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*Mechanics*

# ON OPTIMAL STABILIZATION OF PART OF VARIABLES OF ROTARY MOVEMENT OF A RIGID BODY WITH ONE FIXED POINT IN THE CASE OF SOPHIA KOVALEVSKAYA

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An optimal stabilization problem for part of variables of rotary movement of a rigid body with one fixed point in the Sophia Kovalevskaya's case is discussed in this work. The differential equations of motion of the system are given and it is shown that the system may rotate around $Ox$ with a constant angular velocity. Taking this motion as unexcited, the differential equations for the corresponding excited motion were drawn up. Then the system was linearized and a control action was introduced along one of the generalized coordinates. The optimal stabilization problem for part of the variables was posed and solved. The graphs of optimal trajectories and optimal control were constructed.

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**Dynamics of the Rigid Body.** Consider a rigid body which rotates around a fixed point. Suppose the body is affected only by its own gravity, and its ellipsoid of inertia is a squeezed ellipsoid of rotation, which means $A = B = 2C$. Let as also assume that the center of mass of the body lays in the central plane of the ellipsoid of the inertia (Sophia Kovalevskaya's case) [1]. To examine the dynamics of the body, we choose a coordinate system $O\xi\eta\zeta$, which is fixed to the earth in such a way that the $O\xi$ axis is directed upwards vertically. Let us also fix the axis of the system $Oxyz$ along the main axis of inertia of the body, so that the $Oz$ axis is directed along the dynamical symmetry axis of the body and the $Oxy$ plane coincide with the central plane of the ellipsoid of inertia in such a way that the $Ox$ axis passes through the center of mass of the body $C$. Thus, the $Oy$ axis can be positioned uniquely.

Let $\overrightarrow{OC} = \vec{a}(a;0;0)$ be the radius vector of $C$ about the point $O$. Let the direction cosines of $O\xi$ axis about the axes $Ox$, $Oy$, $Oz$ be denoted as $\gamma_1$, $\gamma_2$, $\gamma_3$.

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Then the dynamical equations of Euler under the condition $A = B = 2C$ will have the following form [1]:

$$\begin{cases} 2C\dfrac{dp}{dt} - Cqr = 0, \\ 2C\dfrac{dq}{dt} + Cpr = Pa\gamma_3, \\ C\dfrac{dr}{dt} = -Pa\gamma_2 \end{cases}$$

or

$$\begin{cases} 2\dfrac{dp}{dt} - qr = 0, \\ 2\dfrac{dq}{dt} + rp = n\gamma_3 \\ \dfrac{dr}{dt} = -n\gamma_2, \end{cases} \tag{1}$$

where $\dfrac{Pa}{C} = n$. Let as add Poisson's equations to the system (1). We will have

$$\begin{cases} \dfrac{d\gamma_1}{dt} = r\gamma_2 - q\gamma_3, \\ \dfrac{d\gamma_2}{dt} = p\gamma_3 - r\gamma_1, \\ \dfrac{d\gamma_3}{dt} = q\gamma_1 - p\gamma_2. \end{cases} \tag{2}$$

Eqs. (1) and (2) together are a system of ordinary differential equations with 6 variables which are $p$, $q$, $r$, $\gamma_1$, $\gamma_2$, $\gamma_3$.

It is easy to show that the systems (1) and (2) have a solution

$$p = \omega = \text{const}, \;\; q = r = 0, \;\; \gamma_1 = 1, \;\; \gamma_2 = \gamma_3 = 0, \tag{3}$$

which means that the body can rotate around $Ox$ with a constant angular velocity $\omega$.

Let us construct the system of excited motion of the body by introducing the following notations:

$$x_1 = p - \omega, \;\; x_2 = q, \;\; x_3 = r, \;\; x_4 = \gamma_1 - 1, \;\; x_5 = \gamma_2, \;\; x_6 = \gamma_3.$$

Thus, we will have:

$$\begin{cases} \dot{x}_1 = 0, \\ \dot{x}_2 = -\dfrac{1}{2}\omega x_3, \\ \dot{x}_3 = -n x_5, \\ \dot{x}_4 = 0, \\ \dot{x}_5 = \omega x_6 - x_3, \\ \dot{x}_6 = x_2 - \omega x_5. \end{cases} \tag{4}$$

Here (4) will be linear approximation of differential equations of motion of the rigid body for the solution (3). In [2] the stability of (3) of the system (1), (2) has been examined and shown that the solution (3) is unstable [2, 3].

**Optimal Stabilization of Motion.** Let us introduce the control action $u$ along the generalized coordinate $x_2$ in (4). Then the system (4) will have the following form:

$$\begin{cases} \dot{x}_1 = 0, \\ \dot{x}_2 = -\frac{1}{2}\omega x_3 + u, \\ \dot{x}_3 = -n x_5, \\ \dot{x}_4 = 0, \\ \dot{x}_5 = \omega x_6 - x_3, \\ \dot{x}_6 = x_2 - \omega x_5. \end{cases} \tag{5}$$

Let us split (5) into two separate systems, which are

$$\begin{cases} \dot{x}_2 = -\frac{1}{2}\omega x_3 + u, \\ \dot{x}_3 = -n x_5, \\ \dot{x}_5 = \omega x_6 - x_3, \\ \dot{x}_6 = x_2 - \omega x_5, \end{cases} \tag{6}$$

and

$$\begin{cases} \dot{x}_1 = 0, \\ \dot{x}_4 = 0. \end{cases} \tag{7}$$

To check the controllability of (6) , let us use Kalman's principle [4]. We have

$$A = \begin{pmatrix} 0 & -\frac{1}{2}\omega & 0 & 0 \\ 0 & 0 & -n & 0 \\ 0 & -1 & 0 & \omega \\ 1 & 0 & -\omega & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$K = \left\{B, AB, A^2B, A^3B\right\}.$$

Therefore,

$$K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -n\omega \\ 0 & 0 & \omega & 0 \\ 0 & 1 & 0 & -\omega^2 \end{pmatrix}.$$

It is obvious that $\det K = n\omega^2 \neq 0$, and hence (6) is fully controllable. Now we are ready to pose the following

**Problem**. *Find an optimal control action $u^0$ such that the solution $x_1 = \cdots = x_6 = 0$ of (5) becomes asymptotically stable for the variables $x_2$, $x_3$, $x_5$, $x_6$ (for part of variables) [5], while minimizing the following functional:*

$$J[\cdot] = \int_0^\infty \left(u^2 + x_2^2 + x_3^2 + x_5^2 + x_6^2\right) dt. \tag{8}$$

We will solve the problem using Lyapunov–Belman's method [6,7]. As there are two parameters in (5), which are $n$ and $\omega$, then Lyapunov's optimal function and the optimal control action will both be functions containing the same parameters. From $\frac{Pa}{C} = n$ it is obvious that the value of $n$ depends on the shape of the rigid body and $\omega$ may have different values.

Belman's expression for the system (6) will have the form

$$B[\cdot] = \frac{\partial V}{\partial x_2}\dot{x}_2 + \frac{\partial V}{\partial x_3}\dot{x}_3 + \frac{\partial V}{\partial x_5}\dot{x}_5 + \frac{\partial V}{\partial x_6}\dot{x}_6 + u^2 + x_2^2 + x_3^2 + x_5^2 + x_6^2$$
$$= \frac{\partial V}{\partial x_2}\left(-\frac{1}{2}\omega x_3 + u\right) - \frac{\partial V}{\partial x_3} n x_5 + \frac{\partial V}{\partial x_5}(\omega x_6 - x_3)$$
$$+ \frac{\partial V}{\partial x_6}(x_2 - \omega x_5) + u^2 + x_2^2 + x_3^2 + x_5^2 + x_6^2. \quad (9)$$

As long as Belman's expression reaches 0, as its minimum value for the optimal control value, then we will have

$$\left.\frac{\partial B}{\partial u}\right|_{u=u^0} = \frac{\partial V}{\partial x_2} + 2u^0 = 0$$

and hence:

$$u^0 = -\frac{1}{2} \cdot \frac{\partial V}{\partial x_2}.$$

Substituting the above expression of $u^0$ into (9), we will have

$$B^0 = -\frac{1}{2} \cdot \frac{\partial V}{\partial x_2}\omega x_3 - \frac{1}{2}\left(\frac{\partial V}{\partial x_2}\right)^2 - \frac{\partial V}{\partial x_3} n x_5 + \frac{\partial V}{\partial x_5}\omega x_6 - \frac{\partial V}{\partial x_5} x_3$$
$$+ \frac{\partial V}{\partial x_6} x_2 - \frac{\partial V}{\partial x_6}\omega x_5 + \frac{1}{4}\left(\frac{\partial V}{\partial x_2}\right)^2 + x_2^2 + x_3^2 + x_5^2 + x_6^2 = 0. \quad (10)$$

For the system (6) let us choose a Lyapunov function in the following form:

$$V(x_2, x_3, x_5, x_6) = \frac{1}{2}\left(c_{22}x_2^2 + c_{33}x_3^2 + c_{55}x_5^2 + c_{66}x_6^2 + 2c_{23}x_2x_3 + 2c_{25}x_2x_5\right.$$
$$\left. + 2c_{26}x_2x_6 + 2c_{53}x_5x_3 + 2c_{36}x_3x_6 + 2c_{56}x_5x_6\right).$$

By substituting Lyapunov's function into (10), we get

$$-\frac{1}{2}\omega x_3(c_{22}x_2 + c_{23}x_3 + c_{25}x_5 + c_{26}x_6) - \frac{1}{4}(c_{22}x_2 + c_{23}x_3 + c_{25}x_5 + c_{26}x_6)^2$$
$$- n x_5(c_{33}x_3 + c_{23}x_2 + c_{35}x_5 + c_{36}x_6)$$
$$+ (\omega x_6 - x_3)(c_{55}x_5 + c_{25}x_2 + c_{35}x_3 + c_{56}x_6)$$
$$+ (x_2 - \omega x_5)(c_{66}x_6 + c_{26}x_2 + c_{36}x_3 + c_{56}x_5) + x_2^2 + x_3^2 + x_5^2 + x_6^2 = 0.$$

In the above equation the coefficients of $x_2^2$, $x_3^2$, $x_5^2$, $x_6^2$, $x_2x_3$, $x_2x_5$, $x_2x_6$, $x_5x_6$,

$x_5x_3$, $x_6x_3$ should be all equal to 0, so we get:

$$\begin{cases} -\frac{1}{4}c_{22}^2+c_{26}+1 &= 0, \\ -\frac{1}{2}\omega c_{23}-\frac{1}{4}c_{23}^2-c_{35}+1 &= 0, \\ -\frac{1}{4}c_{25}^2-nc_{35}-\omega c_{56}+1 &= 0, \\ -\frac{1}{4}c_{26}^2+\omega c_{56}+1 &= 0, \\ -\frac{1}{2}\omega c_{22}-\frac{1}{2}c_{22}c_{23}-c_{25}+c_{36} &= 0, \\ -\frac{1}{2}c_{22}c_{25}-nc_{23}+c_{56}-\omega c_{26} &= 0, \\ -\frac{1}{2}c_{22}c_{26}+\omega c_{25}+c_{66} &= 0 \\ -\frac{1}{2}\omega c_{25}-\frac{1}{2}c_{23}c_{25}-nc_{33}-c_{55}-\omega c_{36} &= 0, \\ -\frac{1}{2}\omega c_{26}-\frac{1}{2}c_{26}c_{23}+\omega c_{35}-c_{56} &= 0, \\ -\frac{1}{2}c_{26}c_{25}-nc_{36}+\omega c_{35}-\omega c_{66} &= 0. \end{cases} \tag{11}$$

The system (11) is a system of algebraic equations for the coefficients $c_{ij}$, which contains also the parameters $n$ and $\omega$. Let us fix a value for the parameter $n$ ($n = 10$ and $n = 50$). Now we can solve the optimal stabilization problem, and for the optimal Lyapunov function we will have:

$$V^0(x_1,x_2,\ldots,x_6) = \frac{1}{2}\left(c_{22}x_2^2+c_{33}x_3^2+c_{55}x_5^2+c_{66}x_6^2+2c_{23}x_2x_3+2c_{25}x_2x_5 \right. \\ \left. +2c_{26}x_2x_6+2c_{35}x_5x_3+2c_{36}x_3x_6+2c_{56}x_5x_6\right),$$

as for the optimal control actions we will have:

$$u^0 = -\left(c_{22}x_2+c_{23}x_3+c_{25}x_5+c_{26}x_6\right). \tag{12}$$

The minimum value of the functional will be as follows:

$$J[\cdot] = \frac{1}{2}\left(c_{22}x_{20}^2+c_{33}x_{30}^2+c_{55}x_{50}^2+c_{66}x_{60}^2+2c_{23}x_{20}x_{30}+2c_{25}x_{20}x_{50} \right. \\ \left. +2c_{26}x_{20}x_{60}+2c_{35}x_{50}x_{30}+2c_{36}x_{30}x_{60}+2c_{56}x_{50}x_{60}\right),$$

where $x_{i0} = x_i(0)$, $i = 2,3,5,6$.

For $n = 10$,

$$c_{22} = -2\cdot 10^{-7}\omega^6+2\cdot 10^{-5}\omega^5-0.0007\omega^4+0.0089\omega^3-0.0149\omega^2 \\ -0.2255\omega+9.6116,$$

$$c_{33} = 5\cdot 10^{-7}\omega^6-8\cdot 10^{-5}\omega^5+0.0051\omega^4-0.1608\omega^3+2.5579\omega^2 \\ -18.178\omega+62.659,$$

$$c_{55} = 6\cdot 10^{-7}\omega^6-9\cdot 10^{-5}\omega^5+0.0536\omega^4-1.5505\omega^3+23.449\omega^2 \\ -171.74\omega+498.06,$$

$$c_{66} = -8\cdot 10^{-6}\omega^6+9\cdot 10^{-5}\omega^5-0.0327\omega^4+0.4815\omega^3-1,8421\omega^2 \\ -4.1251\omega+35.141,$$

$$
\begin{aligned}
c_{23} &= 8\cdot 10^{-6}\omega^5 - 0.0009\omega^4 + 0.0352\omega^3 - 0.6605\omega^2 + 5.3425\omega - 28.62,\\
c_{25} &= 0,1\cdot 10^{-7}\omega^6 - 10^{-4}\omega^5 + 0.0076\omega^4 - 0.1929\omega^3 + 2.6003\omega^2\\
&\qquad -19.369\omega + 85.943,\\
c_{26} &= -10^{-6}\omega^6 + 10^{-4}\omega^5 - 0.0033\omega^4 + 0.0417\omega^3 - 0.0312\omega^2\\
&\qquad -1.3467\omega + 22.506,\\
c_{35} &= -2\cdot 10^{-6}\omega^6 + 0.0003\omega^5 - 0.0196\omega^4 + 0.5518\omega^3 - 8.0446\omega^2\\
&\qquad -56.764\omega + 176.92,\\
c_{36} &= 2\cdot 10^{-6}\omega^6 - 0.0002\omega^5 + 0.0053\omega^4 - 0.0314\omega^3 - 0.8817\omega^2\\
&\qquad +9.8418\omega - 46.894,\\
c_{56} &= -10^{-6}\omega^6 + 4\cdot 10^{-5}\omega^5 + 0.0027\omega^4 - 0.1963\omega^3 + 4.1351\omega^2\\
&\qquad -33.889\omega + 132.19.
\end{aligned}
$$

And for $n = 50$ we will have

$$
\begin{aligned}
c_{22} &= 5\cdot 10^{-7}\omega^6 - 5\cdot 10^{-5}\omega^5 + 0.0019\omega^4 - 0.0337\omega^3 + 0.2625\omega^2\\
&\qquad -0.9003\omega + 25.528,\\
c_{33} &= 3\cdot 10^{-5}\omega^6 - 0.0032\omega^5 + 0.1599\omega^4 - 4.0373\omega^3 + 54.298\omega^2\\
&\qquad -375.6\omega + 1152.7,\\
c_{55} &= 0.0012\omega^6 - 0.1535\omega^5 + 7.7596\omega^4 - 197.85\omega^3 + 2683.8\omega^2\\
&\qquad -18680.0\omega + 55996.0,\\
c_{66} &= 8\cdot 10^{-5}\omega^6 - 0.0088\omega^5 + 0.3441\omega^4 - 6.0166\omega^3 + 47.,389\omega^2\\
&\qquad -158.57\omega + 934.19,\\
c_{23} &= -3\cdot 10^{-6}\omega^6 + 0.0003\omega^5 - 0.015\omega^4 + 0.3728\omega^3 - 5.1356\omega^2\\
&\qquad +40.132\omega - 189.11,\\
c_{25} &= 10^{-5}\omega^6 - 0.0019\omega^5 + 0.093\omega^4 - 2.3969\omega^3 + 34.393\omega^2\\
&\qquad -281.85\omega + 1318.0,\\
c_{26} &= 5\cdot 10^{-6}\omega^6 - 0.0005\omega^5 + 0.019\omega^4 - 0.2941\omega^3 + 1.7179\omega^2\\
&\qquad -2.9522\omega + 151.95,\\
c_{35} &= -0.0002\omega^6 + 0.022\omega^5 - 1.1077\omega^4 + 28.149\omega^3 - 380.74\omega^2\\
&\qquad +2646.0\omega - 8031.1,\\
c_{36} &= -2\cdot 10^{-5}\omega^6 + 0.0026\omega^5 - 0.1183\omega^4 + 2.6507\omega^3 - 32.637\omega^2\\
&\qquad +231.45\omega - 1022.0,\\
c_{56} &= 0.0001\omega^6 - 0.0145\omega^5 + 0.6741\omega^4 - 15.853\omega^3 + 207.61\omega^2\\
&\qquad -1586.6\omega + 7077.8.
\end{aligned}
$$

Thus, having the value of parameter $n$ we can construct optimal Lyapunov function and the optimal control actions as functions of $\omega$.

**Construction of Optimal Trajectories.** Let us complete the solution of the problem for exact values of $n$ and $\omega$. To obtain the numerical solution of the system let us assume $n = 1s^{-2}$, $\omega = 10s^{-1}$. Then the system (11) will have the following form:

$$
\begin{cases}
-\frac{1}{4}c_{22}^2 + c_{26} + 1 &= 0, \\
-5c_{23} - \frac{1}{4}c_{23}^2 - c_{35} + 1 &= 0, \\
-\frac{1}{4}c_{25}^2 - c_{35} - 10c_{56} + 1 &= 0, \\
-\frac{1}{4}c_{26}^2 + 10c_{56} + 1 &= 0, \\
-5c_{22} - \frac{1}{2}c_{22}c_{23} - c_{25} + c_{36} &= 0, \\
-\frac{1}{2}c_{22}c_{25} - c_{23} + c_{56} - 10c_{26} &= 0 \\
-\frac{1}{2}c_{22}c_{26} + 10c_{25} + c_{66} &= 0, \\
-5c_{25} - \frac{1}{2}c_{23}c_{25} - c_{33} - c_{55} - 10c_{36} &= 0, \\
-5c_{26} - \frac{1}{2}c_{26}c_{23} + 10c_{35} - c_{56} &= 0, \\
-\frac{1}{2}c_{26}c_{25} - c_{36} + 10c_{55} - 10c_{66} &= 0.
\end{cases} \tag{13}
$$

From (13) we get

$$
\begin{array}{llll}
c_{22} = 3.7653, & c_{33} = 180.937, & c_{55} = 28.3935, & c_{66} = 30.9565, \\
c_{23} = -20.4556, & c_{25} = -2.6166, & c_{26} = 2.5444, & c_{35} = -1.3302, \\
c_{36} = -22.301, & c_{56} = 0.0618 & &
\end{array}
$$

as the solution of the system, which also makes Lyapunov function positive definit. So Lyapunov's optimal function will be

$$
\begin{aligned}
V^0(x_2, x_3, x_5, x_6) = \frac{1}{2}\big(&3.7653x_2^2 + 180.937x_3^2 + 28.3935x_5^2 \\
&+ 30.9565x_6^2 - 40.9112x_2x_3 - 5.2332x_2x_5 + 5.0888x_2x_6 \\
&-2.6604x_5x_3 - 44.602x_6x_3 + 0.1236x_5x_6\big).
\end{aligned} \tag{14}
$$

So,

$$
\begin{aligned}
u^0 &= -\frac{1}{2}(3.7653x_2 - 20.4556x_3 - 2.6166x_5 + 2.5444x_6) \\
&= -1.8827x_2 + 10.2278x_3 + 1.3083x_5 - 1.2722x_6.
\end{aligned} \tag{15}
$$

Let us substitute the expression of $u^0$ from (15) into (5). Thus, we will get the following system:

$$
\begin{cases}
\dot{x}_1 = 0, \\
\dot{x}_2 = -1.8827x_2 + 5.2278x_3 + 1.3083x_5 - 1.2722x_6, \\
\dot{x}_3 = -x_5, \\
\dot{x}_4 = 0, \\
\dot{x}_5 = 10x_6 - x_3, \\
\dot{x}_6 = x_2 - 10x_5.
\end{cases} \tag{16}
$$

Let us solve (16) for $x_1(0)=x_2(0)=x_3(0)=0.2,\ x_4(0)=x_5(0)=x_6(0)=0.1$. We will obtain the following solutions:

$$
\begin{aligned}
&x_1(t)=0.2,\\
&x_2(t)=-0.6875e^{-1.8117t}\left(e^{0.4463t}-1.3119e^{1.4364t}\right.\\
&\qquad\qquad \left.+0.0211e^{1.7408t}\cos[9.9753t]-0.0104e^{1.7408t}\sin[9.9753t]\right),\\
&x_3(t)=-0.0499e^{-1.8117t}\left(e^{0.4463t}-4.8551e^{1.4364t}\right.\\
&\qquad\qquad \left.-0.1514e^{1.7408t}\cos[9.9753t]+0.1539e^{1.7408t}\sin[9.9753t]\right),\\
&x_4(t)=0.1,\\
&x_5(t)=-0.0682e^{-1.8117t}\left(e^{0.4463t}-1.3348e^{1.4364t}\right.\\
&\qquad\qquad \left.-1.1323e^{1.7408t}\cos[9.9753t]-1.0979e^{1.7408t}\sin[9.9753t]\right),\\
&x_6(t)=0.0043e^{-1.8117t}\left(e^{0.446t}+4.8254e^{1.4364t}\right.\\
&\qquad\qquad \left.+17.35034e^{1.7408t}\cos[9.9753t]-18.1445e^{1.7408t}\sin[9.9753t]\right).
\end{aligned}
\tag{17}
$$

Let us now construct the graphs of the trajectories $x_i(t)$ $(i=1,\ldots,6)$ using Wolfram Mathematics (see Figs. 1–6).

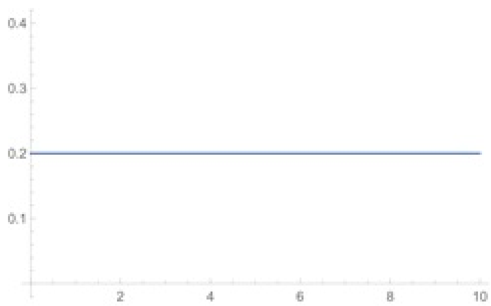

Fig. 1. Graph of function $x_1(t)$.

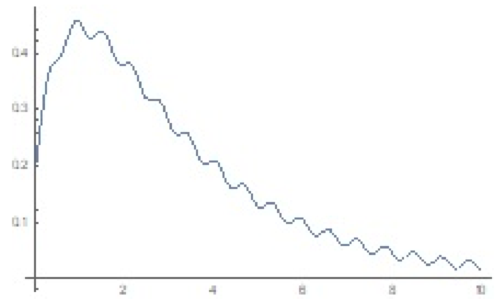

Fig. 2. Graph of function $x_2(t)$.

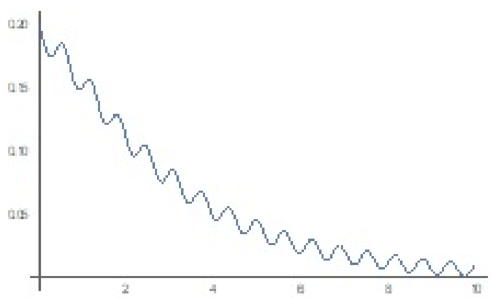

Fig. 3. Graph of function $x_3(t)$.

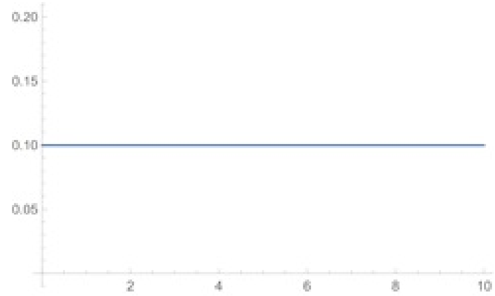

Fig. 4. Graph of function $x_4(t)$.

The graphs show that the optimal trajectories are approaching to the solution (3), which means that the system (5) is asymptotically stable of part of variables $x_2$, $x_3$, $x_5$, $x_6$.

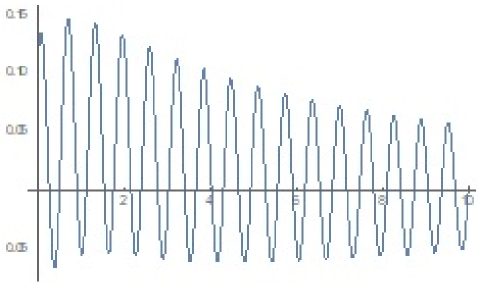

Fig. 5. Graph of function $x_5(t)$.

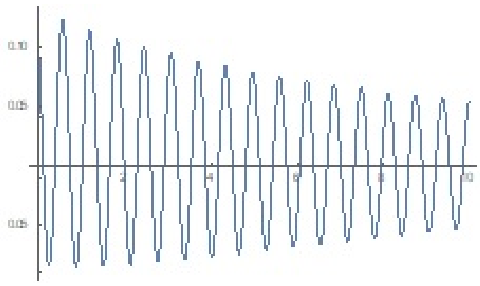

Fig. 6. Graph of function $x_6(t)$.

To get the exact expression of $u^0(t)$, we will substitute $x_i(t)$ $(i = 2,3,5,6)$ from (17) into (15). Then we will have

$$u^0(t) = 0.6893e^{-1.3657t} + 0.8725e^{-0.3753t} + 1.1042e^{-0.0709t}\cos[t] + 1.0495e^{-0.0709t}\sin[t].$$

Let us also construct the graphs of the optimal control $u^0(t)$ using Wolfram Mathematics (see Fig. 7).

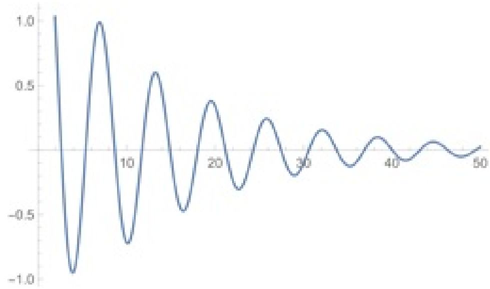


Fig. 7. Graph of function $x_5(t)$.

**Conclusion.** An optimal stabilization problem for part of variables of rotary movement of a rigid body with one fixed point in Sophia Kovalevskaya's case is discussed in this work. The differential equations of motion of the system are given and it is shown that the system may rotate around $Ox$ with constant angular velocity. Accepting this motion as an unexcited motion, the differential equations of the corresponding excited motion were drawn up. Then the system was linearized and a control action was introduced along one of the generalized coordinates. The optimal stabilization problem for part of the variables was stated and solved. The graphs of optimal trajectories were constructed and shown.

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## ՄԵԿ ԱՆՇԱՐԺ ԿԵՏ ՈՒՆԵՑՈՂ ՊԻՆԴ ՄԱՐՄՆԻ ՊՏՏԱԿԱՆ ՇԱՐԺՄԱՆ ԸՍՏ ՓՈՓՈԽԱԿԱՆՆԵՐԻ ՄԻ ՄԱՍԻ ՕՊՏԻՄԱԼ ԿԱՅՈՒՆԱՑՄԱՆ ՄԱՍԻՆ ՍՈՖՅԱ ԿՈՎՈԼԵՎՍԿԱՅԱՅԻ ԴԵՊՔՈՒՄ

Աշխատանքում դիտարկված է անշարժ կետի շուրջը պտտվող պինդ մարմնի օպտիմալ կայունացման խնդիրը ըստ փոփոխականների մի մասի Սոֆյա Կովոլևսկայայի դեպքում: Բերված են մարմնի շարժման դիֆերենցիալ հավասարումները, ցույց է տրված, որ ստացված համակարգը թույլ է տալիս պտույտ *Ox* առանցքի շուրջ հաստատուն անկյունային արագությամբ: Ընդունելով այդ շարժումը որպես չգրգռված շարժում, կազմվել են դրան համապատասխան գրգռված շարժման դիֆերենցիալ հավասարումները: Այնուհետև, սահմանափակվելով գծային մոտավորությամբ, ընդհանրացված կոորդինատներից մեկի ուղղությամբ ներմուծվել է ղեկավարող ազդեցություն: Ստացված ղեկավարվող համակարգի համար ձևակերպվել և լուծվել է օպտիմալ կայունացման խնդիր ըստ փոփոխականների մի մասի: Կառուցվել են օպտիմալ շարժումների և օպտիմալ ղեկավարող ազդեցության գրաֆիկները:

С. Г. ШАГИНЯН

# ОБ ОПТИМАЛЬНОЙ СТАБИЛИЗАЦИИ ПО ЧАСТИ ПЕРЕМЕННЫХ ВРАЩАТЕЛЬНОГО ДВИЖЕНИЯ ТВЕРДОГО ТЕЛА, ИМЕЮЩЕГО НЕПОДВИЖНУЮ ТОЧКУ

В работе рассматривается задача стабилизации по части переменных вращательного движения абсолютно твердого тела вокруг неподвижной точки в случае Софьи Ковалевской. Показано, что тело допускает вращение вокруг оси $Ox$ с постоянной угловой скоростью. Принимая то, что это движение является невозмущенным, составлена соответствующая система дифференциальных уравнений возмущенного движения. Далее, ограничиваясь линейным приближением, по направлению к одной из обобщенных координат введено управляющее воздействие. Для полученной управляющей системы сформулирована и решена задача оптимальной стабилизации по части переменных рассматриваемого движения. Построена оптимальная функция Ляпунова, получены оптимальное управляющее воздействие, уравнения оптимальных движений и минимальное значение функционала, зависящие от угловой скорости невозмущенного движения тела. Построены графики оптимальных движений и оптимального управляющего воздействия.