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FORCED CONVECTION IN NEMATICS LIQUID CRYSTALS IN THE ABSENCE OF REORIENTATION

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The problem of forced convection in a cell of plane-parallel layer of nematic liquid crystal, both the boundaries of which are free and isothermal, has been discussed. However much artificial seem the boundary conditions first proposed by Rayleigh, these permit an obtaining of simple exact solution of the boundary value problem, by means of which some most important features of the problem are elucidated. In particular it proved possible to excite convective motions in the absence of reorientation of the liquid crystal director.

***Keywords*:** nematic liquid crystals, hydrodynamics, director reorientation.

Introduction. Last two decades the problem of convection in the layer of liquid heated from beneath [1–3] attracted an intent attention connected with the use of high power lasers for materials processing (the laser technology [4, 5]). The observed effects are widely known as the convective Rayleigh–Benar and Marangony [6–13] motions. The role played by convection in technological processes connected with melting, welding, cutting and doping of metals is essential. Due to convection, as a result of laser alloying, rapid mixing of dopant material with support medium occurs. The same effect can be used to explain cathode spot's motions [14], as well as form variety of some material's arc-welded joints [15]. The investigation of stability of thermal convection in nematic liquid crystals (NLC) is of great interest because of their unique properties. So, the instability thresholds in NLC notably differ from that for isotropic liquids having the same physical parameters [16–19]. In contrast to the isotropic liquid, in the instability mechanism of NLC the behavior of director, i.e., the unit vector in the direction of preferred orientation of molecules, is predominant. An important implication of this fact is that the stationary convection is observed in the homeotropically aligned (the molecules are oriented perpendicular to the substrates of the cell) NLC specimen in case of its downright heating [20–22]. As was shown recently [23], this kind of layer is always stable both in the case of rigid boundaries and with one free surface. And the dynamics of NLC droplet spreading has been studied in the frameworks of long-wave approximation [24].

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Based on systematic experimental investigations by Benard [6, 7], Rayleigh solved [8] the stability problem of layer equilibrium subject to free boundary conditions and ipso facto laid the foundations of the theory of convective stability. Since then the horizontal layer of liquid now as heretofore is the central object of convective stability research mainly due to the fact that this geometry is readily realizable in experiments and is convenient for thermal and optical measurements. A plane horizontal layer is also of great interest in connection with applications of the theory of convective stability in the metrology, geophysics and astrophysics.

In the present work we deal with the problem of excitation of convective motions in NLC with two free plane isothermal surfaces, that is the forced analogue of the so-called Rayleigh problem. Since this problem can be solved analytically and exactly, it is possible to revel some new qualitatively effects as, e.g., to induce convective motions under certain conditions without the variation of NLC orientation.

The Linearized Equations and Boundary Conditions. Let consider a horizontal layer $(0 \le z \le L)$ of a homeotropically (non-perturbed director $\mathbf{n}_0 = \mathbf{e}_z$) or planar $(\mathbf{n}_0 = \mathbf{e}_x)$ oriented NLC with free surfaces, under gravity $\mathbf{g} = -g\mathbf{e}_z$, that is absorbing the incident light. The temperature T_0 on the boundaries of layer is fixed and there is now temperature gradient in a non-perturbated state. Let two coherent flat light beams (e.g. laser beams) be incident on the layer and create a space periodical pattern of intensity distribution propotional to $|E|^2$. In the presence of weak optical absorption a periodical heat emission in the bulk of the form

$$Q(x) = \frac{L\chi cn}{8\pi}\left|E(x)\right|^2 = \frac{L\chi cn}{8\pi}\left[\left|E_1\right|^2 + \left|E_2\right|^2 + E_1 E_2^* e^{(ikx)} + c.c.\right], \tag{1}$$

where $k = 2\pi\,|\sin\gamma_1 - \sin\gamma_2|/\lambda$ is the wave vector of the inhomogeneous part of heat production; γ_1 and γ_2 are the incidence angles of light wave; λ is the wavelength of light wave in vacuum; χ is the light absorption coefficient $(\chi L \ll 1)$; c is the speed of light in vacuum and n is the average refractive index of the NLC.

The bulk heat source of the system is assumed to be homogeneous relative to the y coordinate, so, that $\partial/\partial y = 0$ everywhere and $v_y = 0$. Here $\mathbf{v}$ is velocity of hydrodynamic motions. Let denote the angle between the nematic director and the z-axis as $\varphi_i + \varphi$. Here φ_i is the angle of director in the non-perturbed state ($\varphi_i = 0$ for initial homeotropic orientation and $\varphi_i = \pi/2$ for a planar orientation of nematic), and φ is the director perturbation $\varphi\big|_{z=0} = 0$.

In the absence of light field the equilibrium state of NLC corresponds to solutions of the form

$$\begin{aligned} &v_0 = 0, \quad T = T_0 = \text{const}, \quad \rho = \rho_0 = \text{const}, \\ &\varphi_0 = 0, \quad p = p_0(z=0) - \rho_0 g z. \end{aligned} \tag{1*}$$

Here p is the hydrodynamic pressure and ρ is the density of NLC. At illumination of liquid layer the system is perturbed, and the stationary, linearized equations for perturbed values of θ, $\delta\rho = \rho - \rho_0 = -\beta\rho_0\theta$ (where β is the coefficient of volume expansion), $\delta p = p - p_0$, v_x, v_z and φ are introduced in the following form [25]

$$\rho\frac{\partial v_x}{\partial t} = -\frac{\partial \delta p}{\partial x} + \eta_x \Delta v_x + \alpha_x \frac{\partial^2 v_x}{\partial x^2} + \alpha_a \frac{\partial^2 \varphi}{\partial z \partial t}, \tag{2}$$

$$\rho\frac{\partial v_z}{\partial t} = -\frac{\partial \delta p}{\partial z} + \eta_z \Delta v_z + \alpha_z \frac{\partial^2 v_z}{\partial z^2} + \alpha_b \frac{\partial^2 \varphi}{\partial x \partial t} + \beta\rho_0 g\theta, \tag{3}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0, \quad \frac{\partial \theta}{\partial t} - r_x \frac{\partial^2 \theta}{\partial x^2} - r_z \frac{\partial^2 \theta}{\partial z^2} = \frac{Q}{\rho_0 c_p L}, \tag{4}$$

$$(\alpha_a - \alpha_b)\frac{\partial \varphi}{\partial t} = K_x \frac{\partial^2 \varphi}{\partial x^2} + K_z \frac{\partial^2 \varphi}{\partial z^2} - \alpha_a \frac{\partial v_x}{\partial z} - \alpha_b \frac{\partial v_z}{\partial x}. \tag{5}$$

The following notation have been made above: for a cell with planar initial orientation, $\eta_z = \eta_2$, $\eta_x = \eta_1$, $\alpha_x = \alpha_1 + \alpha_5$, $\alpha_z = -\alpha_5$, $r_x = r_\parallel$, $r_z = r_\perp$, $K_x = K_3$, $K_z = K_1$, $\alpha_a = \alpha_3$, $\alpha_b = \alpha_2$; and for a cell with homeotropically oriented nematic, $\eta_z = \eta_1$, $\eta_x = \eta_2$, $\alpha_x = -\alpha_5$, $\alpha_z = \alpha_1 + \alpha_5$, $r_x = r_\perp$, $r_z = r_\parallel$, $K_x = K_1$, $K_z = K_3$, $\alpha_a = \alpha_2$, $\alpha_b = \alpha_3$. Here K_1 and K_3 are the coefficients of Frank elasticity, $r_\parallel$ and $r_\perp$ are the parallel and perpendicular components of the thermal conductivity; $\eta_1 = 0.5(\alpha_3 + \alpha_4 + \alpha_6)$ and $\eta_2 = 0.5(-\alpha_2 + \alpha_4 + \alpha_5)$ are the viscosity coefficients of NLC. α_i are the Leslie coefficients; ρ_0 is the non-perturbated density; b is the Biot heat removal coefficient.

Now let formulate the boundary conditions. Assuming after Rayleigh [8] that the layer has free surfaces, then the tangential stresses at the boundaries disappear. Then, these boundaries are supposed to be plane. That is the arising convective disturbances are assumed not to bring to warping of boundaries. As it was previously mentioned, the temperatures on boundaries are fixed. As the temperature at the boundaries was indicated above to be fixed, no temperature perturbations on layer surfaces shall occur. As the director is supposed to be made fast, its deviations at boundaries also vanish. Thus, we have a set of boundary conditions

$$\theta(x, z=0) = \theta(x, z=L) = 0, \qquad v_z(x, z=0) = v_z(x, z=L) = 0, \tag{6}$$

$$\frac{\partial v_x}{\partial z}(x, z=0) = \frac{\partial v_x}{\partial z}(x, z=L) = 0, \qquad \varphi(x, z=0) = \varphi(x, z=L) = 0. \tag{7}$$

To replace the boundary conditions for v_x that are obtained subject to the requirement of absence of tangential stresses at these boundaries, by those for v_z, we take advantage of the continuity equation. After differentiation of the first equation of set (4) with respect to z under the boundary condition for velocity, we obtain $\partial^2 v_z / \partial^2 z = 0$ in case of $z = 0,\ L$.

Analytical Solution of the Forced Convection Equations. Since coefficients in Eqs. (2)–(5) and boundary conditions (6), (7) are independent of time and horizontal coordinate x, there exist solutions that are exponential in time and periodic in (x, z) plane.

The spatially periodic term $E_i(x)E_j^*(x) = a_{ij}e^{ikx}$ in $E_iE_j^*$ tensor characterizing the distribution of light wave intensity, causes the following stationary perturbations

$$v_z(x,z) = V_z e^{ikx}\sin\frac{\pi z}{L}, \qquad \theta(x,z) = \Theta e^{ikx}\sin\frac{\pi z}{L}, \tag{8}$$

$$v_x(x,z) = V_x e^{ikx}\cos\frac{\pi z}{L}, \qquad \varphi(x,z) = \Phi e^{ikx}\sin\frac{\pi z}{L}, \tag{9}$$

satisfying the boundary conditions on free boundaries $z = 0, L$. For amplitudes of these perturbations, we obtain

$$V_x = ikk_0\beta\rho g\Theta[\eta_x k_0^4 + (\eta_x + \eta_z + \alpha_x + \alpha_z)k_0^2k^2 + \eta_z k^4]^{-1},\ V_z = -ik_0^{-1}kV_x, \tag{10}$$

$$\Theta = \frac{\chi cn}{8\pi\rho c_p}(r_x k^2 + r_z k_0^2)^{-1}E_1E_2^*, \quad \Phi = V_x k_0^{-1}(\alpha_a k_0^2 - \alpha_b k^2)(K_z k_0^2 + K_x k^2)^{-1} \tag{11}$$

here $k_0 = \pi/L$. It is seen from second expression of Eq. (11) that when $k = k_0\sqrt{\alpha_a/\alpha_b}$ the viscous moments acting on director compensate each other and, as a result, the reorientation is suppressed although the convection is continued. If we denote $k = 2\pi/\Lambda$, where Λ is the period of interference pattern, then the condition of the absence of reorientation will take on the form

$$L_{cr} = \frac{\Lambda}{2}\sqrt{\alpha_a/\alpha_b}. \tag{12}$$

Based on the example of NLC MBBA, we obtain for the initial homeotropic orientation $L_{cr} = \frac{\Lambda}{2}\sqrt{\alpha_a/\alpha_b} = 4.08\Lambda$ and for the initial planar orientation $L_{cr} = \frac{\Lambda}{2}\sqrt{\alpha_3/\alpha_2} = 0.06\Lambda$. Note, that this effect is feasible only in case of forced convection, because only the laser excitation gives an opportunity of ensuring convection with the desired period.

When $k << k_0(\alpha_a/\alpha_b)^{1/2}$ the response of system decreases with linear reduction of k according to $\Phi \sim (\chi L)kk_0^{-5}$ law. The deviation builds up with k increasing from zero and reaches its first maximum for $k \approx 0.05k_0$ (for planar MBBA). Φ changes its sign in $k = k_0(\alpha_a/\alpha_b)^{1/2} \approx 0.1k_0$ point (for planar MBBA), i.e., φ changes its phase by π, and at $k \approx 0.6k_0$ its amplitude reaches the second maximum. The latter corresponds to the period of regular Benard instability pattern in case of constant vertical temperature gradient. For $k >> 0.6k_0$ the reorientation sharply reduces increasing k according to $\Phi \sim (\chi L)k^{-5}k_0$ law.

Conclusion. Thus, in the present work the Rayleigh problem of excitability of regular convective motions in liquid crystals with two free boundaries exposed to radiation with space periodical intensity distribution has been solved. The laser induced hydrodynamic effects are of interest due to the fact that they permit an induction of regular roller structures, roller structures with dislocations, annular roller structures, as well as some assemblies of cells with hexagonal, cubic and other structures, both the perfect ones and containing various dislocations. It is achieved by interference of a plane light wave with another wave having the front dislocation, as well as the interference of two waves with complicated wave fronts. A ring-type roller structure results at an interference of a plane wave with the one having a conical front. The cells with hexagonal, cubical and other structures are produced at the interference of three, four and greater number of waves, the defects being purposefully introduced in these structures. There are reasons to expect that at slight excess over the threshold the light interference pattern may "impose" its own period and phase upon the steady-state pattern of rollers or cells.

Besides, there is an opportunity for smooth control of the parameters of forced convection pattern. In our opinion, the possibility of controlling the spatial structures is of high interest not only for LC, but also for any unstable systems with finite wave vector $1cm^{-1} \le k \le 10^5 cm^{-1}$ of the grating in the transverse plane. As follows from the foregoing analysis, the laser radiation is a convenient instrument for studying the convective motions.

As was also shown in the present paper in the case of large and small values of the intensity of light and the ratios of the period of interference pattern to the cell thickness, the two-dimensional roller structure proves unstable with gradual transition to a chaotic state. As the amplification of white noise in convenient experiments on turbulence masks the qualitative difference of turbulent flows in various regions of extrinsic parameters, the controllable excitation are of special importance. We are show that the experiment described in the present paper may serve as a model for treatment of the turbulence problem in the above sense.

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