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Mathematics

NONSELFADJOINT DEGENERATE DIFFERENTIAL OPERATOR EQUATIONS OF HIGHER ORDER ON INFINITE INTERVAL

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In the paper the Dirichlet problem for some class of degenerate nonselfadjoint differential operator equations of higher order on the infinite interval are considered.

Existence and uniqueness of the generalized solution of Dirichlet problem is proved, some analogue of the Keldysh theorem for the corresponding one-dimensional operator is established.

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1. Introduction. In this article we consider the degenerate differential operator equation

$$Lu \equiv (-1)^m \left(t^\alpha u^{(m)}\right)^{(m)} + (-1)^{m-1} A\left(t^{\alpha-1} u^{(m)}\right)^{(m-1)} + Pt^\beta u = f(t), \qquad (1)$$

where $m \in \mathbb{N}, t \in (1;+\infty), \alpha \neq 1,3,\ldots,2m-1, \beta \leq \alpha - 2m$, A and P are linear operators (unbounded in general) in the separable Hilbert space $H, f \in L_{2,-\beta}((1,+\infty),H)$, i.e. $\|f\|^2_{L_{2,-\beta}((1,+\infty),H)} = \int_1^{+\infty} t^{-\beta} \|f(t)\|^2_H \, dt < \infty$. The operators A and P have a common complete system of eigenfunctions $\{\varphi_k\}_{k\in\mathbb{N}}$, which form a Riesz basis in H, i.e. any $x \in H$ has the unique representation

$$x = \sum_{k=1}^{\infty} x_k(t)\varphi_k$$

and there are some positive constants c_1 and c_2 such that

$$c_1 \sum_{k=1}^{\infty} |x_k|^2 \leq \|x\|^2 \leq c_2 \sum_{k=1}^{\infty} |x_k|^2.$$

If $m = 1$, then the operator A is a multiplication operator $Au = au$, $a \in \mathbb{R}$, $a \neq 0$, $Pu = -u_{xx}$, $x \in (0,c)$, and so we obtain a degenerate elliptic operator in the rectangle

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$(0,b) \times (0,c)$. The dependence of the character of the boundary conditions with respect to t for $t = +\infty$ from the sign of the number a was first observed in [1] by M.Keldish. The case $m = 1$, $\beta = 0$, $0 \leq \alpha < 2$ was considered in [2,3] and the case $m = 2$, $\beta = 0, 0 \leq \alpha \leq 4$ in [4] (on the finite interval). Observe that weighted Sobolev spaces on the infinite inverval have been studied, for instance, by L.D. Kudryavtzev in [5]. This problem in the case $A = 0$ is considered by L. Tepoyan in [6]. It is important to observe that here the method suggested by A.A. Dezinhas been used (see [7]). Note that in [8] it was studied the question of the number of real roots for the characteristic polynomial in the one-dimensional case of (1) for $a = p = 0$.

First we define the weighted Sobolev spaces $\dot{W}^m_\alpha(1,+\infty)$. then recall the properties of the functions $u \in \dot{W}^m_\alpha(1,+\infty)$ and the embedding theorems. Then we consider the one-dimensional case of equation (1), i.e. the case when the operators A and P are multiplication operators with the constants a and p respectively. We prove the existence and uniqueness of the generalized solution as well as some analogue of the Keldysh theorem (see [1]). Then we investigate the differential-operator equation (1) and, using the general method of A.A. Dezin (see [7]), we prove the unique solvability of the generalized solution of Dirichlet problem for the equation (1).

2. Weighted Sobolev Spaces $\dot{W}^m_\alpha(1,+\infty)$. Let $\dot{C}^m[1,+\infty)$ be the functions $u \in C^m[1,+\infty)$, which satisfy the conditions

$$u^{(k)}(1) = u^{(k)}(+\infty) = 0, \quad k = 0,1,\ldots,m-1. \tag{2}$$

Define $\dot{W}^m_\alpha(1,+\infty)$ as the completion of $\dot{C}^m[1,+\infty)$ in the norm

$$\|u\|^2_{\dot{W}^m_\alpha(1,+\infty)} = \int_1^{+\infty} t^\alpha |u^{(m)}(t)|^2\, dt.$$

Denote the corresponding scalar product in $\dot{W}^m_\alpha(1,+\infty)$ by $\{u,v\}_\alpha = (t^\alpha u^{(m)}, v^{(m)})$, where $(\cdot,\cdot)$ stands for the scalar product in $L_2(1,+\infty)$. Observe that the functions $u \in \dot{W}^m_\alpha(1,+\infty)$ for every $t_0 \in [1,+\infty)$ have the finite values $u^{(k)}(t_0)$, $k = 0,1,\ldots,m-1$, and $u^{(k)}(1) = 0, k = 0,1,\ldots,m-1$ (see [9]). The proofs of the following two Propositions can be found in [6].

Proposition 1. For the functions $u \in \dot{W}^m_\alpha(1,+\infty), \alpha \neq 1,3,\ldots,2m-1$, we have the following estimates

$$|u^{(k)}(t)|^2 \leq C_1 t^{2m-2k-1-\alpha} \|u\|^2_{\dot{W}^m_\alpha(1,+\infty)}, \quad k = 0,1,\ldots,m-1. \tag{3}$$

It follows from Proposition 1, that in the case $\alpha > 2m-1$ (weak degeneracy) $u^{(j)}(+\infty) = 0$ for all $j = 0,1,\ldots,m-1$, while for $\alpha < 2m-1$ (strong degeneracy) not all the conditions $u^{(j)}(+\infty) = 0$ "maintain". For example, if $1 < \alpha < 3$, then after the completion only the condition $u^{(m-1)}(+\infty) = 0$ "maintains" and for $\alpha < 1$ all values $u^{(j)}(+\infty)$ for $j = 0,1,\ldots,m-1$ can be infinite in general.

Denote $L_{2,\beta}(1,+\infty) = \left\{f, \int_1^{+\infty} t^\beta |f(t)|^2\, dt < +\infty\right\}$. Observe that for $\alpha \leq \beta$ we have a continuous embedding $L_{2,\beta}(1,+\infty) \subset L_{2,\alpha}(1,+\infty)$.

Proposition 2. For $\beta \leq \alpha - 2m$ we have a continuous embedding

$$\dot{W}^m_\alpha(1,+\infty) \subset L_{2,\beta}(1,\infty), \tag{4}$$

which is compact for $\beta < \alpha - 2m$.

Note that the embedding (4) for $\beta = \alpha - 2m$ is not compact, and in the case of $\beta > \alpha - 2m$ it fails.

Let $d(m,\alpha) = 4^{-m}(\alpha-1)^2(\alpha-3)^2\cdots(\alpha-(2m-1))^2$. In Proposition 2 (see [6]), using inequality of Hardy (see [5]), it is proved that

$$\int_1^{+\infty} t^{\alpha}|u^{(m)}(t)|^2\,dt \geq d(m,\alpha)\int_1^{+\infty} t^{\alpha-2m}|u(t)|^2\,dt,$$

where the number $d(m,\alpha)$ is exact. It is easy to verify that for $\beta \leq \alpha - 2m$ also an upper bound holds, i.e. we have

$$\|u\|^2_{\dot{W}^m_\alpha(1,+\infty)} \geq d(m,\alpha)\|u\|^2_{L_{2,\beta}(1,+\infty)}. \tag{5}$$

3. One-Dimensional Nonselfadjoint Degenerate Differential Equations. Here we consider the equation

$$Su \equiv (-1)^m\big(t^{\alpha}u^{(m)}\big)^{(m)} + a(-1)^{m-1}\big(t^{\alpha-1}u^{(m)}\big)^{(m-1)} + pt^{\beta}u = f(t), \tag{6}$$

where $m \in \mathbb{N}$, $t \in (1,+\infty)$, $\alpha \neq 1,3,\ldots,2m-1$, $\beta \leq \alpha - 2m$, $a \neq 0$, a and p are real constants, $f \in L_{2,-\beta}(1,+\infty)$.

Define the generalized solution of Dirichlet problem for the equation (6).

Definition 1. A function $u \in \dot{W}^m_\alpha(1,+\infty)$ is called generalized solution of the equation (6), if the equality

$$\{u,v\}_\alpha + a\big(t^{\alpha-1}u^{(m)}, v^{(m-1)}\big) + p(t^{\beta}u,v) = (f,v). \tag{7}$$

holds for an arbitrary $v \in \dot{W}^m_\alpha(1,+\infty)$

Theorem 1. If

$$a(1-\alpha) > 0,\ \gamma = d(m,\alpha) + \frac{a}{2}(1-\alpha)d(m-1,\alpha-2) + p > 0, \tag{8}$$

then the generalized solution of the equation (6) exists and is unique for every $f \in L_{2,-\beta}(1,+\infty)$.

Proof.

Uniqueness. To prove the uniqueness of the generalized solution, we set $f = 0$ and $v = u$ in (7). Suppose that $\alpha < 1$. Then, integrating the equality (7) by parts, we obtain

$$\big(t^{\alpha-1}u^{(m)}, u^{(m-1)}\big) = \frac{1}{2}\big(t^{\alpha-1}|u^{(m-1)}(t)|^2\big)|_{t=+\infty} - \frac{\alpha-1}{2}\int_1^{+\infty} t^{\alpha-2}|u^{(m-1)}(t)|^2\,dt.$$

From the inequality (3) (for $k = m-1$) we conclude that $\big(t^{\alpha-1}|u^{(m-1)}(t)|^2\big)|_{t=+\infty}$ is finite. Now, using the Hardy inequality, we get

$$\int_0^b t^{\alpha-2}|u^{(m-1)}(t)|^2\,dt \geq d(m-1,\alpha-2)\int_0^b t^{\alpha-2m}|u(t)|^2\,dt.$$

Thus, using inequality (5), we obtain

$$0 = \{u,u\}_\alpha + a\big(t^{\alpha-1}u^{(m)}, u^{(m-1)}\big) + p(t^{\beta}u,u) \geq$$

$$\geq \frac{a}{2}\big(t^{\alpha-1}|u^{(m-1)}(t)|^2\big)|_{t=+\infty} + \gamma\int_1^{+\infty} t^{\beta}|u(t)|^2\,dt.$$

Now the uniqueness of the generalized solution immediately follows from the last inequality and the condition (8).

Existence. For the function $f \in L_{2,-\beta}(1,+\infty)$ we define a linear functional $l_f(v) = (f,v), v \in \dot{W}_\alpha^m(1,+\infty)$. From the continuity of the embedding (4), we get

$$|l_f(v)| \leq \|f\|_{L_{2,-\beta}(1,+\infty)}\|v\|_{L_{2,\beta}(1,+\infty)} \leq c\|f\|_{L_{2,-\beta}(1,+\infty)}\|v\|_{\dot{W}_\alpha^m(1,+\infty)}.$$

Hence, the linear functional $l_f(v)$ is bounded in $\dot{W}_\alpha^m(1,+\infty)$. From the Riesz theorem on the representation of the linear continuous functional it follows that the functional $l_f(v)$ can be represented in the form $l_f(v) = (f,v) = \{u^*, v\},\ u^* \in \dot{W}_\alpha^m(1,+\infty)$. The last two terms in the left hand-side of the equality (7) also can be considered as a continuous linear functional with respect to $u \in \dot{W}_\alpha^m(1,+\infty)$ and represented in the form $\{u, Kv\}_\alpha, Kv \in \dot{W}_\alpha^m(1,+\infty)$. Indeed, using the inequality (5), we can write

$$|a\big(t^{\alpha-1}u^{(m)}, v^{(m-1)}\big) + p(t^\beta u, v)| \leq c_1\|u\|_{\dot{W}_\alpha^m(1,+\infty)}\left\{\int_0^b t^{\alpha-2}|v^{(m-1)}(t)|^2\,dt\right\}^{1/2} +$$

$$+c_2\|u\|_{L_{2,\beta}(1,+\infty)}\|v\|_{L_{2,\beta}(1,+\infty)} \leq \frac{2c_1}{|\alpha-1|}\|u\|_{\dot{W}_\alpha^m(1,+\infty)}\|v\|_{\dot{W}_\alpha^m(1,+\infty)}+$$

$$+c_3\|u\|_{\dot{W}_\alpha^m(1,+\infty)}\|v\|_{\dot{W}_\alpha^m(1,+\infty)} = c\|u\|_{\dot{W}_\alpha^m(1,+\infty)}\|v\|_{\dot{W}_\alpha^m(1,+\infty)}.$$

Consequently, we conclude that for every $v \in \dot{W}_\alpha^m(1,+\infty)$ we have

$$\{u,(I+K)v\}_\alpha = \{u^*, v\}_\alpha. \tag{9}$$

Note that the image of the operator $I+K$ is dense in $\dot{W}_\alpha^m(1,+\infty)$. In fact, when we have some $u_0 \in \dot{W}_\alpha^m(1,+\infty)$ such that

$$\{u_0,(I+K)v\}_\alpha = 0$$

for any $v \in \dot{W}_\alpha^m(1,+\infty)$, then we obtain $u_0 = 0$, because we have proved uniqueness of the generalized solution for equation (7).

Assume that $0 < \sigma d(m,\alpha) \leq \gamma$. Then we have

$$\{u,(I+K)u\}_\alpha \geq \sigma\{u,u\}_\alpha + \Big(\big((1-\sigma)d(m,\alpha)+$$

$$+\frac{a}{2}(1-\alpha)d(m-1,\alpha-2)\big)+p\Big)\int_1^{+\infty} t^\beta|u(t)|^2\,dt =$$

$$= \sigma\{u,u\}_\alpha + \big(\gamma - \sigma d(m,\alpha)\big)\int_1^{+\infty} t^\beta|u(t)|^2\,dt \geq \sigma\{u,u\}_\alpha.$$

Finally we get

$$\{u,(I+K)u\}_\alpha \geq \sigma\{u,u\}_\alpha. \tag{10}$$

From (10) it follows that $(I+K)^{-1}$ is defined on $\dot{W}_\alpha^m(1,+\infty)$ and is bounded. Consequently the operators $I+K^*$ and $(I+K^*)^{-1} = ((I+K)^{-1})^*$ are defined. Hence, from (10) we obtain

$$u = (I+K^*)^{-1}u^*.$$

If $\alpha < 1$, then we use the equality $\big(t^{\alpha-1}|u^{(m-1)}(t)|^2\big)\,|_{t=0} = 0$ (see Proposition 1). □

Define an operator $S : D(S) \subset \dot{W}_\alpha^m(1,+\infty) \subset L_{2,\beta}(1,+\infty) \to L_{2,-\beta}(1,+\infty)$ were domain $D(S)$ is dense in $L_{2,\beta}(1,+\infty)$.

Definition 2. We say that the function $u \in \dot{W}^m_\alpha(1,+\infty)$ belongs to $D(S)$, if there exists $f \in L_{2,-\beta}(1,+\infty)$ such that the equality (7) is valid for every $v \in \dot{W}^m_\alpha(1,+\infty)$. Then we write $Su = f$.

The operator S acts from $L_{2,\beta}(1,+\infty)$ to $L_{2,-\beta}(1,+\infty)$. Define $\mathbb{S} := t^{-\beta}S$, $D(\mathbb{S}) = D(S)$. It is easy to check that $\mathbb{S}$ is an operator in the space $L_{2,\beta}(1,+\infty)$, because for $f \in L_{2,-\beta}(1,+\infty)$ we have $f_1 := t^{-\beta} f \in L_{2,\beta}(1,+\infty)$, $\|f\|_{L_{2,-\beta}(1,+\infty)} = \|f_1\|_{L_{2,\beta}(1,+\infty)}$.

Proposition 3. Under the assumptions of Theorem 1, the inverse operator $\mathbb{S}^{-1} : L_{2,\beta}(1,+\infty) \to L_{2,\beta}(1,+\infty)$ is continuous for $\beta \le \alpha - 2m$ and compact for $\beta < \alpha - 2m$.

Proof. First note that for $u \in D(\mathbb{S})$ we have

$$\|u\|_{L_{2,\beta}(1,+\infty)} \le c\|f\|_{L_{2,-\beta}(1,+\infty)} = c\|f_1\|_{L_{2,\beta}(1,+\infty)}.$$

Indeed, when we put $v = u$ in (7), use inequalities (5), (10) and using considerations of Theorem 1 proof, we obtain

$$\sigma d(m,\alpha)\|u\|^2_{L_{2,\beta}(1,+\infty)} \le \sigma d(m,\alpha)\|u\|^2_{\dot{W}^m_\alpha(1,+\infty)} \le \{(I+K)u,u\}_\alpha = (f,u) \le$$
$$\le \|f\|_{L_{2,-\beta}(1,+\infty)}\|u\|_{L_{2,\beta}(1,+\infty)} = \|f_1\|_{L_{2,\beta}(1,+\infty)}\|u\|_{L_{2,\beta}(1,+\infty)}.$$

As a result we get

$$\|\mathbb{S}^{-1} f_1\|_{L_{2,\beta}(1,+\infty)} \le c\|f_1\|_{L_{2,\beta}(1,+\infty)}, \tag{11}$$

and so the continuity of $\mathbb{S}^{-1}$ for $\beta \le \alpha - 2m$ is proved.

For the proof of the compactness of $\mathbb{S}^{-1}$ for $\beta < \alpha - 2m$ we apply the compactness of the embedding (4). □

Let us consider the following equation:

$$Tv \equiv (-1)^m \big(t^\alpha v^{(m)}\big)^{(m)} - a(-1)^{m-1}\big(t^{\alpha-1} v^{(m-1)}\big)^{(m)} + pt^\beta v = g(t), \tag{12}$$

where $\alpha \ge 0, \alpha \ne 1,3,\ldots,2m-1$, $\beta \le \alpha - 2m$, $g \in L_{2,-\beta}(1,+\infty)$, $a \ne 0$, a and p are real constants.

Definition 3. We say, that the function $v \in L_{2,\beta}(1,+\infty)$ is a generalized solution of equation (12), if for every $u \in D(S)$ we have the following equality

$$(Su, v) = (u, g). \tag{13}$$

Let $g_1 := t^{-\beta} g$. Define an operator $\mathbb{T} : L_{2,\beta}(1,+\infty) \to L_{2,\beta}(1,+\infty)$, $\mathbb{T} := t^{-\beta} T$. Observe that the operator $\mathbb{T}$ is the adjoint of the operator $\mathbb{S}$ in $L_{2,\beta}(1,+\infty)$, i.e. $\mathbb{T} = \mathbb{S}^*$.

Theorem 2. Under the assumptions of Theorem 1 the generalized solution of equation (12) exists and is unique for every $g \in L_{2,-\beta}(1,+\infty)$. The inverse operator $\mathbb{T}^{-1} : L_{2,\beta}(1,+\infty) \to L_{2,\beta}(1,+\infty)$ is continuous for $\beta \ge \alpha - 2m$ and compact for $\beta > \alpha - 2m$.

Proof. From the solvability of equation $\mathbb{S}u = f_1$ for any $f_1 \in L_{2,-\beta}(1,+\infty)$ (see Theorem 1) follows uniqueness of the solution of equation (12). Solvability of equation (12) for any $g \in L_{2,-\beta}(1,+\infty)$ follows from the existence of the bounded inverse operator $\mathbb{S}^{-1}$ (see [7]). Since we have $(\mathbb{S}^*)^{-1} = (\mathbb{S}^{-1})^*$, the boundedness and compactness of the operator $\mathbb{S}^{-1}$ imply the boundedness and compactness of the operator $\mathbb{T}^{-1}$ for $\beta \le \alpha - 2m$ and $\beta < \alpha - 2m$ respectively (see Proposition 3). □

R e m a r k . For $\alpha > 1$ and for every generalized solution v of equation (12), we have

$$\left(t^{\alpha-1}|u^{(m-1)}(t)|^2\right)\Big|_{t=0} = 0. \tag{14}$$

In fact, replacing g by Tv in equality (13), integrating by parts the second term and using equality (7), we obtain (14). Note also that for equation (6) the left-hand side of (14) is only bounded. This is some analogue of the Keldish theorem (see [1]).

4. Nonselfadjoint Degenerate Differential Operator Equations. Now we consider the operator equation

$$Lu \equiv (-1)^m\left(t^\alpha u^{(m)}\right)^{(m)} + (-1)^{m-1}A\left(t^{\alpha-1}u^{(m)}\right)^{(m-1)} + Pt^\beta u = f(t), \tag{15}$$

where $m \in \mathbb{N}$, $t \in (1;+\infty)$, $\alpha \neq 1,3,\ldots,2m-1$, $\beta \leq \alpha - 2m$, A and P are linear operators in the separable Hilbert space H, $f \in L_{2,-\beta}((1,+\infty),H)$, the operators A and P have common complete system of eigenfunctions $\{\varphi_k\}_{k\in\mathbb{N}}$, which forms a Riesz basis in H. By the assumption the operators A and P have common complete system of eigenfunctions $\{\varphi_k\}_{k=1}^\infty$, $A\varphi_k = a_k\varphi_k, P\varphi_k = p_k\varphi_k, k \in \mathbb{N}$, which forms a Riesz basis in H, hence, we can write

$$u(t) = \sum_{k=1}^\infty u_k(t)\varphi_k, \quad f(t) = \sum_{k=1}^\infty f_k(t)\varphi_k. \tag{16}$$

Thus, the operator equation (15) can be decomposed into an infinite chain of ordinary differential equations

$$L_k u_k \equiv (-1)^m\left(t^\alpha u_k^{(m)}\right)^{(m)} + a_k\left(t^{\alpha-1}u_k^{(m)}\right)^{(m-1)} + p_k t^\beta u_k = f_k(t),\ k \in \mathbb{N}. \tag{17}$$

It follows from the condition $f \in L_{2,-\beta}((1,+\infty),H)$ that $f_k \in L_{2,-\beta}(1,+\infty), k \in \mathbb{N}$. For one-dimensional equations (17) we can define the generalized solutions $u_k(t)$.

D e f i n i t i o n 4 . A function $u \in L_{2,\beta}((1,+\infty),H)$ is called a generalized solution of the operator equation (16), if it has representation

$$u(t) = \sum_{k=1}^\infty u_k(t)\varphi_k,$$

where the functions $u_k(t), k \in \mathbb{N}$, are the generalized solutions of the one-dimensional equations (17).

For the proof of the next Proposition see [7].

P r o p o s i t i o n 4. The operator equation (15) is uniquely solvable for every $f \in L_{2,-\beta}((1,+\infty),H)$, if and only if the equations (17) are uniquely solvable for every $f_k \in L_{2,-\beta}(1,+\infty), k \in \mathbb{N}$ and uniformly with respect to $k \in \mathbb{N}$, we have

$$\|u_k\|_{L_{2,\beta}(1,+\infty)} \leq c\|f_k\|_{L_{2,-\beta}(1,+\infty)}. \tag{18}$$

Theorem 2 shows us that a sufficient condition for the uniquely solvability of the equations (17) are the conditions (here we assume that $a_k \neq 0$, a_k and p_k are real for $k \in \mathbb{N}$)

$$a_k(1-\alpha) > 0, \gamma_k = d(m,\alpha) + \frac{a_k}{2}(1-\alpha)d(m-1,\alpha-2) + p_k > \varepsilon > 0,\ k \in \mathbb{N}. \tag{19}$$

T h e o r e m 3 . Let the condition (19) be fulfilled. Then the operator equation (15) has a unique generalized solution for every $f \in L_{2,-\beta}((1,+\infty),H)$.

Proof. It is easy to verify that under the conditions (19) uniformly with respect to $k \in \mathbb{N}$ we have the inequalities (18). Thus, using Proposition 4, we conclude that generalized solution exists and is unique. Since the system $\{\varphi_k\}_{k=1}^{\infty}$ forms a Riesz basis in H, we can write

$$\begin{aligned} \|u\|^2_{L_{2,\beta}((1,+\infty),H)} = \int_0^b t^\beta \|u(t)\|^2_H \, dt \le c_1 \int_0^b t^\beta \sum_{k=1}^{\infty} |u_k(t)|^2 \, dt \le \\ \le c_2 \sum_{k=1}^{\infty} \|f_k\|^2_{L_{2,-\beta}(1,+\infty)} \le C\|f\|_{L_{2,-\beta}((1,+\infty),H)}. \end{aligned} \tag{20}$$

It follows from inequality (20) that the inverse operator

$$L^{-1} : L_{2,-\beta}((1,+\infty),H) \to L_{2,\beta}((1,+\infty),H) \quad \text{for} \quad \beta \le \alpha - 2m$$

is bounded. In contrast to the one-dimensional case (see Proposition 3), here for $\beta < \alpha - 2m$ the operator L^{-1} in general is not compact (it will be compact only when the space H is finite-dimensional). □

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