

ON THE SOLUTION OF THE EQUATION $\frac{5}{k} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ ON THE
SET OF NATURAL NUMBERS $N \setminus \{60n+1, n \in N\}$

E. Kh. ASLANYAN *

State Architectural College of Abovyan, Armenia

In the present paper it is shown that for every number $k \not\equiv 1 \pmod{60}$, the equation $\frac{5}{k} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ has at least one solution $(x, y, z) \in N$.

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V. Sierpinsky in his paper [1] suggested a hypothesis, stating that for any number $k > 1$, the equation $5/k = 1/x + 1/y + 1/z$ has a solution (x, y, z) consisting of natural numbers, and the statement was proved in the same paper for all natural numbers $1 < k \leq 1000$.

G. Palma has shown that Sierpinsky's hypothesis is true for all natural numbers $k > 1$, that are not of the form $1260m + 1, m \in N$, as well as for all numbers $k < 922322$ [2].

Later other authors have shown that the set of all numbers $k > 1$, for which Sierpinsky's hypothesis may not be true, is bounded from above, which means that the hypothesis is true for almost all numbers [3–6].

However, the final solution is still unknown.

The main result of the present paper is the following:

Theorem. If k is a natural number with $k \not\equiv 1 \pmod{60}$, then the equation

$$\frac{5}{k} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \tag{1}$$

has a natural solution (x, y, z) .

Proof. We will consider separately several events covering all possible cases of $k \not\equiv 1 \pmod{60}$.

Case 1. If k is an even number, i.e. $k = 2n$, then the triple $x = 2n; y = n; z = n$, is a solution for (1).

* E-mail: absecollege@rambler.ru

Case 2. Assume now that $k \equiv 1 \pmod{10}$. Obviously, $\{10n + 1, n \in N\} = \{30n - 9, n \in N\} \cup \{30n + 1, n \in N\} \cup \{30n - 19, n \in N\}$, and we have in this case three possibilities:

Case 2a. If $k = 30n - 19$ for some $n \in N$, then the triple $x = 3(2n - 1)$; $y = 3(2n - 1)(30n - 19)$; $z = (2n - 1)(30n - 19)$ provides a solution of (1).

Case 2b. If $k = 30n - 9$ for some $n \in N$, then the triple $x = 6n - 1$; $y = 3(6n - 1)(30n - 3)$; $z = (6n - 1)(10n - 3)$ is a solution of (1).

Case 2c. Let $k \in \{30n + 1, n \in N\}$, it is easy to see that $\{30n + 1, n \in N\} = \{60n - 29, n \in N\} \cup \{60n + 1, n \in N\}$.

Since we have assumed that $k \not\equiv 1 \pmod{60}$, then $k = 60n - 29$ for some $n \in N$ and the triple $x = 60n - 29$; $y = 15n - 7$; $z = (60n - 29)(15n - 7)$ is the solution for (1).

Case 3. If $k \equiv 3 \pmod{10}$, Then $k = 10n - 7$, for some natural n and $x = 2n - 1$; $y = (2n - 1)(10n - 7)$; $z = (2n - 1)(10n - 7)$ is a solution for (1).

Case 4. Assume $k \equiv 5 \pmod{10}$, so $k = 5(2n - 1)$ for some $n \in N$. Then the solution of (1) is $x = 2(2n - 1)$; $y = 3(2n - 1)$; $z = 6(2n - 1)$.

Case 5. Let now $k \equiv 7 \pmod{10}$. We get $k = 10n - 3$, then the triple $x = 2n$; $y = 2n(10n - 3)$; $z = n(10n - 3)$ is solution for (1).

Case 6. Finally, let $k \equiv 9 \pmod{10}$. If $k = 10n - 1$, then the following triple provides a solution for (1): $x = 4n$; $y = 4n$; $z = 2n(10n - 1)$.

So we show that the equation $5/k = 1/x + 1/y + 1/z$ has natural solutions when $k \notin 60n + 1$, $n \in N$ set. \square

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REFERENCES

1. **Sirpinski W.** Sur les Décompositions de Nombres Rationnels en Fractions Primaires. // *Mathesis*, 1956, v. 65, p. 16–32.
2. **Palamá G.** Sudi & Congettura di Sierpínski Relative alla Possibilitáin Numeri Naturali della $5/k = 1/x + 1/y + 1/z$. // *Boll. Un. Mat. Ital.*, 1958, v. 3, p. 65–72.
3. **Vaughan R.C.** On a Problem of Erdős, Straus and Schinzel. // *Mathematika*, 1970, v. 17, p. 193–198.
4. **Viola C.** On the Diophantine Equations $\prod_0^k x_i - \sum_0^k x_i = n$ and $\sum_0^k \frac{1}{x_i} = \frac{a}{n}$. // *Acta Arith.*, 1972, v. 22, p. 339–352.
5. **Shen Z.** On the Diophantine Equations $\sum_0^k \frac{1}{x_i} = \frac{a}{n}$. // *Chinese Ann. Math. Ser. B*, 1986, v. 7, № 2, p. 213–220.
6. **Elsholtz C.** Sums of k Unit Fractions. // *Trans. Amer. Mat. Soc.*, 2001, p. 3209–3227.