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**C O M M U N I C A T I O N S**

*P h y s i c s*

# MOMENTUM OF AN ELECTROMAGNETIC WAVE IN TIME-VARYING DIELECTRIC MEDIA

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In the context of the Abraham–Minkowski controversy, the problem of the propagation of electromagnetic waves in a linear dielectric medium with a time-varying dielectric constant is considered. It is shown that the momentum of an electromagnetic wave in the form of Minkowski is preserved with an instantaneous change in the dielectric permittivity of the medium. At the same time, the Abraham momentum is not conserved, despite the spatial homogeneity of the problem. This circumstance is interpreted as a manifestation of the Abraham force.

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**Introduction.** While electromagnetic waves propagating in a vacuum are a self-sufficient form of materia, propagating in a medium, they acquire a "mechanical" component. This is in the sense that electromagnetic waves in a material medium are characterized not only by electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields, but also by an electric displacement $\boldsymbol{D}$ and a magnetic induction $\boldsymbol{B}$. And last two, as is known, in addition to the electric and magnetic fields, partly also represent the material medium through the polarization density $\boldsymbol{P}$ and magnetization density $\boldsymbol{M}$:

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P}, \qquad \boldsymbol{B} = \boldsymbol{E} + 4\pi\boldsymbol{M}.$$

The problem of the Abraham–Minkowski controversy is actually connected with this dual nature of electromagnetic waves propagating in a ponderable medium. In the general case, we are talking about the correct expression for the energy–momentum tensor of an electromagnetic field in a medium. In a more particular case, the problem

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can be formulated as follows: how does the momentum of an electromagnetic wave (photon) change when crossing media with different dielectric permittivities and magnetic permeabilities? From the expression proposed by Abraham [1, 2] it follows a momentum of $U/nc$ for the momentum of a light pulse of energy $U$ propagating in a medium of refractive index $n$. While the tensor constructed by Minkowski [3] predicts a momentum of $Un/c$.

Many studies [4–8] have been developed in favor of one or another approaches, but due to the absence of undeniable arguments supporting one over another of the proposed formulations of the energy–momentum tensor of an electromagnetic field in a medium, debate is ongoing.

In this article, we make a preliminary report on some observations about the momentum of an electromagnetic wave propagating in a dielectric medium with a time-varying permittivity.

**Sudden Change of the Dielectric Permittivity.** The problem of propagation of electromagnetic waves in a linear dielectric medium with a time-varying permittivity, was first discussed by F.R. Motgenthaler in 1958 [9]. In this section, following the article [10], we present some formulas that will be needed below. Gaussian units are adopted.

Consider a linear, homogeneous, isotropic, non-magnetic $(\mu = 1)$, and non-dispersive dielectric medium. Let us assume that the dielectric constant at some time instant $t_0$ changes discretely from $\varepsilon_1$ (at $t < t_0$) to $\varepsilon_2$ (at $t > t_0$). For these constant values of the dielectric constant, the solution to the source-free Maxwell macroscopic equations can be expressed in terms of plane monochromatic waves. For a wave with an initial frequency $\omega_{in} = ck/\sqrt{\varepsilon_1}$ and a constant amplitude $D_{in}$

$$D_{in}(\boldsymbol{r},t) \equiv D(\boldsymbol{r},t<0) = D_{in} e^{-i(\boldsymbol{kr}-\omega_{in}t)} \tag{1}$$

the sudden change of the permittivity gives rise to a superposition of two (transmitted $(T)$ and reflected $(R)$) waves

$$D_{out}(\boldsymbol{r},t) \equiv D(\boldsymbol{r},t>0) = D_T e^{-i(\boldsymbol{kr}-\omega_T t)} + D_R e^{-i(\boldsymbol{kr}-\omega_R t)} \tag{2}$$

propagating with opposite frequencies $\omega_T = -\omega_R = ck/\sqrt{\varepsilon_2} \equiv \omega_{out}$ and distinct amplitudes $D_T$ and $D_R$. The frequencies of the initial and transmitted waves are related via the relation [9]

$$\omega_{in} n_1 = \omega_{out} n_2, \tag{3}$$

where $n_1 = \sqrt{\varepsilon_1}$ and $n_2 = \sqrt{\varepsilon_2}$ represent the refractive indices of the medium before and after the change, respectively.

By calculating the Poynting vectors of the three waves $(in,\ T,\ R)$, one can readily find out that the waves on the right-hand side of Eq. 2 describe actual transmission and reflection in space.

The relationship between the amplitudes of $in$, $T$, and $R$ waves is established using the continuity conditions at $t_0$ for the electric displacement and magnetic induction:

$$D_R = D_{in} \frac{\omega_{out} - \omega_{in}}{2\omega_{out}} = \frac{1}{2} D_{in} \left(1 - \frac{n_2}{n_1}\right), \tag{4}$$

$$D_T = D_{in}\frac{\omega_{out}+\omega_{in}}{2\omega_{out}} = \frac{1}{2}D_{in}\left(1+\frac{n_2}{n_1}\right). \tag{5}$$

It should be noted that other boundary conditions are also used in the literature [11–13], which require the continuity of the electric and magnetic fields. There is no contradiction here, since these boundary conditions are mainly used to describe the propagation of electromagnetic waves in plasma, while plasma can in no way be regarded as a non-dispersive medium.

**Momentum Conservation.** Energy consideration of $in, T$ and $R$ waves has been carried out in many articles, e.g. in [9, 10], where was revealed the character of energy exchange between electromagnetic waves and a dielectric medium with dielectric permittivity changing suddenly. In the same problem, consider the exchange of momentum between a dielectric medium and electromagnetic waves.

Given the Minkowski form for electromagnetic wave momentum density

$$\boldsymbol{g}^M = \frac{1}{4\pi c}\boldsymbol{D}\times\boldsymbol{B}, \tag{6}$$

for $in, T$ and $R$ waves in Eqs. (1) and (2), we get

$$\boldsymbol{g}^M_{in} = \frac{|D_{in}|^2}{4\pi\omega_{in}\varepsilon_1}\boldsymbol{k}, \tag{7}$$

$$\boldsymbol{g}^M_{T} = \frac{|D_{T}|^2}{4\pi\omega_{out}\varepsilon_2}\boldsymbol{k}, \tag{8}$$

$$\boldsymbol{g}^M_{R} = -\frac{|D_{R}|^2}{4\pi\omega_{out}\varepsilon_2}\boldsymbol{k}. \tag{9}$$

According to Abraham's formula for an electromagnetic momentum density

$$\boldsymbol{g}^A = \frac{1}{4\pi c}\boldsymbol{E}\times\boldsymbol{H}, \tag{10}$$

the very same $in, T$ and $R$ waves are characterized by the momenta

$$\boldsymbol{g}^A_{in} = \frac{|D_{in}|^2}{4\pi\omega_{in}\varepsilon_1^2}\boldsymbol{k}, \tag{11}$$

$$\boldsymbol{g}^A_{T} = \frac{|D_{T}|^2}{4\pi\omega_{out}\varepsilon_2^2}\boldsymbol{k}, \tag{12}$$

$$\boldsymbol{g}^A_{R} = -\frac{|D_{R}|^2}{4\pi\omega_{out}\varepsilon_2^2}\boldsymbol{k}. \tag{13}$$

Taking into account Eqs. 3–5, it is easy to check that the Minkowski momentum is conserved – $\boldsymbol{g}^M_{in} = \boldsymbol{g}^M_R + \boldsymbol{g}^M_T$, while the Abraham momentum does not – $\boldsymbol{g}^A_{in} \neq \boldsymbol{g}^A_R + \boldsymbol{g}^A_T$.

Proceeding from the spatial homogeneity of the problem under consideration, on the basis of this particular result, it seems that in the context of the Abraham–Minkowski controversy, a decisive inference can be made in favor of Minkowski's expression for the energy–momentum tensor of an electromagnetic field in a dielectric medium. However, another interpretation is also possible. Due to the spatial homogeneity of the problem, according to the well-known theorem of Noether, it follows not the law of conservation of electromagnetic momentum, but the law of conservation of the momentum of the entire system, including the medium. Accordingly, if the momentum of the electromagnetic wave is not conserved within the framework of Abraham's approach, then this is due to that while changing in the dielectric permeability, the medium and the electromagnetic wave exchange not only energy, but also momentum. At the same time, this means that the interplay between the electromagnetic wave and the medium during the change of the dielectric permeability is of a force nature. In fact, this is a manifestation of the Abraham force [7, 8].

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## ԷԼԵԿՏՐԱՄԱԳՆԻՍԱԿԱՆ ԱԼԻՔԻ ԻՄՊՈՒԼՍԸ ԺԱՄԱՆԱԿԻ ՄԵՋ ՓՈՓՈԽՎՈՂ ԴԻԵԼԵԿՏՐԱԿԱՆ ՄԻՋԱՎԱՅՐԵՐՈՒՄ

Աբրահամ–Մինկովսկու հակասության համապեքստում դիտարկված է ժամանակի մեջ փոփոխվող դիէլեկտրական թափանցելիությամբ գծային դիէլեկտրական միջավայրում էլեկտրամագնիսական ալիքների տարածման խնդիրը: Ցույց է տրված, որ միջավայրի դիէլեկտրական հաստատունի ակնթարթային փոփոխության պայմաններում Մինկովսկու տեսքով էլեկտրամագնիսական ալիքի իմպուլսը պահպանվում է: Միևնույն ժամանակ Աբրահամի իմպուլսը չի պահպանվում՝ չնայած խնդրի տարածական համասեռությանը: Այս հանգամանքը մեկնաբանվում է որպես Աբրահամի ուժի դրսևորում:

К. К. ГРИГОРЯН

## МОМЕНТ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ В ДИЭЛЕКТРИЧЕСКИХ СРЕДАХ, МЕНЯЮЩИХСЯ ВО ВРЕМЕНИ

В контексте полемики Абрахама–Минковского рассмотрена задача распространения электромагнитных волн в линейной диэлектрической среде с изменяющейся во времени диэлектрической проницаемостью. Показано, что импульс электромагнитной волны в форме Минковского сохраняется при мгновенном изменении диэлектрической проницаемости среды. В то же время импульс Абрахама не сохраняется, несмотря на пространственную однородность задачи. Это обстоятельство трактуется как проявление силы Абрахама.