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Physics

EFFECT OF DONOR IMPURITY ON AHARONOV–BOHM OSCILLATIONS IN A DOUBLE QUANTUM RING WITH GAUSSIAN CONFINEMENT

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In this paper the effect of donor impurity on the Aharonov–Bohm oscillations of the electronic states in a double quantum ring with Gaussian confinement has been studied. Three different impurity positions: namely in the inner ring, in the outer ring and in the barrier between the rings, have been considered. It is shown that the energies of the two lowest states are almost constant, while for the higher levels the Aharonov–Bohm oscillations are observed. The obtained results indicate on the possibility of the effective control of the quantum states by means of donor impurity and external magnetic field.

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Introduction. Semiconductor heterostructures utilizing quantum dots (QD) and nanocrystal structures have been subjected to an extensive research field, because of their unique physical properties, useful for applications in opto- and nano-electronics [1]. Advances with respect to growth as well as high-resolution electron-beam lithography techniques allow the fabrication of novel structures called quantum rings (QR) [2]. QR have find a use for various practical applications in the last few years. Photodetectors based on semiconductor nanorings have been fabricated in the mid-infrared and THz spectral ranges [3, 4]. Nanorings also demonstrate superior potential for high density magnetic memory [5]. Lasers incorporating nano-rings in the gain medium have also been reported [6].

Using droplet epitaxial technique, authors of [7] performed self-assembly of concentric double quantum rings (DQR) with high uniformity and excellent rotational symmetry. Formation and characterization of QR complexes open a new route to measurement of quantum interference effects [8, 9] promised by the ring geometry.

An understanding of the nature of impurity states in semiconductor structures is one of the crucial problems in semiconductor physics, because impurities can

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dramatically alter the properties and performance of a quantum device [10–12]. The investigation of shallow-impurity states in these systems are presented in several works [13, 14]. Many works related to optical transitions, electronic states and impurity states binding energy have been reported recently [15–17]. In almost all the references cited above, the calculations have been made in the effective-mass approximation and using variational techniques, which in general provide a good interpretation of experimental results associated with effects such as geometrical confinement and applied electric fields, hydrostatic pressure and temperature [18]. However, for considering the effect of magnetic field on electronic properties of QR the variational procedure becomes not appropriate, because of the changing the angular quantum number of the ground state due to the crossings of the levels. Recently, some works have been reported on the investigation of the effects of static magnetic field, as well as intense electromagnetic radiation on optical properties of quantum rings using the exact diagonalization method [19, 20]. Our previous works are devoted to the theoretical investigation on electronic band structure of superlattices composed of Gaussian-shaped DQR with smooth potential profile due to interdiffusion of compound materials in the heterojunction [21, 22]. It has been shown that the modelling of the smooth potential profile by an appropriate analytic function provides reasonable results for the electronic band structure on the one hand and simplifies numerical calculations on the other hand [23].

In the present work the effect of the off center donor impurity on the energy Aharonov–Bohm oscillations and the probability distribution for a single electron in a GaAs-GaAlAs DQR with Gaussian confinement has been investigated using the exact diagonalization procedure. The paper is organized as follows: in the section II the theoretical model is described, section III is devoted to the discussion of the results, the conclusion and the acknowledgment are presented in sections IV and V respectively.

Theory. Let us consider a two dimensional DQR with a confining potential, expressed by two shifted Gaussians:

$$V(r) = V_0(1-(\exp(-\alpha^2(r-r_1)^2)+\exp(-\beta^2(r-r_2)^2)), \tag{1}$$

were V_0 is the maximal depth of the potential well, $r_{1(2)}$ is the center of the inner (outer) ring, α and β are the Gaussian parameters describing the abruptness of the potential profile. Fig. 1, a and b represent the three- and two-dimensional profiles of the confining potential (1) for the values of parameters $\alpha = \beta = 0.025\ \AA^{-1}$, $r_1 = 100\ \AA$, $r_2 = 200\ \AA$ and $V_0 = 246\ meV$. This kind of potential almost prevents the penetration of an electron in the central region of QR providing clear observation of Aharonov–Bohm oscillations. The comparatively small barrier between the rings allows the electron to travel from one ring to another, which is desirable for understanding the effect of the impurity on the Aharonov–Bohms oscillations with different periods.

In the framework of effective mass approximation the Hamiltonian of the system in the external magnetic field with induction vector perpendicular to the ring plane has the following form in cylindrical coordinates:

$$H = -\frac{\hbar^2}{2m^*}\left(\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\cdot\frac{\partial^2}{\partial\varphi^2}\right)+\frac{i\hbar\omega_B}{2}\cdot\frac{\partial}{\partial\varphi}+\frac{m^*\omega_B^2}{8}r^2+V(r)- \\ -\frac{e^2}{\varepsilon\sqrt{r^2+r_i^2-2rr_i\cos\varphi}}, \tag{2}$$

were r_i is the impurity radius vector, ε is the dielectric constant, ω_B is the cyclotron frequency, m^* and e are the effective mass and the charge of electron, respectively. Here we omit the Zeeman term because of negligible small value of Land*é* g-factor for the considered material.

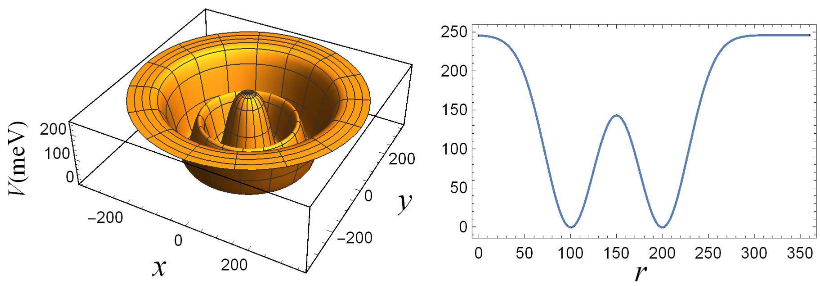

Fig. 1. The 3D (left) and 2D (right) plots of the confining potential of DQR. The Cartesin coordinates "x" and "y" and the radial coordinate "r" are expressed in angstroms.

We express the wave function of electron by the following expansion:

$$\Psi(r,\varphi)=\sum_{n,l}C(n,l)\psi(r,\ \varphi;\ n,\ l), \tag{3}$$

were $C(n,l)$ are the expansion coefficients and $\psi(r,\ \varphi;\ n,\ l)$ are the normalized electron wave functions in a cylindrical disk of large enough radius R_0 with impenetrable rectangular potential barriers:

$$\psi(r,\ \varphi;\ n,\ l)=\frac{1}{\sqrt{2\pi}}e^{il\phi}\frac{\sqrt{2}J(l,k(n,l)r/R_0)}{R_0J(l+1,k(n,|l|))}. \tag{4}$$

In Eq. (4) $J(\xi,\varsigma)$ is the first kind Bessel function of the ξ-th order, $k(p,q)$ is the p-th root of the Bessel function of q-th order.

Substituting the expression (3) in the Eq. (2) one arrives to a system of algebraic equations, which has non-trivial solutions if the corresponding determinant is zero:

$$\|\langle n,l|H-E|n',l'\rangle\|=0. \tag{5}$$

Solving Eq. (5) one can get the energy eigenvalues and the eigenvectors for all the confined states.

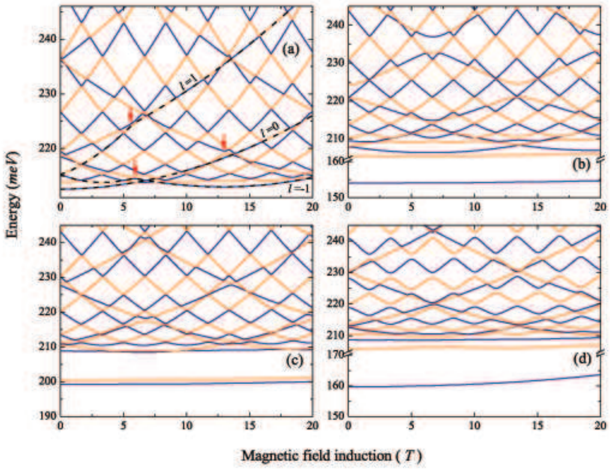

Fig. 2. Dependencies of electron energy on magnetic field induction in the absence of impurity (a), in the presence of an impurity positioned in the inner ring (b), in the presence of an impurity positioned in the barrier region between the rings (c) and in the presence of an impurity positioned in the outer ring (d). The red arrows indicate on the anticrossings, which are not the consequence of the symmetry breaking.

Results and Discussion. The numerical calculations are performed for GaAs-GaAlAs heterostructure with the following parameter values: $m^* = 0.067m_0$, $\varepsilon = 12.9$, $V_0 = 246\ meV$ [24].

On Fig. 2 the dependencies of the electron energy on external magnetic field induction are presented, in the absence (a) and the presence (b)–(d) of donor impurity. Fig. 2 (b) has been obtained for an impurity positioned in the middle of the inner ring: $r_i = 100\ \AA$. Fig. 2 (c) corresponds to an impurity in the barrier region between the rings: $r_i = 150\ \AA$ and Fig. 2 (d) is for an impurity in the outer ring: $r_i = 200\ \AA$.

Evident oscillations of the energy levels can be observed in all the cases, excepted the lowest two levels in Fig. 2 (b)–(d). These levels correspond to the states confined by the impurity. One can see that for $r_i = 100\ \AA$ and $200\ \AA$ the ground state energy is much lower than others, indicating to the strong confinement of the electron, while for $r_i = 150\ \AA$ the lowest two non-oscillating levels are close to each other.

The second regularity concerns to the period of energy oscillations. Namely, in all figures two different periods of oscillations related with the Aharonov–Bohm effect for the inner and the outer rings are observed. Moreover, one can observe anti-crossings for the oscillations with larger (smaller) period when impurity is located in the inner (outer) ring (Fig. 2 (b), (d)). For the impurity in the barrier region there are no considerable anti-crossings, because of the negligibly small effect of impurity on the symmetry of the electron's wave function (Fig. 2 (c)). In the case of the absence of impurity the levels with different values of l cross each other (in this case the angular momentum is a conserving quantity and each level can be described by the quantum number l), because of the cylindrical symmetry of the structure. However it is important to mention, that for some levels which correspond to the same value of l also anti-crossings are observed. This anti-crossings are mentioned by red arrows in Fig. 2 (a). For clarity we repeat the calculations of the energies without impurity for each fixed value of the angular quantum number. Three curves corresponding to the values of $l = 0$, $l = 1$ and $l = -1$ are shown in Fig. 2 (a) by black dashed lines.

The electronic probability densities for the first five levels ($n = 1,2,3,4,5$) for three fixed values of magnetic field induction in DQR in the presence of donor impurity are presented in Fig. 3 and Fig. 4.

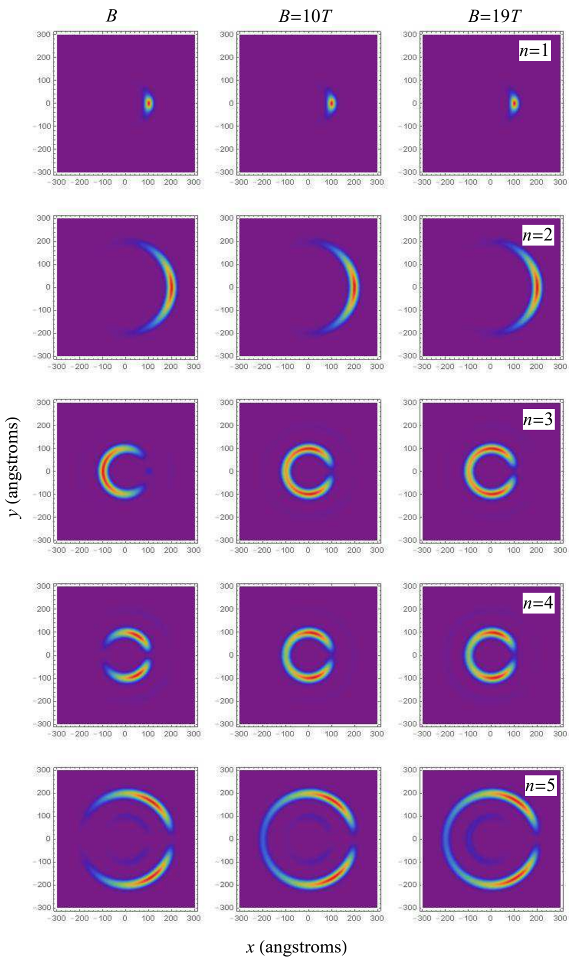

Fig. 3. Electronic probability densities in the five lowest states for different values of magnetic field induction when impurity is in the inner ring.

Fig. 3 corresponds to the case when the impurity is positioned in the inner ring, while Fig. 4 corresponds to the case when the impurity is in the outer ring. From the upper two rows of the Figs. 3 and 4 one can observe a strong confinement of electron with the impurity for the lowest two levels, which leads to a very weak dependence of the corresponding energies on magnetic field (see Fig. 2 (b), (d)). Interestingly, when the impurity is in the inner (outer) ring the ground state corresponds to the localization of electron in the inner (outer) ring, while the first excited state corresponds to the localization of electron in the outer (inner) ring. In the 3-rd and the 4-th states electron is localized in the same ring as impurity but have considerable angular distribution in contrast to the 1-st and the 2-nd states. And finally, the 5-th state reveals considerable probability in both of the rings, which is more pronounced for the case when impurity is in the outer ring (the 5-th row in Fig. 4).

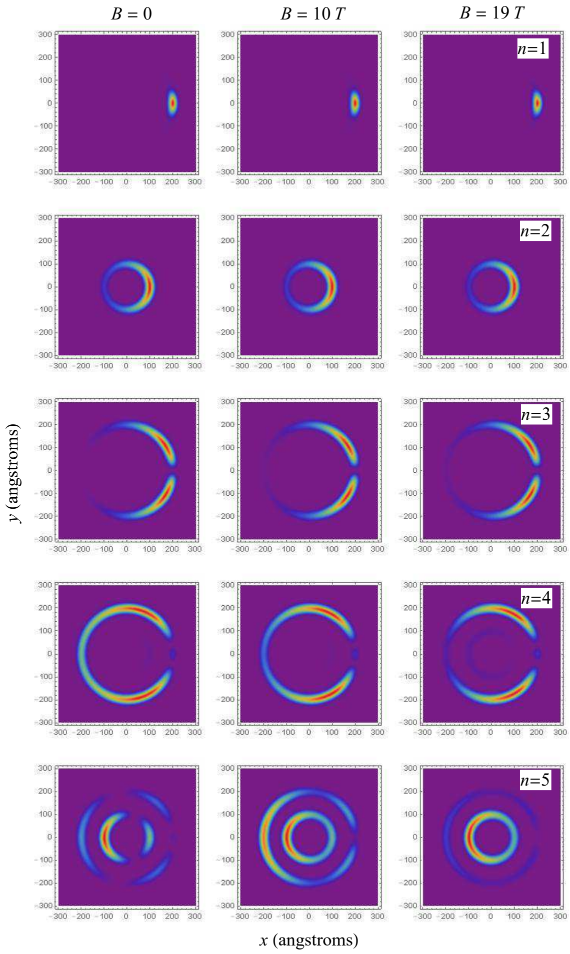

Fig. 4. Electronic probability densities in the five lowest states for different values of magnetic field induction when impurity is in the outer ring.

Summarizing, the effect of donor impurity on the Aharonov–Bohm oscillations of the electron energy levels in a DQR with Gaussian-shaped confining potential has been studied in the framework of exact diagonalization method. Three different impurity positions: namely, in the inner ring, in the outer ring and in the barrier between the inner and the outer rings, have been considered.

In the absence of impurity energy levels reveal crossings excepted levels with different values of the radial quantum number but with the same value of the angular quantum number. This fact indicates on the impossibility of the existence of two different states with the same value of energy and angular quantum number. In the presence of impurity the ground state energy level does not oscillate with the increase of magnetic field induction due to the symmetry breaking of the electron wave function. When impurity is located in the inner ring the higher levels with larger (smaller) period of oscillations reveal anti-crossings (crossings) and vice versa, the impurity in the outer ring leads to anti-crossings (crossings) for the levels with smaller (larger) oscillation period. The ground impurity state is strongly localized near the impurity in the same ring as the impurity, while the first excited state is localized in the ring which does not contain impurity. The obtained results show that by changing the impurity position one can efficiently tune the crossings and anti-crossings of the energy levels and the electron localization. The latter, in its turn, makes possible the control of optoelectronic characteristics of novel devices based on quantum rings.

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