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M e c h a n i c s

ON THE LOCALIZATION OF SHEAR VIBRATIONS IN A COMPOSITE ELASTIC SEMI-INFINITE FLAT WAVEGUIDE

M. V. BELUBEKYAN [1] *, S. L. SAHAKYAN [2] **

[1] *Institute of Mechanics NAS of the Republic of Armenia*
[2] *Chair of Numerical Analysis and Mathematical Modeling YSU, Armenia*

In this paper we consider semi-infinite flat waveguides with different boundary conditions on the planes and on the edges that bound the waveguide. The possibility of localizing shear waves in the vicinity of the junction of neighbouring parts of a semi-infinite flat waveguide is established.

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***Keywords*:** shear oscillations, waveguide.

Introduction. The study of purely shear waves in a flat layer began with Love's work in 1911 [1]. Subsequently many problems were solved for elastic waveguides with various boundary conditions and in a dynamic formulation (problems with initial conditions). A survey of these papers is given in the monograph [2] and in [3]. In [4] localized shear waves are considered in the vicinity of the edge of a semi-infinite waveguide. The paper [5] is devoted to the case when the plane boundary of the semi-infinite part of the waveguide passes into a periodically varying boundary. In [6] resonance oscillations in a plane finite composite waveguide were investigated. In [7] the propagation of shear waves in elastic waveguide with the periodically changed boundary conditions is investigated. The problem of localized shear waves in the vicinity of the junction of the two parts of the waveguide with a symmetric arrangement of the boundary conditions relative to the median plane of the layer was considered in [8].

Statement of the Problem. Let the flat waveguide consist of two parts. In a rectangular Cartesian coordinate system, the first part of the waveguide with the index (1) occupies the region $-a \leq x < 0,\ 0 \leq y < h,\ -\infty < z < \infty$, the second part with the index (2) occupies the region $0 < x < \infty,\ 0 \leq y < h,\ -\infty < z < \infty$ (Fig. 1). Consider the pure shear elastic vibrations (anti-flat deformation):

$$u_i = 0,\ \ v_i = 0,\ \ w_i = w_i(x,y,t), \quad i = 1,2. \tag{1}$$

* E-mail: mbelubekyan@yahoo.com ** E-mail: ssahakyan@ysu.am

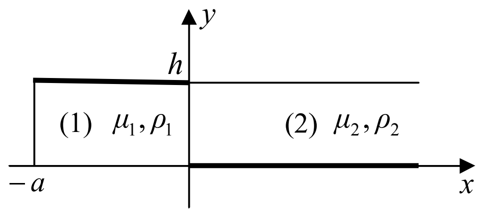

Fig. 1. The flat waveguide.

Wave propagation equations for the waveguide parts are of the form [2, 9]:

$$c_i^2 \Delta w_i = \frac{\partial^2 w_i}{\partial t^2}, \quad c_i^2 = \frac{\mu_i}{\rho_i}, \quad i = 1, 2, \tag{2}$$

where Δ is the two-dimensional Laplace operator; μ_i is the shear modulus; ρ_i is the density of the waveguide material; c_i is the velocity of the bulk shear wave. It is assumed that the waveguide surface $y = 0$ is free $\left(\sigma_{yz}^{(1)} = 0\right)$ for $x < 0$ and fixed for $x > 0$, and the surface $y = h$ is free for $x > 0$ and fixed for $x < 0$, i.e.

$$\left.\frac{\partial w_1}{\partial y}\right|_{y=0} = 0, \quad \left.w_2\right|_{y=0} = 0, \tag{3}$$

$$\left.w_1\right|_{y=h} = 0, \quad \left.\frac{\partial w_2}{\partial y}\right|_{y=h} = 0. \tag{4}$$

At the junction of the two parts of the waveguide (at the point of articulation) the conditions for continuity of displacements and shear stresses σ_{xz} should be satisfied:

$$\left.w_1\right|_{x=0} = \left.w_2\right|_{x=0}, \quad \mu_1 \left.\frac{\partial w_1}{\partial x}\right|_{x=0} = \mu_2 \left.\frac{\partial w_2}{\partial x}\right|_{x=0}. \tag{5}$$

The ending edge of the waveguide is free [10, 11]:

$$\left.\frac{\partial w_1}{\partial x}\right|_{x=-a} = 0, \tag{6}$$

and when $x \to +\infty$, the condition of the damping of oscillations must be satisfied:

$$\lim_{x \to +\infty} w_2 = 0. \tag{7}$$

Obtaining of the Corresponding System of Equations. The solutions of Eq. (2) for the waveguide parts that satisfy the boundary conditions (3), (4) are represented as follows:

$$w_1 = e^{iwt} \sum_{n=0}^{\infty} f_n(x) \cos \lambda_n y, \quad \lambda_n = \frac{\pi + 2\pi n}{2h}, \tag{8}$$

$$w_2 = e^{iwt} \sum_{m=0}^{\infty} g_m(x) \cos \lambda_m y, \quad \lambda_m = \frac{\pi + 2\pi m}{2h}. \tag{9}$$

The substitution (8), (9) into the Eq. (2) leads to a sequence of ordinary differential equations for the functions $f_n(x)$, $g_m(x)$. General solutions of these equations are obtained in the form:

$$f(x) = A_n \sin \lambda_n p_n x + B_n \cos \lambda_n p_n x, \tag{10}$$

$$g_m(x) = C_m e^{-\lambda_m q_m x} + D_m e^{-\lambda_m q_m x}, \tag{11}$$

where $A_n,\ B_n,\ C_m,\ D_m$ are arbitrary constants and

$$p_n = \sqrt{\frac{w^2}{\lambda_n^2 c_1^2} - 1}, \quad g_m = \sqrt{1 - \frac{w^2}{\lambda_m^2 c_2^2}}. \tag{12}$$

Given the condition (6), the solution (10) will be rewritten as follows:

$$f_n(x) = F_n \cos[\lambda_n p_n (a + x)], \tag{13}$$

where F_n are new arbitrary constants. Taking into account the damping condition (7), the solutions (11) will take the following form:

$$g_m(x) = C_m e^{-\lambda_m q_m x}, \tag{14}$$

and from (5), (13), (14) it follows

$$\begin{gathered} \sum_{n=0}^{\infty} F_n \cos \lambda_n p_n a \cos \lambda_n y = \sum_{m=0}^{\infty} C_m \sin \lambda_m y, \\ -\mu_1 \sum_{n=0}^{\infty} F_n \lambda_n p_n \sin \lambda_n p_n a \cos \lambda_n y = -\mu_2 \sum_{m=0}^{\infty} C_m \lambda_m q_m \sin \lambda_m y. \end{gathered} \tag{15}$$

Taking into account the expansion in the Fourier series

$$\sin \lambda_m y = \sum_{n=0}^{\infty} b_{mn} \cos \lambda_n y, \tag{16}$$

from (15) we obtain the following system of infinite equations:

$$\begin{cases} \sum\limits_{m=0}^{\infty} b_{mn} C_m = F_n \cos \lambda_n p_n a, \\ \sum\limits_{m=0}^{\infty} b_{mn} \lambda_m q_m C_m = F_n \dfrac{\mu_1}{\mu_2} \lambda_n p_n \sin \lambda_n p_n a. \end{cases} \tag{17}$$

Excluding the unknowns F_n from the system (17), we arrive at an infinite system equations with unknowns C_m:

$$\sum_{m=0}^{\infty} b_{mn} \left(\tan P_n \xi - \frac{\mu_2}{\mu_1} \cdot \frac{Q_m}{P_n} \right) C_m = 0, \ \ n = 0, 1, 2, \dots, \tag{18}$$

where

$$\begin{gathered} P_n = \sqrt{\eta^2 - (1+2n)^2}, \ \ Q_m = \sqrt{(1+2m)^2 - \kappa^2 \eta^2}, \\ \eta = \frac{2h\omega}{\pi c_1}, \ \ \kappa = \frac{c_1}{c_2}, \ \ \xi = \frac{a\pi}{2h}, \end{gathered} \tag{19}$$

$$b_{mn} = \begin{cases} \dfrac{2}{(1+m+n)\pi}, & \text{if } \ m+n \ \text{ is an even number;} \\ \dfrac{2}{(m-n)\pi}, & \text{if } \ m+n \ \text{ is an odd number.} \end{cases} \tag{20}$$

The Solution of the Problem. Corresponding truncated systems will be considered. Then, in the m-th order approximation $(m = 0, 1, 2, \dots)$ from the condition that the solution of the truncated system is non-trivial, we obtain a characteristic equation for determining the dependence of η on ξ:

$$\begin{vmatrix} b_{00}\left(\tan P_0\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_0}{P_0}\right) & b_{10}\left(\tan P_0\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_1}{P_0}\right) & \cdots \\ \cdots & b_{m0}\left(\tan P_0\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_m}{P_0}\right) & \\ b_{01}\left(\tan P_1\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_0}{P_1}\right) & b_{11}\left(\tan P_1\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_1}{P_1}\right) & \cdots \\ \cdots & b_{m1}\left(\tan P_1\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_m}{P_1}\right) & \\ b_{0m}\left(\tan P_m\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_0}{P_m}\right) & b_{1m}\left(\tan P_m\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_1}{P_m}\right) & \cdots \\ \cdots & b_{mm}\left(\tan P_m\xi - \frac{\mu_2}{\mu_1}\cdot\frac{Q_m}{P_m}\right) & \end{vmatrix} = 0. \quad (21)$$

From (19), (20) and (21) it follows that on the zero approximation $(m = 0)$, the characteristic equation is an analogue of the characteristic equation of the Love wave [1]:

$$\tan\xi\sqrt{\eta^2-1} - \frac{\mu_2\sqrt{1-\kappa^2\eta^2}}{\mu_1\sqrt{\eta^2-1}} = 0. \quad (22)$$

This equation has real solutions only when $\kappa < 1$, i.e. when $c_1 < c_2$. These solutions set the frequency for the appropriate mode of oscillations. The frequency ω determined from (22) satisfies the inequality $\pi c_1/2h < \omega < \pi c_2/2h$ and depends on h and a. Moreover, for each value of ξ (the ratio a to h) there are several modes. The critical values, at which the birth of new modes begins are determined from Eq. (22) when its right-hand side is zero. As the parameter ξ increases, the frequencies of all modes decrease monotonically, tending asymptotically to $\pi c_1/2h$. Figs. 2 and 3 show the behavior of oscillation frequency modes at the zero approximation.

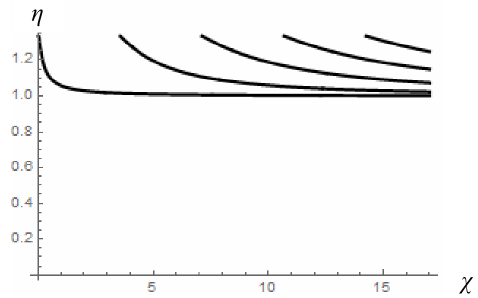

Fig. 2. Frequency graph according to Eq. (22)
when $\kappa = 0.75$, $\mu_2/\mu_1 = 0.2$.

In the m-th order approximation, the frequency ω determined from (21) satisfies the inequality $\pi(1 + 2m)c_1/2h < \omega < \pi c_2/2h$ and depends on the quantities h and a.

For example, in the first order approximation(m=1) the frequency ω satisfies the inequality $3\pi c_1/2h < \omega < \pi c_2/2h$. Fig. 4 shows the behavior of oscillation frequency modes at the first approximation $(m = 1)$, when $\kappa = 0.25$ and $\mu_2/\mu_1 = 0.5$.

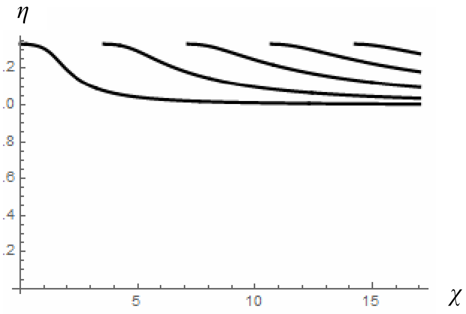

Fig. 3. Frequency graph according to Eq. (22)
when $\kappa = 0.75$, $\mu_2/\mu_1 = 5$.

The comparison with the previous figures shows that new frequency modes have appeared here, which asymptotically tend towards $3\pi c_1/2h$ monotonously descending, and the previous modes have been "cut off" by this asymptote.

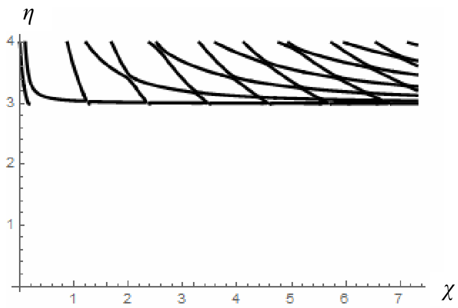

Fig. 4. Frequency graph according to the first approximation.

Conclusion. The possibility of localizing shear oscillations in the vicinity of the junction of different parts of the waveguide has been established. With an increase in the approximation order, the number of oscillation frequency modes also increases, and all the previous approximation modes are "cut off" by the new asymptote.

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Մ. Վ. ԲԵԼՈՒԲԵԿՅԱՆ, Ս. Լ. ՍԱՀԱԿՅԱՆ

ԲԱՂԱԴՐՅԱԼ ԱՌԱՁԳԱԿԱՆ ԿԻՍԱԱՆՎԵՐՋ ՀԱՐԹ ԱԼԻՔԱՏԱՐՈՒՄ ՍԱՀՔԻ ՏԱՏԱՆՈՒՄՆԵՐԻ ՏԵՂԱՅՆԱՑՄԱՆ ՄԱՍԻՆ

Այս հոդվածում դիտարկվում են հարթ կիսաանվերջ ալիքատարներ՝ դրանք սահմանափակող հարթություններում և դրանց եզրերում տարբեր սահմանային պայմանների դեպքում: Հաստատվել է ալիքատարների հարևան մասերի անցման (կցման) շրջակայքում սահքի ալիքների տեղայնացման հնարավորությունը:

М. В. БЕЛУБЕКЯН, С. Л. СААКЯН

О ЛОКАЛИЗАЦИИ СДВИГОВЫХ КОЛЕБАНИЙ В СОСТАВНОМ ПЛОСКОМ ПОЛУБЕСКОНЕЧНОМ УПРУГОМ ВОЛНОВОДЕ

В настоящей статье рассматриваются плоские полубесконечные волноводы при разных граничных условиях на плоскостях и на краях, ограничивающих волновод. Устанавливается возможность локализации сдвиговых колебаний в окрестности стыка разных частей полубесконечного плоского волновода.