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TRIDIMENSIONAL WAVES AT THE INTERFACE OF TWO ELASTIC MEDIA ON CONTACT WITHOUT FRICTION

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The question of presence of Stoneley's surface waves in three-dimensional statement is considered. Conditions are given at the interface of two half-space corresponding to the contact of two half-space without friction. The investigated problems are simplified by the introduction of potential function with analogue of the plane deformation problems. The characteristic equation is obtained concerning the phase speed of the surface wave, for which the special cases are considered.

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Introduction. The problem of surface waves propagation at the interface of two medium in full contact condition is known as the Stoneley problem [1]. The investigations of Stoneley's surface waves have been done in numerous works [2–13]. In the present paper is we consider the existence problem question of the presence of Stoneley's surface waves in three-dimensional case.

1. Suppose we have two elastic half-spaces separated by plane $y = 0$ (in the coordinate system $0xyz$). The half-spaces are characterized by coefficients of Lame $\lambda,\ \mu$ and by density ρ. The physical-mechanical characteristics of the half-space holding the domain $y \in (0,+\infty),\ x,z \in (-\infty,+\infty)$ will be denoted by index one as the physical-mechanical characteristics of the half-space $y \in (-\infty,0),\ x,z \in (-\infty,+\infty)$ by index two. The elastic waves propagation equations in the isotropic medium with the displacements $\vec{u}(u,v,w)$ has the form [14]:

$$c_{ts}^2 \Delta \vec{u}^{(s)} + (c_{ls}^2 - c_{ts}^2)\operatorname{grad}\operatorname{div}\vec{u}^{(s)} = \frac{\partial^2 \vec{u}^{(s)}}{\partial t^2}, \tag{1}$$

where c_l and c_t are the speed of longitudinal and transversal waves propagation corresponding to $s = 1$ and 2 respectively.

At the interface of two isotropic half-space we will admit the following boundary conditions:

$$v^{(1)} = v^{(2)},\ \sigma_{22}^{(1)} = \sigma_{22}^{(2)},\ \sigma_{12}^{(1)} = 0,\ \sigma_{23}^{(s)} = 0. \tag{2}$$

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In (2) σ_{ij} are the stress component of the tensor. The boundary conditions (2) corresponds to the contact of two half-spaces without friction.

We consider the following problem: find a solution of the $3D$ Eq. (1), which satisfies to the boundary conditions (2) and the attenuation condition as $y \to \pm\infty$.

2. To solve the problem of the surface waves propagation, by analogy with the problems of plane deformation, the potential functions $\varphi(x,y,z,t)$ and $\psi(y,z,t)$ are introduced [14]:

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \ w = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{3}$$

Using (1) and (3) and the attenuation conditions

$$\lim_{y\to\pm\infty} \vec{u}^{(s)} = 0, \ \lim_{y\to\pm\infty} \varphi^{(s)} = 0, \ \lim_{y\to\pm\infty} \psi^{(s)} = 0, \tag{4}$$

the displacements u, v, w and the potential functions φ and ψ are defined by [10]:

$$\begin{aligned}
&u^{(1)} = -i\left[A_1 k_1 e^{-\nu_{11}y} + (B_1 k_1 + C_1 k_3)e^{-\nu_{21}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&v^{(1)} = -k\left[A_1 \nu_{11} e^{-\nu_{11}y} + B_1 \nu_{21}^{-1} e^{-\nu_{21}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&w^{(1)} = -i\left[A_1 k_3 e^{-\nu_{11}y} + (B_1 k_3 - C_1 k_1)e^{-\nu_{21}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&\varphi^{(1)} = \left[A_1 e^{-\nu_{11}y} + B_1 e^{-\nu_{21}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&\psi^{(1)} = C_1 e^{-\nu_{21}y}\exp(i(ckt - k_1 x - k_3 z));
\end{aligned} \tag{5}$$

$$\begin{aligned}
&u^{(2)} = -i\left[A_2 k_1 e^{-\nu_{12}y} + (B_2 k_1 + C_2 k_3)e^{-\nu_{22}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&v^{(2)} = -k\left[A_2 \nu_{12} e^{-\nu_{12}y} + B_2 \nu_{22}^{-1} e^{-\nu_{22}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&w^{(2)} = -i\left[A_2 k_3 e^{-\nu_{12}y} + (B_2 k_3 - C_2 k_1)e^{-\nu_{22}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&\varphi^{(2)} = \left[A_2 e^{-\nu_{12}y} + B_2 e^{-\nu_{22}y}\right]\exp(i(ckt - k_1 x - k_3 z)),\\
&\psi^{(2)} = C_2 e^{-\nu_{22}y}\exp(i(ckt - k_1 x - k_3 z)),
\end{aligned} \tag{6}$$

where
$\nu_{11}^2 = k_1^2(1+\xi^2)(1-\eta_1\theta_1), \nu_{21}^2 = k_1^2(1+\xi^2)(1-\eta_1), \ k^2 = k_1^2(1+\xi^2),$ $\nu_{12}^2 = k_1^2(1+\xi^2)(1-\eta_2\theta_2), \ \nu_{22}^2 = k_1^2(1+\xi^2)(1-\eta_2), \ \xi = \dfrac{k_3}{k_1},$ $\eta_1 = \dfrac{c^2}{c_{t1}^2}, \ \eta_2 = \dfrac{c^2}{c_{t2}^2}, \theta_1 = \dfrac{c_{t1}^2}{c_{l1}^2}, \theta_2 = \dfrac{c_{t2}^2}{c_{l2}^2},$ k_1 and k_2 are the wave numbers, $c < \min(c_{t1}, \ c_{t2})$ is the unknown phase speed of the space waves, $A_s, \ B_s$ and C_s are unknown constants of integration.

According to Hooke's rule and notations (3), the boundary conditions (2) can be written as:

$$\begin{aligned}
&v^{(1)} = v^{(2)}, \ (c_{l1}^2 - 2c_{t1}^2)\left(\frac{\partial^2\varphi^{(1)}}{\partial x^2} + \frac{\partial^2\psi^{(1)}}{\partial x \partial z}\right) + c_{l1}^2\frac{\partial v^{(1)}}{\partial y} =\\
&= \alpha(c_{l2}^2 - 2c_{t2}^2)\left(\frac{\partial^2\varphi^{(2)}}{\partial x^2} + \frac{\partial^2\psi^{(2)}}{\partial x \partial z}\right) + \alpha c_{l2}^2\frac{\partial v^{(2)}}{\partial y}, \ \ \alpha = \frac{\rho_2}{\rho_1},\\
&\frac{\partial v^{(s)}}{\partial x} + \left(\frac{\partial^2\varphi^{(s)}}{\partial x \partial y} + \frac{\partial^2\psi^{(s)}}{\partial y \partial z}\right) = 0, \ \ \frac{\partial v^{(s)}}{\partial z} + \left(\frac{\partial^2\varphi^{(s)}}{\partial y \partial z} + \frac{\partial^2\psi^{(s)}}{\partial x \partial y}\right) = 0.
\end{aligned} \tag{7}$$

Substituting (5) and (6) into the boundary conditions (7), we get a system of homogeneous algebraic equations with respect to the coefficients $A_s, \ B_s$ and C_s. From the condition of compatibility of the system we obtain the following equation, where the unknown is the speed phase of the surface wave:

$$
\begin{aligned}
&(1-\eta_2\theta_2)^{1/2}\Big[(2+\xi^2-2\theta_1)\Big(2\big((1-\gamma\eta_2)(1-\gamma\theta_1\eta_2)\big)^{1/2}-(2-\gamma\eta_2)\Big)+\\
&+(2-\gamma\eta_2)(1+\xi^2)\gamma\eta_2\theta_1\Big]-\alpha\gamma\theta_1(1-\gamma\eta_2\theta_1)^{1/2}\big[2\gamma\theta_2^{-1}(\xi^2+2\theta_2)\times\\
&\times\big((1-\eta_2)(1-\theta_2\eta_2)\big)^{1/2}-(2-\eta_2)+\xi^2(\eta_2-\theta_2^{-1})\big]=0,\ \ \gamma=\frac{c_{t2}^2}{c_{t1}^2}.
\end{aligned}
\tag{8}
$$

3. For the Eq. (8) the following particular cases are considered:

a) Let $\xi=0, \rho_1=0$. Then from (8) we get

$$
S(\eta_2)\equiv 4\gamma\Big((1-\eta_2)(1-\theta_2\eta_2)\Big)^{1/2}-(2-\eta_2)=0. \tag{9}
$$

The Eq. (9) is the characteristic equation for the determination of the dimensionless phase speed $\left(\eta_2=\dfrac{c^2}{c_{t2}^2}\right)$ for the surface wave of the half-space (plane problem) with the following boundary conditions: $v=0,\ \sigma_{22}=0,\ \sigma_{21}=0$ at $y=0$.

In Figs. 1 and 2 the graphs of the $S(\eta_2)$ functions for different values of γ are displayed.

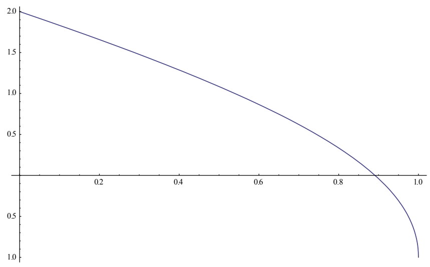

Fig. 1. $\gamma=1, \eta_2\approx 0.8911$.

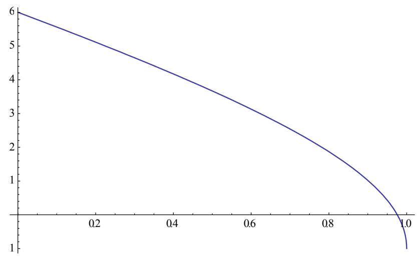

Fig. 2. $\gamma=2, \eta_2\approx 0.9758$.

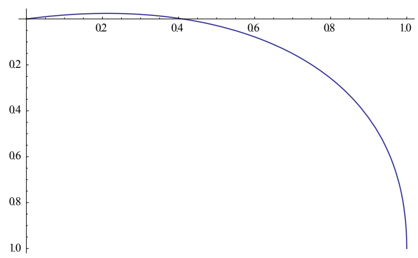

Fig. 3. $\theta_1=0.33,\ \eta_1\approx 0.4115$.

b) Let $\xi=0, \rho_2=0$. The Eq. (8) takes the following form:

$$
S(\eta_1)\equiv 2(1-\theta_1)\Big(2\big((1-\eta_1)(1-\theta_1\eta_1)\big)^{1/2}-2+\eta_1\Big)+\eta_1\theta_1(2-\eta_1)=0. \tag{10}
$$

The Eq. (10) is the characteristic equation for the determination of the dimensionless phase speed $\left(\eta_1=\dfrac{c^2}{c_{t1}^2}\right)$ of a surface wave (plane problem).

Fig. 3 shows the graph of function $S(\eta_1)$ for $\theta_1 = 0.33$ ($\eta_1 \approx 0{,}4115$).

c) Let $c_{t1} = c_{t2} = c_t, c_{l1} = c_{l2} = c_l, \theta_1 = \theta_2 = \theta, \eta_1 = \eta_2 = \eta, \gamma = 1$. The Eq. (8) reduces to the form:

$$\begin{aligned} S(\eta) &\equiv (2-\eta)\big(\eta\theta - 2 + 2\theta + \alpha\theta + \xi^2(\eta\theta - 1)\big) - \alpha\xi^2(\eta\theta - 1) + \\ &+ 2\big((1-\eta)(1-\theta\eta)\big)^{1/2}\big(2 - 2\theta - 2\alpha\theta + \xi^2(1-\alpha)\big) = 0. \end{aligned} \tag{11}$$

The values of k_{eff} are shown in Table:

ξ	0	0.5	0.8	1	2	3	4	5
η	0.9965	0.9430	0.9109	0.8971	0.5046	0.8473	0.8435	0.8419

The Table displays the numerical results, which were calculated according to the Eq. (11) for the parameter $\eta = \dfrac{c^2}{c_t^2}$, characterizing the square of the dimensionless phase speed of the surface wave, depending on the parameters $\alpha = \dfrac{\rho_2}{\rho_1} = 4$ and $\xi = \dfrac{k_3}{k_1}$ in the case $\theta = \dfrac{c_t^2}{c_l^2} = \dfrac{1}{3}$.

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R E F E R E N C E S

1. **Stoneley R.** Elastic Waves on the Surface of Separition of Two Solids. // Proccedings of the Royal Society, S.A., 1924, v. 106, № A73, p. 416–428.
2. **Gogoladze V.G.** Reflection and Refraction of Elastic Waves. The General Theory of Boundary Waves. // Proceedings of the Seismological Institute of the Academy of Siences of the USSR, 1947, № 125, p. 1–48 (in Russian).
3. **Cagniard L.** Reflection and Refraction of Progressive Seismic Waves. NY: Mc-Graw-Hill, 1962, 281 p.
4. **Achenbach J.D., Epstein H.J.** Dynamic Interaction of a Layer and a Half-Space. // Proc. Amer. Soc. Civil Eng. Mech. J. Eng. Mech., 1967, v. 93, № 5, p. 27–42.
5. **Grichenko V.T., Meleshko V.V.** Harmonic Waves in Elastic Bodies. Kiev: Naukova Dumka, 1981, 283 p. (in Russian).
6. **MiklowitžJ.** The Theory of Elastic Waves and Waveguides. Amsterdam–NY–Oxford: North-Holland, 1984, 618 p.
7. **Belubekyan M.V.** On the Stounly Waves Existence Condition under the Slider Contact. // Proceedings of National Academy of Sciences of Armenia. Mechanics, 1990, v. 43, № 4, p. 52–56 (in Russian).
8. **Belubekyan M.V.** Surface Waves in Elastic Media. In Collection of Articles: The Problems of Mechanics of Deformable Solids. Yer: Inst. of Mechanics of NAS RA, 1997, p. 79–100.
9. **Ambartsumian S.A., Belubekyan M.V., Kazarian K.B.** Magnetoelastic Surface Waves at the Interface of Conductive Media. // Mezhvuz. Sbornik Nauch. Tr. Yer.: YSU Press, 1986, vip. 4, p. 5–10 (in Russian).

10. **Belubekyan V.M., Belubekyan M.V.** Three-Dimensional Problem of Reyleight Wave Propagation. // Reports of NAS RA, 2005, v. 105, № 4, p. 362–369 (in Russian).
11. **Knowles J.K.** A Note on Surface Waves. // J. of Geophysical Research., 1966, v. 21, № 22, p. 5480–5481.
12. **Sargsyan S.V., Melkonyan A.V.** On Three-Dimensional Problem of Wave Propagation Stoneley. In Collection of Articles Dedicated to the 90th Anniversary of Academician Ambartsumian S.A.: The Problems of Mechanics of Deformable Solids. Yer.: Inst. of Mechanics of NAS RA, 2012, p. 245–249 (in Russian).
13. **Sargsyan S.V., Melkonyan A.V.** Three-Dimensional Problem of Wave Propagation Stoneley. Proceedings of International Conference "Topical Problems of Continuum Mechanics" Dedicated to the 100th Anniversary of Academician Nagush Kh. Arutyunyan. Yer., 2012, v. 2, p. 273–276 (in Russian).
14. **Nowatckii V.** Theory of Elasticity. M.: Mir, 1975, 872 p.