k_\beta = 0.2 k_0$ .

As both  $k_\alpha$  and  $k_\beta$  are nonzero the contour of constant energy of the polaron is anisotropic as for electron with SOI, but without electron-phonon interaction [21].

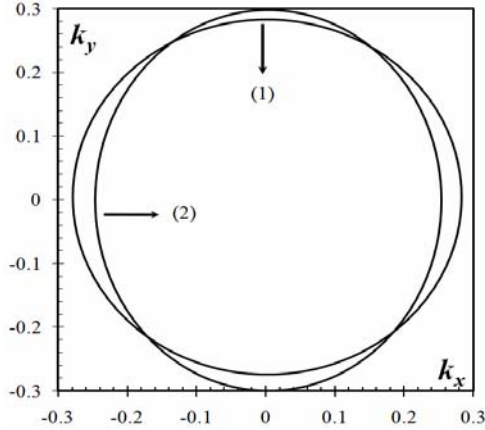
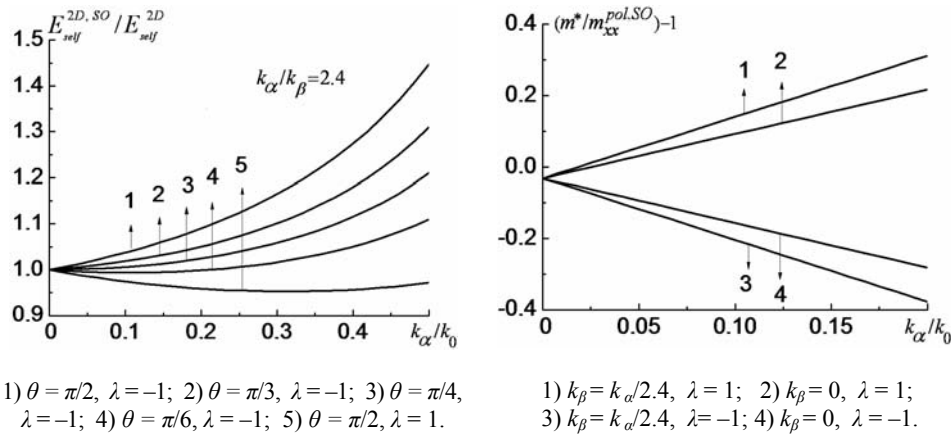


Fig. 1. Contours of constant energy for electron without SOI (1) and for polaron with SOI (2) in the momentum plane  $(k_x, k_y)$ . The contour plots determined by the condition  $E_\lambda(\mathbf{k}) = 0.1\hbar\omega_{LO}$ .

The numerical results for two-dimensional polaron self energy with SOI are presented in Fig. 2, a. It can be seen that the effect of joint Rashba and Dresselhaus SOI on the polaron state give rise to the angular dependence of the self-energy. It is found that  $E_{self}^{2D}$  increases significantly (more than 40%) for direction  $\theta = \pi/2$  with spin branch  $\lambda = -1$ , while it is lowered by the SOI, for the same direction with spin branch  $\lambda = 1$ .

It has been shown theoretically that the polaron effective mass with Rashba SOI is not constant even near the band minimum and the mass correction is positive for the upper Rashba branch and negative for the lower Rashba branch [22]. The same behavior we have obtained in the presence of both Rashba and Dresselhaus SOI (Fig. 2, b). The only difference is that the Dresselhaus SOI additionally enhances the SOI effect on the polaron effective mass.



1)  $\theta = \pi/2, \lambda = -1$ ; 2)  $\theta = \pi/3, \lambda = -1$ ; 3)  $\theta = \pi/4, \lambda = -1$ ; 4)  $\theta = \pi/6, \lambda = -1$ ; 5)  $\theta = \pi/2, \lambda = 1$ .

1)  $k_\beta = k_\alpha/2.4, \lambda = 1$ ; 2)  $k_\beta = 0, \lambda = 1$ ; 3)  $k_\beta = k_\alpha/2.4, \lambda = -1$ ; 4)  $k_\beta = 0, \lambda = -1$ .

Fig. 2. The polaron self-energy (a) and the polaronic effective mass correction (b) at  $k=0$  as a function of the Rashba SOI strength for  $\alpha_{ph} = 0.07$ .

In conclusion, we have shown that the effect of joint Rashba and Dresselhaus SOI leads to quite polaronic effect in two-dimensional electron gas. The polaron self energy as well as effective mass correction varies differently in different spin branches.

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