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LOCALIZED BENDING VIBRATIONS OF PIEZOCERAMIC TRANSVERSE POLARIZED PLATE

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Problem of the piezoceramic plate polarized along the normal of the middle plane of the plate is solved, based on the assumptions of the hypothesis of Kirchhoff, taking into account the components characterizing the electric field. The equations of planar and bending vibrations are obtained. Localized bending vibrations are considered, and the effect of the electric field on the frequency of localized vibrations is investigated.

MSC2010: 74H45.

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Introduction. The problems of vibrations of piezoceramic plates were studied in [1–6]. In [2, 3] bending vibrations of piezoceramic plate were studied, based on the assumptions of Kirchhoff's hypothesis. In [4] the problem of bending vibrations of piezoceramic plate of class 6 *mm* was investigated polarized along the normal of the middle plane of the plate. The components of plate displacement are presented, taking into account the assumptions of refined theory and with allowance for the components characterizing the electric field.

In [5] the piezoceramic plate was considered, where the components of displacement are presented based on the theory of S. Ambartsumian.

In present paper localized bending vibrations of piezoceramic plate are considered, on the basis of Kirchhoff's hypothesis, taking into account additional components characterizing the electric field.

Problem Setting. Consider piezoceramic plate of constant thickness $2\,h$ and polarized along the normal of the middle plane of the plate. The plate is located in the Cartesian coordinate system, so, that its middle plane coincides with the plane XOY and the plate occupies the region $0 \le x \le a,\ 0 \le x \le \infty,\ -h \le z \le h$.

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The equations of state of an elastic body are written in the form [1]:

$$
\begin{aligned}
&\varepsilon_{11}=\frac{(\sigma_{11}-\upsilon\sigma_{22})}{E}-\frac{\upsilon'}{E'}\sigma_{33}+d_{31}E_3, \quad \varepsilon_{22}=\frac{(\sigma_{22}-\upsilon\sigma_{11})}{E}-\frac{\upsilon'}{E'}\sigma_{33}+d_{31}E_3,\\
&\varepsilon_{33}=-\frac{\upsilon'}{E'}(\sigma_{11}+\sigma_{22})+\frac{1}{E}\sigma_{33}+d_{33}E_3, \quad \varepsilon_{12}=\frac{2(1+\upsilon)}{E}\sigma_{12},\\
&\varepsilon_{13}=\frac{1}{G'}\sigma_{13}+d_{15}E_1, \qquad \varepsilon_{23}=\frac{1}{G'}\sigma_{23}+d_{15}E_2,
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&D_1=\in_{11}E_1+d_{15}\sigma_{13}, \quad D_2=\in_{11}E_2+d_{15}\sigma_{23},\\
&D_3=\in_{33}E_3+d_{13}(\sigma_{11}+\sigma_{22})+d_{33}\sigma_{33},
\end{aligned}
\tag{2}
$$

where $\in_{11}, \in_{33}$ are permittivity coefficients at zero mechanical stresses; d_{ij} are piezoelectric constants; ε_{ij} are components of deformation; σ_{ij} are components of stress tensor; E_i are components of vector of electric intensity; D_i are components of vector of electric displacement field; E, E' are Young's modulus; G' are shear modulus; υ, υ' are Poisson's coefficients.

According to the assumption of Kirchhoff's hypothesis from (1) and (2) and neglecting $\sigma_{13}, \sigma_{23}, \sigma_{33}$ the equations of state are written as:

$$
\begin{aligned}
&\sigma_{11}=\frac{E}{1-\upsilon^2}(\varepsilon_{11}+\upsilon\varepsilon_{22})-\frac{E}{1-\upsilon}d_{31}E_3,\\
&\sigma_{22}=\frac{E}{1-\upsilon^2}(\varepsilon_{22}+\upsilon\varepsilon_{11})-\frac{E}{1-\upsilon}d_{31}E_3, \quad \sigma_{12}=\frac{E}{2(1+\upsilon)}\varepsilon_{12},
\end{aligned}
\tag{3}
$$

$$
D_1=\in_{11}E_1, \quad D_2=\in_{11}E_2, \quad D_3=\in_{33}E_3+d_{13}(\sigma_{11}+\sigma_{22}). \tag{4}
$$

Equations of electrodynamics for piezomedium have the following view in the electrostatic approach:

$$
\operatorname{div}\vec{D}=0, \qquad \operatorname{rot}\vec{E}=0 \quad (\vec{E}=-\operatorname{grad}\varphi). \tag{5}
$$

For displacement component, at any point of the plate, according to the assumptions of Kirchhoff's hypothesis, we have:

$$
u_1=u-z\frac{\partial w}{\partial x}+d_{15}\int_0^z E_1 d\xi, \quad u_2=v-z\frac{\partial w}{\partial x}+d_{15}\int_0^z E_2 d\xi, \quad u_3=w, \tag{6}
$$

here u, v, w are displacement of the middle plane of the plate.

For the problem of bending vibration of piezoceramic plate, displacements of the middle plane of the plate are obtained, according to the Kirchhoff's hypothesis assumption, where the integrals members are not taken into account [2, 3].

In [5] displacement components are considered taking into account the refined theory of S. Ambartsumian, and in special case displacements of the middle plane of the plate are obtained according to the Kirchhoff's assumption, where the components are not taken into account characterizing electric field.

Stress components and components of the electric induction vector, taking into account (6), are written by the following way:

$$
\begin{aligned}
\sigma_{11} = \frac{E}{1-\upsilon^2}\left(\frac{\partial u}{\partial x}+\upsilon\frac{\partial v}{\partial y}-z\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right)-d_{15}\int_0^z\left(\frac{\partial^2\varphi}{\partial x^2}+\upsilon\frac{\partial^2\varphi}{\partial y^2}\right)d\xi\right)+\\
+\frac{E}{1-\upsilon}d_{31}\frac{\partial\varphi}{\partial z},\\
\sigma_{22} = \frac{E}{1-\upsilon^2}\left(\frac{\partial v}{\partial y}+\upsilon\frac{\partial u}{\partial x}-z\left(\frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\right)-d_{15}\int_0^z\left(\frac{\partial^2\varphi}{\partial y^2}+\upsilon\frac{\partial^2\varphi}{\partial x^2}\right)d\xi\right)+\\
+\frac{E}{1-\upsilon}d_{31}\frac{\partial\varphi}{\partial z},\\
\sigma_{12} = \frac{E}{2(1+\upsilon)}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}-2z\frac{\partial^2 w}{\partial x\partial y}-2d_{15}\int_0^z\frac{\partial^2\varphi}{\partial x\partial y}d\xi\right).
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
D_1 = -\in_{11}\frac{\partial\varphi}{\partial x},\qquad D_2 = -\in_{11}\frac{\partial\varphi}{\partial y},\qquad D_3 = -\in_{33}\frac{\partial\varphi}{\partial z}+\\
+\frac{d_{13}E}{1-\upsilon}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}-z\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-d_{15}\int_0^z\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)d\xi\right)+\frac{2d_{13}^2E}{1-\upsilon}\frac{\partial\varphi}{\partial z}.
\end{aligned}
\tag{8}
$$

Motion equation of piezoceramic body:

$$
\frac{\partial\sigma_{ij}}{\partial x_j} = \rho\frac{\partial^2 u_i}{\partial t^2}.
\tag{9}
$$

Internal forces $T_i = \int_{-h}^{h}\sigma_{ii}dz,\ i=1,2$, and moments relative to the median plane are:

$$
\begin{aligned}
T_1 = \frac{2hE}{1-\upsilon^2}\left(\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)+\frac{Ed_{13}}{1-\upsilon}\varphi\Big|_{-h}^{h}-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^2}{\partial x^2}+\upsilon\frac{\partial^2}{\partial y^2}\right)(I_1+m_1),\\
T_2 = \frac{2hE}{1-\upsilon^2}\left(\frac{\partial v}{\partial y}+v\frac{\partial u}{\partial x}\right)+\frac{Ed_{13}}{1-\upsilon}\varphi\Big|_{-h}^{h}-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^2}{\partial y^2}+\upsilon\frac{\partial^2}{\partial x^2}\right)(I_1+m_1),\\
S = \int_{-h}^{h}\sigma_{12}dz = \frac{hE}{1+\upsilon}\left(\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)-\frac{Ed_{15}}{1+\upsilon}\cdot\frac{\partial^2}{\partial x\partial y}(I_1+m_1).
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
M_1 = \int_{-h}^{h}z\sigma_{11}dz = -\frac{2h^3E}{3(1-\upsilon^2)}\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right)+\frac{Ed_{13}}{1-\upsilon}\left(\varphi\Big|_{-h}^{h}-\frac{1}{h}\int_{-h}^{h}\varphi dz\right)-\\
-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^2}{\partial x^2}+\upsilon\frac{\partial^2}{\partial y^2}\right)(I_2+m_2),\\
M_2 = \int_{-h}^{h}z\sigma_{22}dz = -\frac{2h^3E}{3(1-\upsilon^2)}\left(\frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\right)+\frac{Ed_{13}}{1-\upsilon}\left(\varphi\Big|_{-h}^{h}-\frac{1}{h}\int_{-h}^{h}\varphi dz\right)-\\
-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^2}{\partial y^2}+\upsilon\frac{\partial^2}{\partial x^2}\right)(I_2+m_2),\\
H = \int_{-h}^{h}z\sigma_{12}dz = -\frac{2h^3E}{3(1+\upsilon)}\left(\frac{\partial^2 w}{\partial x\partial y}\right)-\frac{Ed_{15}}{1+\upsilon}\cdot\frac{\partial^2}{\partial x\partial y}(I_2+m_2),
\end{aligned}
\tag{11}
$$

where the following notes are accepted:

$$
\begin{aligned}
I_1 = h\left(\int_0^h\varphi dz+\int_0^{-h}\varphi dz\right),\ I_2 = \frac{h^2}{2}\left(\int_0^h\varphi dz-\int_0^{-h}\varphi dz\right),\\
m_1 = -\int_{-h}^{h}z\varphi dz,\ m_2 = -\int_{-h}^{h}\frac{z^2}{2}\varphi dz.
\end{aligned}
\tag{12}
$$

Integrating the motion equations (9) on z by the limits from $-h$ to h, we will get:

$$\begin{aligned} &\frac{\partial T_1}{\partial x}+\frac{\partial S}{\partial y}+\sigma_{13}\Big|_{-h}^{h}=2\rho h\frac{\partial^2 u}{\partial t^2},\\ &\frac{\partial S}{\partial x}+\frac{\partial T_2}{\partial y}+\sigma_{23}\Big|_{-h}^{h}=2\rho h\frac{\partial^2 v}{\partial t^2},\\ &\frac{\partial N_1}{\partial x}+\frac{\partial N_2}{\partial y}+\sigma_{33}\Big|_{-h}^{h}=2\rho h\frac{\partial^2 w}{\partial t^2}. \end{aligned} \tag{13}$$

After multiplying the first two motion equations (9) by z and integrating by the limits $-h$ to h, we obtain:

$$\frac{\partial M_1}{\partial x}+\frac{\partial H}{\partial y}=N_1,\ \ \frac{\partial H}{\partial x}+\frac{\partial M_2}{\partial y}=N_2. \tag{14}$$

Substituting the internal forces values and the moments in the first two equations (13), (14), we have:

$$\begin{aligned} &\frac{2hE}{1-\upsilon^2}\left(\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial x\partial y}\right)+\frac{hE}{1+\upsilon}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right)+\frac{Ed_{13}}{1-\upsilon}\cdot\frac{\partial}{\partial x}\left(\varphi|_{-h}^{h}\right)-\\ &-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^3}{\partial x^3}+\upsilon\frac{\partial^3}{\partial x\partial y^2}\right)(I_1+m_1)+\sigma_{13}|_{-h}^{h}=2\rho h\frac{\partial^2 u}{\partial t^2}-\rho d_{15}\frac{\partial^3}{\partial x\partial t^2}(I_1+m_1),\\ &\frac{2hE}{1-\upsilon^2}\left(\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^2 u}{\partial x\partial y}\right)+\frac{hE}{1+\upsilon}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}\right)+\frac{Ed_{13}}{1-\upsilon}\cdot\frac{\partial}{\partial x}\left(\varphi|_{-h}^{h}\right)-\\ &-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^3}{\partial y^3}+\upsilon\frac{\partial^3}{\partial y\partial x^2}\right)(I_1+m_1)+\sigma_{23}|_{-h}^{h}=2\rho h\frac{\partial^2 v}{\partial t^2}-\rho d_{15}\frac{\partial^3}{\partial y\partial t^2}(I_1+m_1), \end{aligned} \tag{15}$$

$$\begin{aligned} &-\frac{2h^3E}{3(1-\upsilon^2)}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)+\frac{Ehd_{13}}{1-\upsilon}\left(\varphi|_{-h}^{h}-\frac{1}{h}\int_{-h}^{h}\varphi dz\right)-\\ &\qquad-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^3}{\partial x^3}+\frac{\partial^3}{\partial x\partial y^2}\right)(I_2+m_2)=N_1,\\ &-\frac{2h^3E}{3(1-\upsilon^2)}\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial y\partial x^2}\right)+\frac{Ehd_{13}}{1-\upsilon}\left(\varphi|_{-h}^{h}-\frac{1}{h}\int_{-h}^{h}\varphi dz\right)-\\ &\qquad-\frac{Ed_{15}}{1-\upsilon^2}\left(\frac{\partial^3}{\partial y^3}+\frac{\partial^2}{\partial y\partial x^2}\right)(I_2+m_2)=N_2. \end{aligned} \tag{16}$$

The system (15) represents the equations of planar vibrations of a piezoceramic plate. Substituting the last equation from (13) in (16), we obtain the equation of bending vibrations:

$$D\Delta^2 w+2\rho h\frac{\partial^2 w}{\partial t^2}-\frac{Ehd_{13}}{1-\upsilon}\Delta\left(\varphi|_{-h}^{h}-\frac{1}{h}\int_{-h}^{h}\varphi dz\right)+\frac{Ed_{15}}{1-\upsilon^2}\Delta^2(I_2+m_2)-\sigma_{33}|_{-h}^{h}=0, \tag{17}$$

where $D=\dfrac{2h^3E}{3(1-\upsilon^2)}$.

Averaging the equation of electrodynamics, the following relations are obtained:

$$-\left(\in_{11}+\frac{d_{13}d_{15}E}{1-\upsilon}\right)\Delta\varphi-\left(\in_{33}+\frac{d_{13}^2 2E}{1-\upsilon}\right)\frac{\partial^2\varphi}{\partial z^2}-\frac{d_{13}E}{1-\upsilon}\Delta w=0. \tag{18}$$

Let's consider the equation of bending vibrations. On the surface faces of the plate $z=\pm h$ has the following condition:

$$\sigma_{13}=0,\ \ \sigma_{23}=0,\ \ \sigma_{33}=0,\ \ \sigma=0. \tag{19}$$

Then the equations of bending vibrations and for the potential will be written:

$$
\begin{gathered}
D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} + \frac{Ehd_{13}}{1-\upsilon}\Delta\left(\frac{1}{h}\int_{-h}^{h}\varphi dz\right) + \frac{Ed_{15}}{1-\upsilon^2}\Delta^2(I_2+m_2) = 0, \\
\Delta\varphi + a_1\frac{\partial^2\varphi}{\partial z^2} = -a_2\Delta w,
\end{gathered}
\tag{20}
$$

where the following notes are accepted:

$$
a_1 = \frac{\in_{33}(1-\upsilon)+d_{13}^2 E}{\in_{11}(1-\upsilon)+d_{13}d_{15}E}, \quad a_2 = \frac{d_{13}E}{\in_{11}(1-\upsilon)+d_{13}d_{15}E}.
$$

In this case the equations of bending vibrations are separated from the equations of planar vibrations.

Suppose that the following boundary conditions are given at the edges of the plate and in the limit $y\to\infty$:

$$
\begin{gathered}
\text{at } x=0 \quad w=0,\ M_1=0,\ \varphi=0; \qquad \text{at } x=a \quad w=0,\ M_1=0,\ \varphi=0; \\
\text{at } y=0 \quad M_2=0,\ \frac{\partial M_2}{\partial y}+2\frac{\partial H}{\partial y}=0,\ \varphi=0; \quad \text{at } y\to\infty \quad w=0,\ \varphi=0.
\end{gathered}
\tag{21}
$$

The solution of the Eqs. (20) of the system we will seek in the form

$$
w(x,y,t) = w_0(y)\sin\alpha_n x\exp(i\omega t), \quad \varphi(x,y,z,t) = \Phi(y,z)\sin\alpha_n x\exp(i\omega t), \tag{22}
$$

where $\alpha_n = \pi n/a$.

Substituting the seeking solutions in differential equations for φ potential, we will get

$$
\left(\frac{\partial^2\Phi}{\partial y^2} - \alpha_n^2\Phi\right) + a_1\frac{\partial^2\Phi}{\partial z^2} = -a_2(w_0'' - \alpha_n^2 w_0). \tag{23}
$$

Let's seek (23) differential equation $\Phi(y,z) = \varphi_0(y,z) + R(y)$. The seeking solution inserting in (23), for $\Phi(y,z)$ we will get

$$
\begin{gathered}
\Phi(y,z) = (A_1\exp(by) + A_2\exp(-by))(B_1\sin(\beta_m z) + B_2\cos(\beta_m z)) + \\
+ C_1(y)\exp(\alpha_n y) + C_2(y)\exp(-\alpha_n y),
\end{gathered}
\tag{24}
$$

where $C_1(y) = -\dfrac{a_2}{2\alpha_n}(w_0' + \alpha_n w_0) + C_{10}$; $C_2(y) = -\dfrac{a_2}{2\alpha_n}(w_0' + \alpha_n w_0) + C_{20}$; $\beta_m = \dfrac{2\pi m}{h}$; $b^2 = \alpha_n^2 + \lambda^2 a_1$.

Satisfying boundary conditions on the border of $z = \pm h$, for φ potential we will get

$$
\varphi(x,y,z) =)(C_1(y)\exp(\alpha_n y) + C_2(y)\exp(-\alpha_n y))(1 - \cos(\beta_m z)\sin\alpha_n x\exp(i\omega t)). \tag{25}
$$

Substituting the seeking solution in the equation of bending vibration, we will get

$$
w_0'' - 2\alpha_n^2\left(1 + \frac{\gamma}{1-\chi}\right)w_0'' + \alpha_n^4\left(1 - \frac{\Omega^2 - 2\gamma}{1-\chi}\right) = 0, \tag{26}
$$

where

$$
\gamma = \frac{3d_{13}a_2(1+\upsilon)}{2h^2\alpha_n^2},\ \ \chi = d_{15}a_2\left(1 + \frac{3}{h^2\beta^2}\right),\ \ \Omega^2 = \frac{2\rho h\omega^2}{\alpha_n^4 D}. \tag{27}
$$

The solution of (26) differential equation will have following view

$$w_0(y) = A\exp(\alpha_n p_1 y) + B\exp(-\alpha_n p_1 y) + C\exp(\alpha_n p_2 y) + D\exp(-\alpha_n p_2 y), \qquad (28)$$

where $\theta_{1,2} = \pm\sqrt{1-\zeta+\sqrt{\zeta^2+\hat{\Omega}^2}}$, $\theta_{3,4} = \pm\sqrt{1-\zeta-\sqrt{\zeta^2+\hat{\Omega}^2}}$, $\zeta = \dfrac{\gamma}{1-\chi}$, $\hat{\Omega}^2 = \dfrac{\Omega^2}{1-\chi}$.

According to the condition, from (21) we will get

$$w_0(y) = B\exp(-\alpha_n p_1 y) + D\exp(-\alpha_n p_2 y). \qquad (29)$$

According to the damping condition, when $y \to \infty$, $w = 0$, $\varphi = 0$, the seeking constants C_{10}, A, D are equal to zero. Then satisfying boundary conditions on $y = 0$ we will get system of mathematical equations towards the seeking unknown constants.

In order the system of algebraic equations has non-zero solution, it is necessary that its determinant was equal to zero. From the condition that the determinant is equal to zero, we will get dispersion equation which depends on the frequency vibration

$$\begin{gathered} K(\hat{\Omega}) \equiv \theta_1^2\theta_3^2 + 2(1-\upsilon)\theta_1\theta_3 - \upsilon(\upsilon-2\zeta) - r(\theta_1\theta_3+\theta_1+\theta_3+\upsilon) = 0, \\ r = 2\zeta - \frac{(1-\upsilon)\chi}{1-\chi}. \end{gathered} \qquad (30)$$

According to the damping condition from (30) follows that $\theta_1 > 0$, $\theta_3 > 0$, it follows from here $0 < \hat{\Omega}^2 < 1-2\zeta$. In limited cases the following inequalities must be satisfied. In the case when $\hat{\Omega}^2 = 0$, $K(0) > 0$, we will get $\theta_1 = 0$, $\theta_3 = \sqrt{1-2\zeta}$,

$$1-2\zeta+2(1-\upsilon)\sqrt{1-2\zeta} - \upsilon(\upsilon-2\zeta) - r\left(\sqrt{1-2\zeta}+1+\upsilon\right) > 0. \qquad (31)$$

In the case when $\hat{\Omega}^2 = 0$, $K(1-2\zeta) < 0$, we will get $\theta_1 = \sqrt{2(1-\zeta)}$, $\theta_3 = 0$.

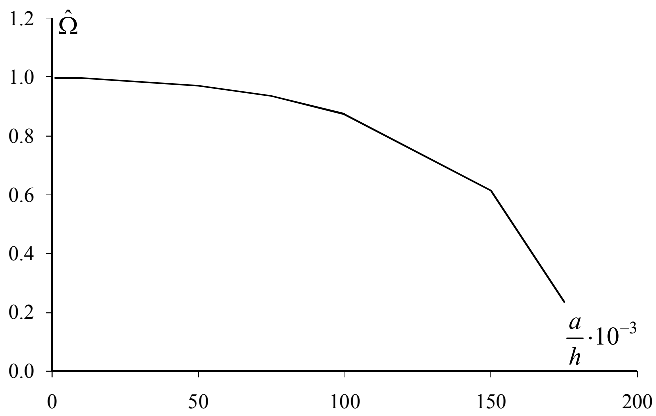
$$-\upsilon(\upsilon-2\zeta) - r\left(\sqrt{2(1-\zeta)}+\upsilon\right) < 0. \qquad (32)$$

In the case, when piezoeffect is missing, (31) and (32) inequalities are satisfied, we will get known result [7]. For piezoceramic plate 2 h with constant thickness and polarized along the normal of the middle plane of the plate, numerical analysis are done for defining the frequency of localized vibrations. The meaning of the frequency of localized vibrations dependant on dimensionless values γ and χ at $\upsilon = 0,3$ are presented in the Table.

Frequencies of localized vibrations

γ	χ	$K(0)$	$K(1-2\xi)$	Ω
0.10	0.15	1.62	–0.20	0.79
0.15	0.10	1.07	–0.39	0.72
0.20	0.25	0.69	–0.38	0.52
0.25	0.30	0.19	–0.56	0.55

In the case when piezoeffect is missing, the meaning of the frequency of localized vibrations is $\Omega = 0.99$. From the Table follows, that for piezoceramic plate 2 h with constant thickness and polarized along the normal of the middle plane of the plate exist frequency of localized vibrations and with the increase non dimensional values y, χ, the size of frequency of localized vibrations decreases.



Frequency of localized vibrations for piezocrystal PTZ-4.

From Figure is seen, that frequency of localized vibrations decreases with the increasing of geometric parameters of plate.

Conclusion. For piezoceramic plate polarized along the normal of the middle plane of the plate, the equations of planar and bending vibrations are obtained. In frequent case bending vibrations are considered, for which dispersion equation is obtained for determining frequency of localized vibrations. It is shown that for bending vibrations of piezoceramic plate polarized along the normal of the middle plane of the plate the frequency of localized vibrations decreases at the increasing of dimensionless values y, χ.

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