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VIBRATIONS OF PIEZOELECTRIC LAYER OF CLASS 6 *mm* WITH RIGIDLY CLAMPED AND FREE EDGES UNDER INITIAL CONDITIONS

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The paper considers the problem of vibration of a piezoelectric layer of the class 6 *mm* with initial conditions in the form of impact of an external electric field or displacement, when one edge is rigidly grounded and the other is free. The layer displacement and internal electric field are determined.

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Introduction. The propagation of electroelastic waves from a piezoelectric material of the class 6 *mm* is considered in [1–3] on the basis of the hyperbolic equations of the electromagnetic field without using quasistatic approximations.

In [4], the problem of controlling the displacements of piezoelectric plates with the help of an electric field is considered.

In [5–8], magnetoelastic waves with initial conditions in the form of an initial deflection, an initial magnetic field, or an electromagnetic impulse are investigated. An infinite conducting plate in a constant longitudinal magnetic field with initial conditions is investigated in [5, 6]. In one case, the initial displacement is specified, and in the other, the initial impulse.

The propagation of one-dimensional magnetoelastic waves in an ideally conducting medium is studied in [7]. The initial conditions are set in the form of an initial magnetic impulse. Various problems of controlling with the mechanical system are considered in the monographs [9, 10].

In this paper, the problem of shear vibration of a piezoelectric layer of class 6 *mm* with the initial conditions in the form of impact of an external electric field and displacement of the layer, when one edge is rigidly grounded and the other is free. It is necessary to determine the initial impact of the external electric field and displacement, at which at the preferred moment of time we obtain the expected displacement of the layer.

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Problem Setting. Let a piezoactive layer made of a material of class 6 *mm* with the Cartesian coordinate system occupies an area $0<x<l,\ -\infty<y<\infty,\ -\infty<z<\infty$. The equation for propagation of purely shear electroelastic waves can be written in the following form [1–3]:

$$c_{44}\Delta w - e_{15}\left(\frac{\partial E_1}{\partial x}+\frac{\partial E_2}{\partial y}\right)=\rho\frac{\partial^2 w}{\partial t^2},\qquad \frac{\partial E_2}{\partial x}-\frac{\partial E_1}{\partial y}=-\mu\frac{\partial H_3}{\partial t},$$
$$\frac{\partial H_3}{\partial y}=e_{15}\frac{\partial^2 w}{\partial x\partial t}+\varepsilon_{11}\frac{\partial E_1}{\partial t},\qquad \frac{\partial H_3}{\partial x}=-e_{15}\frac{\partial^2 w}{\partial y\partial t}-\varepsilon_{11}\frac{\partial E_2}{\partial t}, \tag{1}$$

where w is the elastic displacement in the direction of the axis z; E_1, E_2 are components of the electric field strength along the directions of the axes x and y; H_3 is the component of the magnetic field in the direction of the axis z; μ is the magnetic permeability; c_{44} is the shear modulus; ρ is the elastic material density; ε_{11} is the dielectric constant and e_{15} is the piezo module material; $\Delta\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is Laplace operator.

Let's consider the one-dimensional case when the equation (1) does not depend on the coordinate y. Then the system (1) will be as follows

$$\begin{cases} c_{44}\frac{\partial^2 w}{\partial x^2}-e_{15}\frac{\partial E_1}{\partial x}=\rho\frac{\partial^2 w}{\partial t^2}, \\ e_{15}\frac{\partial^2 w}{\partial x\partial t}+\varepsilon_{11}\frac{\partial E_1}{\partial t}=0, \end{cases}\qquad \begin{cases} \frac{\partial E_2}{\partial x}=-\mu\frac{\partial H_3}{\partial t}, \\ \frac{\partial H_3}{\partial x}=-\varepsilon_{11}\frac{\partial E_2}{\partial t}. \end{cases} \tag{2}$$

The second system in (2) serves to determine the components of the electromagnetic field.

The boundary conditions are as follows:

$$x=0,\quad w=0,$$
$$x=l,\quad \sigma_{xz}=0, \tag{3}$$

where $\sigma_{xz}=c_{44}\frac{\partial w}{\partial x}-e_{15}E_1$.

It is required to find a solution to the first system in (2) that satisfies the boundary conditions (3) and the initial conditions.

At the initial moment $t=0$ of the final time interval, the following two variants of the initial conditions are given

$$t=0;\quad w\big|_{t=0}=f(x);\quad \frac{\partial w}{\partial t}\bigg|_{t=0}=g(x);\quad E_1=E_0(x). \tag{4}$$

From equation (2) for $E_1(x,t)$ we get

$$E_1(x,t)=-\frac{e_{15}}{\varepsilon_{11}}\cdot\frac{\partial w}{\partial x}+q(x),$$

and from the first initial condition (4) we have

$$q(x)=E_0(x)+\frac{e_{15}}{\varepsilon_{11}}\cdot\frac{df}{\partial x}.$$

Hence, for $E_1(x,t)$ we obtain

$$E_1(x,t) = E_0 + \frac{e_{15}}{\varepsilon_{11}}\left(\frac{df}{dx} - \frac{\partial w}{\partial x}\right). \tag{5}$$

Whence, for σ_{xz} we have

$$\sigma_{xz} = c_{44}(1+\chi)\frac{\partial w}{\partial x} - \frac{e_{15}^2}{\varepsilon_{11}}\cdot\frac{df}{dx} - e_{15}E_0, \tag{6}$$

where the following notation is adopted: $\chi = \dfrac{e_{15}^2}{c_{44}\varepsilon_{11}}$.

The first system of equation in (2), after eliminating $E_1(x,t)$, will be written in the following form

$$c_{44}(1+\chi)\frac{\partial^2 w}{\partial x^2} - \rho\frac{\partial^2 w}{\partial t^2} = e_{15}\frac{d}{dx}\left(E_0 + \frac{e_{15}}{\varepsilon_{11}}\cdot\frac{df}{dx}\right) \tag{7}$$

with boundary conditions

$$\begin{gathered} x = 0, \qquad w = 0, \\ x = l, \qquad c_{44}(1+\chi)\frac{\partial w}{\partial x} - \frac{e_{15}^2}{\varepsilon_{11}}\cdot\frac{df}{dx} - e_{15}E_0 = 0, \end{gathered} \tag{8}$$

and with initial conditions

$$t = 0, \quad w\big|_{t=0} = f(x), \quad \frac{\partial w}{\partial t}\bigg|_{t=0} = g(x), \quad E_1 = E_0(x). \tag{9}$$

In the case of matching the boundary and initial conditions $f(0) = 0$, $g(0) = 0$.

The problem (7)–(9) requires solving an inhomogeneous equation with an inhomogeneous boundary condition at $x = l$). Therefore, a direct application of the variable separation method is impossible.

To make the boundary conditions homogeneous, a transformation is suggested:

$$u(x,t) = c_{44}(1+\chi)w - \frac{e_{15}^2}{\varepsilon_{11}}f - e_{15}\int_0^x E_0(\xi)d\xi \tag{10}$$

or

$$w(x,t) = \frac{1}{c_{44}(1+\chi)}\left(u + \frac{e_{15}^2}{\varepsilon_{11}}f + e_{15}\int_0^x E_0(\xi)d\xi\right). \tag{11}$$

The function $u(x,t)$, taking into account the matching condition and the condition (9), must satisfy the following conditions

$$x = 0, \qquad u = 0, \qquad x = l, \qquad \frac{\partial u}{\partial x} = 0. \tag{12}$$

Substituting equation (11) into equation (7), we obtain

$$\begin{gathered} \frac{\partial^2 u}{\partial x^2} - \frac{1}{c_t^2}\cdot\frac{\partial^2 u}{\partial t^2} = 0, \\ c_t^2 = \frac{c_{44}(1+\chi)}{\rho}. \end{gathered} \tag{13}$$

The general solution of the homogeneous equation (13) with homogeneous boundary conditions (12) can be obtained by the method of separation of variables in the form

$$u(x,t)=\sum_{n=1}^{\infty}\left(A_n\sin(\lambda_n c_t t)+B_n\cos(\lambda_n c_t t)\right)\sin(\lambda_n x),\quad \lambda_n=\frac{\pi+2\pi n}{2l},\tag{14}$$

where $A_n,\ B_n$ arbitrary constants to be determined from the initial conditiions. According to equation (11), for the elastic displacement function we obtain

$$w(x,t)=\frac{1}{c_{44}(1+\chi)}\times\left(\sum_{n=1}^{\infty}\left(A_n\sin(\lambda_n c_t t)+B_n\cos(\lambda_n c_t t)\right)\sin(\lambda_n x)+\frac{e_{15}^2}{\varepsilon_{11}}f+e_{15}\int_0^x E_0(\xi)d\xi\right).\tag{15}$$

To satisfy the initial conditions (9), we expand the functions $f(x),\ E_0(x),\ g(x)$ in a raw:

$$f(x)=\sum_{n=1}^{\infty}a_n\sin(\lambda_n x),\quad E_0(x)=\sum_{n=1}^{\infty}b_n\cos(\lambda_n x),\quad f(x)=\sum_{n=1}^{\infty}c_n\sin(\lambda_n x).\tag{16}$$

Equation (15) satisfying the initial conditions, gives a system of algebraic equations for arbitrary constants

$$\begin{gathered}B_n+\frac{e_{15}^2}{\varepsilon_{11}}a_n+\frac{e_{15}}{\lambda_n}b_n=c_{44}(1+\chi)a_n,\\ \lambda_n A_n=c_{44}(1+\chi)c_n.\end{gathered}\tag{17}$$

Form here we get

$$B_n=c_{44}a_n-\frac{e_{15}}{\lambda_n}b_n,\qquad A_n=\frac{c_n\sqrt{c_{44}(1+\chi)\rho}}{\lambda_n}.\tag{18}$$

The electric field is determined according to equation (5) taking into account equations (18) and (15).

Special Cases.

First Case. Consider the particular case $E_0=0,\ g=0$. It follows that $b_n=0,\ c_n=0$, then we have $B_n=c_{44}a_n,\ A_n=0$. For $w(x,t)$ we obtain the following

$$w(x,t)=\frac{1}{(1+\chi)}\sum_{n=1}^{\infty}a_n\left(\cos(\lambda_n c_t t)+\chi\right)\sin(\lambda_n x).\tag{19}$$

Let's assume that at some time t_* the layer displacement is equal to zero $w(x,t_*)=0$, hence $\left(\cos(\lambda_n c_t t_*)+\chi=0\right)$.

Then the electric field will be written in the following form

$$E_1(x,t)=\frac{e_{15}}{e_{11}(1+\chi)}\sum_{n=1}^{\infty}\lambda_n a_n\left(1-\cos(\lambda_n c_t t)\right)\cos(\lambda_n x).\tag{20}$$

In the case when the electric field $E_1(x,t)$ is equal to zero, it explicitly does not depend on χ.

Second Case. Let the initial conditions be given in the following form $f(x)=0,\quad g(x)=0,\quad E_0(x)=\sum_{n=1}^{\infty}b_n\cos(\lambda_n x)$.

By specifying the electric field at the initial moment of time, it is possible to obtain the required value of the layer displacement. In this case $B_n = -\dfrac{e_{15}}{\lambda_n} b_n$, and for $w(x,t)$ we get the following

$$w(x,t) = \frac{1}{c_{44}(1+\chi)} \sum_{n=1}^{\infty} \frac{e_{15}}{\lambda_n} b_n \left(1 - \cos(\lambda_n c_t t)\right) \sin(\lambda_n x). \quad (21)$$

When $E_0(x) = b_1 \sin(\lambda_1 x)$, at the point $x = a$, $w(x,t)$, can be written as

$$w(a,T) = \frac{1}{c_{44}(1+\chi)} \cdot \frac{e_{15}}{\lambda_1} b_1 \left(1 - \cos(\lambda_1 c_t T)\right) \sin(\lambda_1 a). \quad (22)$$

Here the electric field excites an elastic displacement. Let at the given moment of time $t = T$, displacement of layer be maximum or minimum at $x = a$.

At $T^* = \dfrac{\pi}{\lambda_1 c_t}$ the layer displacement will be maximum and the value will depend on the initial electric field.

Based on equation (22), we can insist that varying the values of the electric field we can obtain the expected displacement of the layer at the point a at the fixed moment of time.

Third case. Let's study the next case, $f(x) = a_1 \sin(\lambda_1 x)$, $E_0(x) = b_1 \cos(\lambda_1 x)$, $g(x) = 0$. In this case, satisfying the initial conditions, we'll have:

$$A_n = 0,$$
$$B_1 = c_{44} a_1 - \frac{e_{15}}{\lambda_1} b_1, \quad B_n = 0, \quad n \neq 1.$$

The layer displacement will be written as follows:

$$w(x,t) = \frac{1}{c_{44}(1+\chi)} \left(\left(c_{44} \cos(\lambda_1 c_t t) + \frac{e_{15}^2}{\varepsilon_{11}} \right) a_1 + \frac{e_{15}}{\lambda_1} b_1 (\cos(\lambda_1 c_t t) - 1) \right) \sin(\lambda_n x).$$

Let's assume that at a fixed moment of time $t = t_*$ displacement of the layer is zero. From there we'll have

$$a_1 = b_1 \frac{e_{15}}{\lambda_1} \cdot \frac{1 - \cos(\lambda_1 c_t t_*)}{c_{44} \cos(\lambda_1 c_t t_*) + \dfrac{e_{15}^2}{\varepsilon_{11}}}.$$

From there we can obtain the value of the electromagnetic field, at which the displacement of the plate at the fixed moment of time is equal to zero.

Conclusion. The problem of vibrations of a piezoelectric layer of class 6 *mm* with the initial conditions in the form of the impact of an external electric field or displacement, when one edge is rigidly grounded and the other is free, is considered. The problem is solved based on the hypothetical equations of electromagnetic field without using the quasi static approximation. The displacement of the layer and initial electric field are determined. It is obtained that by varying the values of the initial displacement and the initial electric field, one can obtain the expected value of the layer displacement at the fixed moment of time.

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ՍԿԶԲՆԱԿԱՆ ՊԱՅՄԱՆՆԵՐՈՎ 6 *mm* ԴԱՍԻ ՊՅԵԶՈԷԼԵԿՏՐԻԿ ՇԵՐՏԻ ՏԱՏԱՆՈՒՄՆԵՐԸ, ԿՈՇՏ ԱՄՐԱԿՑՎԱԾ ԵՎ ԱԶԱՏ ԵԶՐԵՐԻ ԴԵՊՔՈՒՄ

Դիտարկվել է 6 *mm* դասի պյեզոէլեկտրիկ շերտի տատանումները սկզբանական պայմանների դեպքում հանձինս սկզբանական ճքվածքի կամ սկզբանական էլեկտրական դաշտի, երբ շերտի մի եզրը կոշտ ամրակցված է, իսկ մյուսը ազատ: Խնդիրը ուսումնասիրվել է էլեկտրամագնիսական դաշտի հիպերբոլական հավասարումների հիման վրա: Որոշվել են շերտի ճքվածքի և արտաքին էլեկտրական դաշտի մեծությունները:

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КОЛЕБАНИЯ ПЬЕЗОЭЛЕКТРИЧЕСКОГО СЛОЯ 6 mm-КЛАССА С ЖЕСТКО ЗАЩЕМЛЕННЫМ И СВОБОДНЫМ КРАЯМИ ПРИ НАЧАЛЬНЫХ УСЛОВИЯХ

Рассматривается задача колебаний пьезоэлектрического слоя 6 mm-класса при начальных условиях в виде воздействия внешнего электрического поля или перемещения слоя, когда один его край жестко защемлен, а другой свободен. Задача рассматривается на основе гиперболических уравнений электромагнитного поля. Определены перемещение слоя и внешнее электрическое поле.