

ELECTRON GAS IN ASYMMETRIC BICONVEX THIN QUANTUM LENS:  
REALIZATION OF KOHN'S THEOREM

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The problem of electromagnetic absorption in the electronic gas, which is localized in asymmetric biconvex thin quantum lens, is discussed. It is shown, that in the case of thin lens, in the plane of quantum lens electron gas is localized in two-dimensional parabolic confining potential. Thus, in this system the conditions for the realization of the generalized Kohn's theorem are satisfied.

**Keywords:** Kohn's theorem, asymmetric biconvex thin quantum lens.

**Introduction.** Consideration of the many-particle systems in the nanostructures are the subject of intense research, since the results may have important application in the design of nanoelectronic devices of new generation [1]. One of the most beautiful many-particle effects is the independence of the resonant frequency of the long wave radiation absorption of the electron gas localized in a parabolic quantum dot (QD) [2–4]. Realization of the confining parabolic potential may be due to the diffusion of the components of QD and environment [5]. On the other hand, the parabolic confining potential may be due to the specific geometry of the QD, in particular strongly oblate ellipsoid [6].

In the paper, the possibility of realization of generalized Kohn's theorem for the case of the asymmetric biconvex thin quantum lens (ABTQL) containing the electron gas is shown.

**Theory.** Consider the few-body electron gas localized in ABTQL with impenetrable walls. Then the confining potential for each particle has the following form [7]:

$$U_{conf} = \begin{cases} 0, & \text{inside ABTQL,} \\ \infty, & \text{outside ABTQL.} \end{cases} \quad (1)$$

Schrödinger equation for the one electron case in cylindrical coordinate is written as:

$$-\frac{\hbar^2}{2\mu} \left( \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \cdot \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} \right) \Psi(\rho, \varphi, z) + U_{conf}(\rho, z) \Psi(\rho, \varphi, z) = E \Psi(\rho, \varphi, z). \quad (2)$$

The condition of the thinness of the biconvex lens means, that  $\rho_0 \gg h_1 + h_2$ , where  $h_1$  and  $h_2$  are semi-axes for each convex part of ABTQL and  $\rho_0$  is radial semi-axis. Based on this fact we can solve the problem with the help of adiabatic approximation. The geometrical specificity of ABTQL is such that the motion of particle along the *OZ*-axis occurs much faster

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than in the plane perpendicular to it. The wave function of the charge carrier can be presented in the following form:

$$\Psi(\rho, \varphi, z) = f(\vec{\rho})\chi(z(\rho)). \quad (3)$$

For a fixed value of  $\rho$  along the  $z$ -axis, the particle will be localized in the 1D infinitely deep quantum well with the boundaries

$$z^+ = \sqrt{R_1^2 - \rho^2} + h_1 - R_1, \quad z^- = -\sqrt{R_2^2 - \rho^2} - h_2 + R_2. \quad (4)$$

Here  $R_1$  and  $R_2$  are the curvature radii of two spheres intersection. Thus, the  $z(\rho)$  changes in the interval

$$-\sqrt{R_2^2 - \rho^2} - h_2 + R_2 \leq z \leq \sqrt{R_1^2 - \rho^2} + h_1 - R_1. \quad (5)$$

Therefore, at the fixed value of the coordinate  $\rho$  the motion of each particle is localized in 1D potential well with the effective width

$$a(\rho) = \sqrt{R_1^2 - \rho^2} + \sqrt{R_2^2 - \rho^2} + (h_1 + h_2) - (R_1 + R_2). \quad (6)$$

For the axial part of the wave function and energy we have

$$\chi(z(\rho)) = (2/a(\rho))^{1/2} \sin \frac{\pi n}{a(\rho)} \left( z + \sqrt{R_2^2 - \rho^2} + h_2 - R_2 \right), \quad (7)$$

$$E_n^{(z)} = \frac{\pi^2 \hbar^2 n^2}{2\mu a^2(\rho)} \equiv U_n^{eff}(\rho).$$

As the number of the electrons  $N$  is small and the size-quantization in the  $Z$ -direction is sufficiently strong, the majority of the electrons will be localized in the geometrical center of ABTQL. The condition of the electron localization in the QD center is mathematically expressed by the relation  $\rho \ll \rho_0$ . Then the effective potential energy for the slow motion can be expanded into series by the small parameter  $\rho/\rho_0$ . So:

$$U_n^{eff} = \frac{\pi^2 \hbar^2 n^2}{2\mu (h_1 + h_2)^2} \left( 1 + \frac{R_1 + R_2}{h_1 + h_2} \cdot \rho^2 R_1 R_2 \right) = \frac{\pi^2 \hbar^2 n^2}{2\mu (h_1 + h_2)^2} + \frac{\mu \Omega_n^2 \rho^2}{2}, \quad (8)$$

where  $\Omega_n^2 = \frac{\pi^2 \hbar^2 (R_1 + R_2)}{\mu^2 (h_1 + h_2)^3 R_1 R_2} n^2$ .

If we have few-body gas with the pair interaction:

$$V_{int}(\vec{r}_1, \dots, \vec{r}_N) = \frac{1}{2} \sum_{i,j}^N v(|\vec{r}_i - \vec{r}_j|), \quad (9)$$

then the Hamiltonian of the system will have the following form:

$$\hat{H}(1, \dots, N) = \frac{1}{2\mu} \sum_{j=1}^N \hat{p}_j^2 + \sum_{j=1}^N U_{conf}(\vec{r}_j) + V_{int}(\vec{r}_1, \dots, \vec{r}_N). \quad (10)$$

We assume that the interaction of particles with ABTQL walls along the  $OZ$ -axis is so strong that one can neglect the interparticle interaction in this direction. Therefore, the operator of interaction between electrons  $V_{int}$  is a function of coordinates in  $XOY$  plane:

$$V_{int} \equiv V_{int}(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_N) = \frac{1}{2} \sum_{i,j}^N v(|\vec{\rho}_i - \vec{\rho}_j|). \quad (11)$$

Taking into account (3) the wave function of the system can be presented as:

$$\Psi(\vec{r}_1, \dots, \vec{r}_N) = \chi_{n_1, \dots, n_N}(z_1(\rho_1), \dots, z_N(\rho_N)) F(\vec{\rho}_1, \dots, \vec{\rho}_N), \quad (12)$$

where

$$\chi_{n_1, \dots, n_N}(z_1(\rho_1), \dots, z_N(\rho_N)) = \prod_{j=1}^N \sqrt{2/a(\rho_j)} \sin \frac{\pi n_j}{a(\rho_j)} \left( z_j + \sqrt{R_2^2 - \rho_j^2 + h^2 - R^2} \right). \quad (13)$$

Respectively for the energy spectra, we have:

$$E_{n_1, \dots, n_N}^{(z)} = \sum_{j=1}^N \frac{\pi^2 \hbar^2 n_j^2}{2\mu a^2(\rho_j)}. \quad (14)$$

After substituting the expression for the wave function in the Schrödinger equation and taking into account (14) for the energy, we will come to the equation for  $F(\vec{\rho}_1, \dots, \vec{\rho}_N)$

$$\left\{ \sum_{j=1}^N \left( \frac{\hat{p}_{xj}^2 + \hat{p}_{yj}^2}{2\mu} \right) + \sum_{j=1}^N \frac{\pi^2 \hbar^2 n_j^2}{2\mu a^2(\rho_j)} + \sum_{i,j=1; i \neq j}^N \frac{v(|\vec{\rho}_i - \vec{\rho}_j|)}{2} \right\} \times \quad (15)$$

$$\times F(\vec{\rho}_1, \dots, \vec{\rho}_N) = EF(\vec{\rho}_1, \dots, \vec{\rho}_N).$$

As we mention above, the second term in the Eq. (15) plays the role of effective potential energy for the “slow” subsystem. Expanding the expression under the sum by the small parameter  $\rho/\rho_0$  and after some transformations, we get:

$$\left\{ \frac{1}{2\mu} \sum_{j=1}^N (\hat{p}_{xj}^2 + \hat{p}_{yj}^2) + \frac{\mu}{2} \sum_{j=1}^N \Omega_j^2 \rho_j^2 + \frac{1}{2} \sum_{i,j=1; i \neq j}^N v(|\vec{\rho}_i - \vec{\rho}_j|) \right\} \times \quad (16)$$

$$\times F(\vec{\rho}_1, \dots, \vec{\rho}_N) = (E - S_N^{(0)}) F(\vec{\rho}_1, \dots, \vec{\rho}_N); \text{ where } S_N^{(0)} = \sum_{j=1}^N \frac{\pi^2 \hbar^2 n_j^2}{2\mu(h_1 + h_2)^2}.$$

As the gas is few-body, we assume that the strong vertical quantization brings gas to the states with quantum numbers  $n_1 = n_2 = \dots = n_N = 1$ . In this case for the  $\{\Omega_1, \Omega_2, \dots, \Omega_N\}$  we have

$$\Omega_1 = \Omega_2 = \dots = \Omega_N = \frac{\pi \hbar}{\mu} \left( \frac{R_1 + R_2}{R_1 R_2 (h_1 + h_2)^3} \right)^{1/2}. \quad (17)$$

Finally, we come to the 2D Hamiltonian:

$$\hat{H}^{2D} = \frac{1}{2\mu} \sum_{j=1}^N (\hat{p}_{xj}^2 + \hat{p}_{yj}^2) + \frac{\mu \Omega}{2} \sum_{j=1}^N (x_j^2 + y_j^2) + \sum_{i,j=1; i \neq j}^N \frac{v(|\vec{\rho}_i - \vec{\rho}_j|)}{2}. \quad (18)$$

Then, following [4] we carry out the change of variables in (18) as follows:

$$\vec{R} = \sum_{j=1}^N \frac{\vec{\rho}_j}{\sqrt{N}}; \vec{x}_1 = \frac{\vec{\rho}_1 - \vec{\rho}_2}{\sqrt{1 \cdot 2}}; \vec{x}_2 = \frac{\vec{\rho}_1 + \vec{\rho}_2 - 2\vec{\rho}_3}{\sqrt{2 \cdot 3}}; \vec{x}_{N-1} = \frac{\vec{\rho}_1 + \vec{\rho}_2 + \dots - (N-1)\vec{\rho}_N}{\sqrt{(N-1) \cdot N}}. \quad (19)$$

After the transition to the new variables, the initial Hamiltonian  $\hat{H}^{2D}$  is divided into two independent parts:  $\hat{H}_1(\vec{R}) = -\frac{\hbar^2}{2\mu} \cdot \frac{d^2}{dR^2} + \frac{\mu \Omega^2 R^2}{2}$  depends only on the coordinate of the center of mass, and  $\hat{H}_2(\vec{x}_1, \dots, \vec{x}_N)$  depends only on relative coordinates.

We suppose that the long-wave radiation incident on the system. For the perturbation operator in the long-wave approximation, we can write:

$$\hat{H}' = e\vec{E}(t) \sum_j \vec{\rho}_j = \sqrt{N} e\vec{E}(t) \vec{R}. \quad (20)$$

As it follows from (20),  $\hat{H}'$  does not contain relative coordinates  $\{\vec{x}_1 \vec{x}_2 \dots \vec{x}_N\}$ . Consequently, absorption occurs as in the single-particle case under the influence of field with amplitude  $\sqrt{N}E_0$ . In other words, in many-particle system the one-particle transitions are realized [8].

**Conclusion.** So, within the framework of adiabatic approximation we have shown that in the case of ABTQL for the relatively low levels of the few-body electron gas for confining potential the parabolic confinement is realized. In such a system the conditions are fulfilled for the realization of the generalized Kohn's theorem.

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