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TWO-DIMENSIONAL POLARIZATION PATTERNS FOR HOLOGRAPHIC RECORDING

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Two-dimensional (2D) polarization pattern is achieved by the interference of two pairs of beams with perpendicular planes of incidence and orthogonal circular polarizations. Imposing a phase shift of $\pi/2$ between consecutive beams contains the amplitude modulation of the optical field in the superposition region and, thus, pure 2D polarization patterns are created. The recording of this interference field in a polarization sensitive material creates reconfigurable 2D periodic micro-structure with peculiar diffraction properties.

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Introduction. Under conditions of rapid progress in formation technologies the research and development in new active optical media intended for reflection of optical information and also control of light propagation direction and properties in photonic devices are essential. Because of their optically active properties the polarization holographic gratings are promising for production of highly functional optical devices such as the light modulators, polarization multiplexers and demulti-plexers, valves, polarized radiation splitters [1]. Owing to the vectorial nature of light, peculiar one-dimensional (1D) polarization light patterns have been created, and the use of this holographic technique in conjunction with polarization-sensitive materials enabled to design optical devices with special performance [1–4].

The holographic approaches have been utilized along with other methods for preparation of two-dimensional (2D) and three-dimensional microstructures and photonic crystals in different photosensitive materials. Several configurations have been used to exploit multiple beam interference [5, 6] or multiple exposures to two interfering beams for different sample orientation [7]. Nevertheless, the pure 2D polarization patterns were not produced with any of these approaches. Actually, in the first case the optical field typically exhibits both the intensity and polarization modulation. In the second case, the superposition of 1D polarization holograms at different angles produces 2D patterns of optical anisotropies that are not pure polarization holograms. More recently, the spatial light modulator (SLM)-assisted

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approaches made it possible to create complex light field configurations, but the cases of pure polarization light fields have never been considered [8–11].

In the present paper we report on pure 2D polarization pattern obtained at the interference of multiple asymmetrically polarized beams assisted by SLM; the phase difference of $\pi/2$ introduced between each consecutive beam suppresses the amplitude modulation in the optical field, creating thus a 2D polarization pattern with intriguing chiral structure. The 2D pattern can be recorded on a film of polarization-sensitive material, in which the circular birefringence can be photoinduced.

Results and Discussions. 2D light patterns may be formed at superposition of off-axis plane waves with different polarizations and phase shifts, that, in general, are combinations of the amplitude and polarization modulation of light field. Here we analyze, according to the figure caption, some special cases of superposition of two pairs of plane waves with orthogonal circular polarization (OCP) and relative phase shifts of $\pi/2$. No modulation of light intensity occurs in the superposition region, and the Jones formalism has been adopted to evaluate the optical fields. The normalized Jones vector of the total field of OCP waves (see figure caption) is

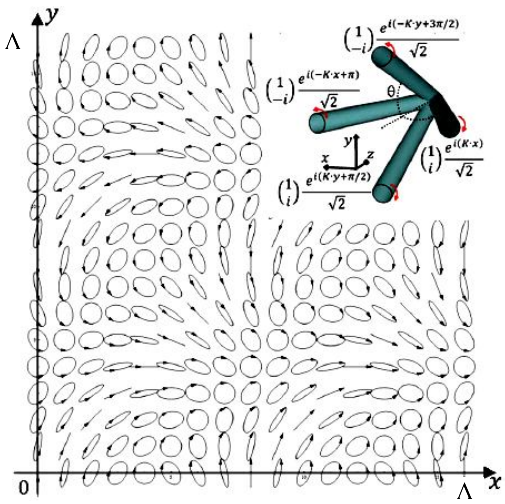
$$E = E_1 + E_2 + E_3 + E_4 = \frac{\sqrt{2}}{2}\begin{bmatrix} i\ \sin Kx - \sin Ky \\ i\ \cos Kx - \cos Ky \end{bmatrix},$$

where

$$E_1 = \frac{1}{2\sqrt{2}}\exp(iKx)\begin{bmatrix} 1 \\ i \end{bmatrix}; \qquad E_2 = \frac{1}{2\sqrt{2}}\exp(-iKx)\begin{bmatrix} -1 \\ i \end{bmatrix};$$

$$E_3 = \frac{1}{2\sqrt{2}}\exp(iKy)\begin{bmatrix} i \\ -1 \end{bmatrix}; \quad E_4 = -\frac{1}{2\sqrt{2}}\exp(-iKy)\begin{bmatrix} i \\ 1 \end{bmatrix}.$$

Here $K = 2\pi/\Lambda$, $\Lambda = \lambda/(2\sin\theta)$ is the spatial periodicity, θ is the angle of interfering beams incidence, and λ is the wavelength of light.



2D polarization pattern for OCP configuration.

Inset: Schemes of four plane waves with orthogonal circular polarizations and their representations in terms of Jones vectors.

The 2D polarization patterns for OCP configuration we get in the Figure give a very interesting spatial distribution of the polarization state including linear, elliptical, and circular polarizations arranged in periodic vortex structures with opposite helicity. To probe the quality of polarization patterns we have investigated the optical properties of 2D polarization holograms obtained by recording these light fields on a photosensitive material. The first three Stocks parameters are

$$S_0 = 1, \qquad S_1 = -\frac{1}{2}[\cos(2Kx) + \cos(2Ky)],$$

$$S_2 = \frac{1}{2}[-\sin(2Kx) + \sin(2Ky)] - i\sin K(x+y).$$

Here the absence of additional phase difference between E_x and E_y was taken into account. Then, following to [1], the Jones matrix describing the polarization hologram with periodic modulation of the anisotropy is obtained using the refractive index $T = \exp(ikL(n_0 + n_1))$. Here L is the hologram thickness, $k = 2\pi\frac{1}{\lambda}$ is wave vector and n_1 is defined by

$$n_1 = \begin{bmatrix} \bar{n}S_0 + \Delta n S_1 & \Delta n S_2 \\ \Delta n S_2 & \bar{n}S_0 - \Delta n S_1 \end{bmatrix}.$$

Here $\bar{n} = (\Delta n_\parallel + \Delta n_\perp)/2$, $\Delta n = (\Delta n_\parallel - \Delta n_\perp)/2$, $\Delta n_\parallel$ and $\Delta n_\perp$ are photoinduced changes in refractive indexes. The components of the T matrix depend on the type of interference field used during the recording of the anisotropic hologram. For above calculated Stocks parameters, we have

$$n_1 = \begin{bmatrix} \bar{n} + \frac{1}{2}\Delta n[\cos(2Kx) + \cos(2Ky)] & \frac{1}{2}\Delta n[-\sin(2Kx) + \sin(2Ky)] - i\Delta n \sin K(x+y) \\ \frac{1}{2}\Delta n[-\sin(2Kx) + \sin(2Ky)] - i\Delta n \sin K(x+y) & \bar{n} - \frac{1}{2}\Delta n[\cos(2Kx) + \cos(2Ky)] \end{bmatrix}.$$

It can be represented as $n_1 = n_1' + n_1'' + n_1'''$, were

$$n_1' = \begin{bmatrix} \bar{n} + \frac{1}{2}\Delta n\cos(2Kx) & -\frac{1}{2}\Delta n\sin(2Kx) \\ -\frac{1}{2}\Delta n\sin(2Kx) & \bar{n} - \frac{1}{2}\Delta n\cos(2Kx) \end{bmatrix}, \quad n_1'' = \begin{bmatrix} \frac{1}{2}\Delta n\cos(2Ky) & \frac{1}{2}\Delta n\sin(2Ky) \\ \frac{1}{2}\Delta n\sin(2Ky) & -\frac{1}{2}\Delta n\cos(2Ky) \end{bmatrix},$$

$$n_1''' = \begin{bmatrix} 0 & -i\Delta n\sin K(x+y) \\ -i\Delta n\sin K(x+y) & 0 \end{bmatrix}.$$

At the construction the Jones matrix for this 2D polarization pattern the field after the grating is obtained to be

$$E_{out} = TE_{in}.$$

The Jones matrix is represented as $T = \exp(ikL(n_0 + n_1)) = T' \times T'' \times T'''$. For $K = 0$

$$T_0' = \begin{bmatrix} \exp[ikL(n_0 + \bar{n} + \Delta n/2)] & 0 \\ 0 & \exp[ikL(n_0 + \bar{n} - \Delta n/2)] \end{bmatrix}. \qquad (1)$$

Let denote $\varphi_0 = kLn_0$, $\varphi_S = kL\bar{n}$ and $G = \pi\frac{\Delta n L}{\lambda}$, then, instead of (1), we have

$$T_0' = \exp[i(\varphi_0 + \varphi_S)]\begin{bmatrix} \exp[iG] & 0 \\ 0 & \exp[-iG] \end{bmatrix}. \tag{2}$$

The Jones matrix for the $K \neq 0$ is obtained by rotating (2) through $\delta = 2Kx$ angle. This yields

$$T' = R(-\delta)\, T_0' R(\delta), \quad R(\delta) = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix},$$

$$T' = \exp[i(\varphi_0 + \varphi_S)]\begin{bmatrix} \cos G + i\sin G \cos(2Kx) & -i\sin G\sin(2Kx) \\ -i\sin G\sin(2Kx) & \cos G - i\sin G\cos(2Kx) \end{bmatrix}.$$

For the second matrix it is found at the $K = 0$

$$T_0'' = \begin{bmatrix} \exp[iG] & 0 \\ 0 & \exp[-iG] \end{bmatrix},$$

and for $K \neq 0$ it is obtained by rotating through $\delta = -2Ky$ angle:

$$T'' = \begin{bmatrix} \cos G + i\sin G \cos(2Ky) & i\sin G\sin(2Ky) \\ i\sin G\sin(2Ky) & \cos G - i\sin G\cos(2Ky) \end{bmatrix}.$$

The procedure for the third matrix $T''' = \exp N$(were $N = i\,Ln_1'''$) is somewhat more complicated, as we cannot do the same thing with n_1''' (it could not bring to diagonal form). So, in this event we have to refer to the general case. By definition we have for the exponent from matrix N,

$$\exp N = \sum_{m=0}^{\infty} \frac{1}{m!} N^m .$$

Let we transform matrix N:

$$N = aJ, \; J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \; a = k\,L\Delta n \sin K(x+y) = 2G\sin K(x+y),$$

where J is the unit matrix. Then for the third matrix we have

$$T''' = \sum_{m=0}^{\infty} \frac{a^{2m}}{(2m)!} I + \sum_{m=0}^{\infty} \frac{a^{2m+1}}{(2m+1)!} J = I\cosh a + J\sinh a.$$

Now we can write $T' = T_0' + T_{+1}' + T_{-1}'$ and $T'' = T_0'' + T_{+1}'' + T_{-1}''$, where

$$T_0' = T_0'' = T_0 = \begin{bmatrix} \cos G & 0 \\ 0 & \cos G \end{bmatrix} = I\cos G; \quad T_{\pm1}' = \frac{i\sin G}{2}\exp(\pm i2Kx)\begin{bmatrix} 1 & \pm i \\ \pm i & -1 \end{bmatrix};$$

$$T_{\pm1}'' = \frac{i\sin G}{2}\exp(\mp i2Ky)\begin{bmatrix} 1 & \pm i \\ \pm i & -1 \end{bmatrix}; \qquad T_0''' = I\cosh a; \; T_{\pm1}''' = J\sinh a.$$

The Jones matrix of such cycloidal diffractive wave plate we can write in the form

$$T = T' \times T'' \times T''' = (T_0' + T_{+1}' + T_{-1}') \times (T_0'' + T_{+1}'' + T_{-1}'') \times (T_0''' + T_{+1}''' + T_{-1}''').$$

Hence, there must be five strong waves after the grating – one undiffracted wave (0th order) and four diffracted waves – in the +1 and –1 orders (from x, y). There must be also some weak waves – two ±1 orders from mixed gratings and other high orders. T_0 matrix determines the 0th order wave and $T_{\pm1}$ matrixes determine four diffracted waves. To obtain the intensities and polarizations of the waves, the Jones vector of the reconstructed wave E_{in} shall be multiplied by the matrix T:

$$T=[T_0^2+T_0(T'_{+1}+T'_{-1}+T''_{+1}+T''_{-1})+T'_{+1}T''_{+1}+T'_{+1}T''_{-1}+T'_{-1}T''_{+1}+T'_{-1}T''_{-1}]\times(T_0'''+T'''_{+1}+T'''_{-1})=$$
$$=T_{00}+T_{+1x}+T_{-1x}+T_{+1y}+T_{-1y}+T_{+1(x+y)}+T_{-1(x+y)}+T'_{+1(x+y)}+T'_{-1(x+y)}+\text{Remaining terms.}$$

Here we have

$$T_{00}=I\cos^2 G\cosh a;\;\; T_{\pm1x}=\frac{i}{4}\sin(2G)\cosh a\cdot\exp(\pm i2Kx)\begin{bmatrix}1 & \pm i\\ \pm i & 1\end{bmatrix};$$

$$T_{\pm1y}=\frac{i}{4}\sin(2G)\cosh a\cdot\exp(\mp i2Ky)\begin{bmatrix}1 & \pm i\\ \pm i & -1\end{bmatrix};\;\; T_{\pm1(x+y)}=J\cos^2 G\sinh a\approx$$

$$\approx\mp iG_1\cos^2 G\exp[\pm iK(x+y)]J\ (\text{for small } G_1);\;\; T'_{+1}T''_{+1}=0;\;\; T'_{-1}T''_{-1}=0;$$

$$T'_{+1}T''_{-1}T_0'''=T'_{+1(x+y)};\;\; T'_{-1}T''_{+1}T_0'''=T'_{-1(x+y)};$$

$$T'_{\pm1(x+y)}=-\frac{1}{2}\sin^2 G\cosh a\cdot\exp[\pm i2K(x+y)]\begin{bmatrix}1 & \mp i\\ \pm i & 1\end{bmatrix}.$$

As for the “Remaining terms”: $[T_0(T'_{+1}+T'_{-1}+T''_{+1}+T''_{-1})]\times(T'''_{+1}+T'''_{-1})$ are the second order diffractions and second order terms in G; $[T'_{+1}T''_{+1}+T'_{+1}T''_{-1}+T'_{-1}T''_{+1}+T'_{-1}T''_{-1}]\times(T'''_{+1}+T'''_{-1})$ are the second order and zero order additional diffractions and third order terms in G.

If we take incident field E_{in} to be linearly polarized along Ox axis with unit intensity

$$E_{in}=\begin{bmatrix}1\\0\end{bmatrix},$$

then the 0th order field after the grating is given by

$$E_0=\cos^2 G\cosh a\,.$$

It emerges with the same polarization and its intensity is proportional to $\cos^2 G$. The waves of ±1 orders are

$$E_{\pm1x}=\frac{i}{4}\sin(2G)\cosh a\cdot\exp(\pm i2Kx)\begin{bmatrix}1\\ \pm i\end{bmatrix},$$

$$E_{\pm1y}=\frac{i}{4}\sin(2G)\cosh a\cdot\exp(\mp i2Ky)\begin{bmatrix}1\\ \pm i\end{bmatrix},$$

$$E_{\pm1(x+y)}=\mp iG_1\cos^2 G\cdot\exp[\pm iK(x+y)]\begin{bmatrix}0\\1\end{bmatrix}.$$

Thus, we get right-hand and left-hand circular polarized waves that intersects (x, y) plane on the x and y axes. We get also linear waves polarized, perpendicular to the incident one and intersects (x, y) plane on the bisectors of x, y axes.

In conclusion, the formation of pure 2D polarization patterns as a result of interference of four $\pi/2$ phase shifted, asymmetrically polarized light beams has been shown. Orthogonal circular configuration has been investigated. The Jones matrices for diffracted beams have been calculated to obtain their intensities.

Particularly, we have found that if the delay is π, then there is no zero and ± 1 orders and there will be some new diffraction orders. It is interesting also that the zero order diffraction is achromatic (independent on the wavelength).

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