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TRANSFER OF LOADS FROM THREE HETEROGENEOUS ELASTIC STRINGERS WITH FINITE LENGTHS TO AN INFINITE SHEET THROUGH ADHESIVE LAYERS

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This paper considers the problem for an elastic infinite plate (sheet), which on parallel finite parts of its upper surface is strengthened by three finite stringers, two of which are located on the same line, having different elastic properties. The stringers are deformed under the action of horizontal forces. The interaction between infinite sheet and stringers takes place through thin elastic adhesive layers having other physical-mechanical properties and geometric configuration. The problem of determining unknown shear forces acting between the infinite sheet and stringers is reduced to a system of Fredholm integral equations of second kind with respect to unknown functions, which are specified on three finite intervals. It is shown that in the certain domain of the change of the characteristic parameters of the problem this system of integral equations can be solved by the method of successive approximations. Particular cases are considered, the character and behaviour of unknown shear forces are investigated. Further, for various values of changing characteristic parameters of the problem the multiple numerical results and its analysis are presented.

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Introduction. The problems associated with of load transfer from thin-walled elastic elements in the form of elastic stringers or overlays to more massive elastic bodies (modeled in the form of classical or non-classical regions of elasticity theory) through adhesive layers is one of the actual in both theoretical and applied aspects in the region of the contact and mixed problems of elasticity theory. Without going into details, we note that some of them which is closely associated with this article. In paper [1] considers the problem of loads transfer from two parallel elastic

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stringers with finite lengths to an infinite sheet through adhesive layers. The problems for an elastic strip and infinite sheet through adhesive layers, when two finite stringers are arranged on the same line, with different approach to the solution are considered in [2, 3]. The paper [4] considers the problem for an infinite sheet with two finite stringers when only one of the stringers is connected through an adhesive layer. In [5–8], using various approaches, problems are investigated for various elastic bodies, which are strengthened by a single finite stringer through adhesive layer. In [9, 10] transfer of loads from finite number of finite elastic stringers to an elastic infinite sheet (or half-plane) and to an infinite strip through an adhesive layers are considered. Some contact problems for an elastic infinite sheet strengthened by parallel finite stringers without adhesive layer are considered in [11]. In this article, a problem is considered for an elastic infinite sheet, which along two parallel lines of its upper surface is strengthened by three finite stringers two of which are located on the same line, having different elastic properties. The interaction between sheet and stringers is assumed to be carried out through thin adhesive layers with different physical-mechanical properties and geometric configuration.

Statement of the Problem and Obtaining the System of Integral Equations. Let an elastic infinite plate (sheet) of small constant thickness h, the Young's modulus E and the Poisson's ratio ν, which is in a generalized plane stress state (xOy is its middle plane), on its upper surface along $y = b$ and $y = -d$ parallel lines being $\ell = b + d$ (b, $d > 0$) distance from each other on the $[a_1, b_1]$, $[a_2, b_2]$ ($a_2 > b_1$) and $[c_1, d_1]$ finite intervals is strengthened by three finite stringers modulus of elasticity equal to E_1 for $x \in [a_1, b_1]$ ($b_1 > a_1$), E_2 for $x \in [a_2, b_2]$ ($b_2 > a_2$) and equal to E_3 for $x \in [c_1, d_1]$ ($d_1 > c_1$), respectively. It is supposed that the stringers have a rectangular cross-sections with small areas $F_1 = b_1^* h_1$, $F_2 = b_1^* h_2$ and $F_3 = b_3^* h_3$, respectively, where b_1^* ($b_1^* \ll b_1 - a_1$; $b_2 - a_2$), and b_3^* ($b_3^* \ll d_1 - c_1$), are the widths of the stringers and h_j $(j = \overline{1,3})$ are their small constant thicknesses. The interaction between

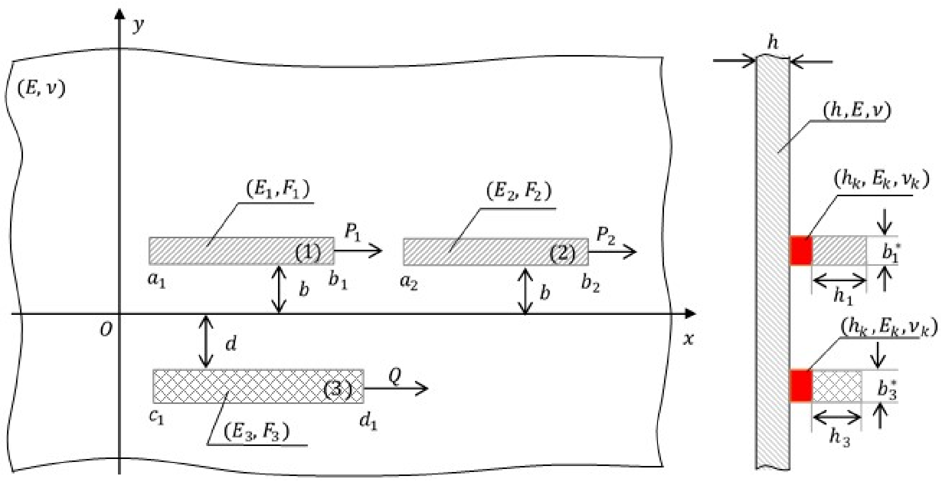

Fig. 1.

infinite sheet and stringers takes place through thin, uniform, elastic adhesive layers with Young's modulus E_k, Poisson's ratio ν_k, and small constant thickness h_k. The problem is to specify the law of distribution of unknown forces acting between the sheet and the stringers, when the concentrated forces P_1, P_2 and Q are applied at one end points of the stringers $x = b_1$, $x = b_2$ and $x = d_1$, respectively, and are directed to parallel along the Ox axis (see Fig. 1).

It is assumed that during the deformation for the stringers the model of uniaxial strain state in combination with the model of contact along the line is realized [12], and for the adhesive layer there is the pure shear condition [5], i.e. as in [5, 11, 12] bending is neglected and the interaction between sheet and stringers is idealized as a line loading of the sheet [1–10].

In view of above assumptions, let write the horizontal displacements $u_1(x,b)$ and $u_2(x,-d)$of the points of the elastic infinite plate (sheet), when tangential (shear) forces with intensity $p_1(x)$, $p_2(x)$ and $q(x)$ act on the $[a_1,b_1]$, $[a_2,b_2]$ and $[c_1,d_1]$, finite intervals of its upper surface along $y = b$ and $y = -d$ parallel lines, respectively, as in [1], in the following form:

$$u_1(x,b) = \frac{1}{\pi A^*}\int_{a_1}^{b_1}\left(\ln\frac{1}{|x-s|}+C\right)p_1(s)\,ds + \frac{1}{\pi A^*}\int_{a_2}^{b_2}\left(\ln\frac{1}{|x-s|}+C\right)p_2(s)\,ds + \frac{1}{\pi A^*}\int_{c_1}^{d_1}(N(x-\nu)+C)\,q(\nu)\,d\nu, \quad (1)$$

$$u_2(x,-d) = \frac{1}{\pi A^*}\int_{c_1}^{d_1}\left(\ln\frac{1}{|x-\nu|}+C\right)q(\nu)\,d\nu + \frac{1}{\pi A^*}\int_{a_1}^{b_1}(N(x-s)+C)\,p_1(s)\,ds + \frac{1}{\pi A^*}\int_{a_2}^{b_2}(N(x-s)+C)\,p_2(s)\,ds, \quad (2)$$

where

$$N(x) = \ln\frac{1}{\sqrt{x^2+\ell^2}} - \frac{\kappa\ell^2}{x^2+\ell^2}, \quad A^* = \frac{4Eh}{(1+\nu)(3-\nu)}, \quad \kappa = \frac{1+\nu}{3-\nu},$$

$p_1(x) = b_1^*\tau^{(1)}(x,b)$, $\tau^{(1)}(x,b)$ is the shear stresses, acting under of the stringer on the $[a_1,b_1]$ finite part, $p_2(x) = b_1^*\tau^{(2)}(x,b)$, $\tau^{(2)}(x,b)$ is the shear stresses, acting under of the stringer on the $[a_2,b_2]$ finite part, and $q(x) = b_3^*\tau^{(3)}(x,-d)$, $\tau^{(3)}(x,-d)$ is the shear stresses, acting under of the stringer on the $[c_1,d_1]$ finite part, C is arbitrary constant.

Note that, the horizontal displacements $u(x,y)$ of the points of an infinite sheet, arising in the upper half-plane, when shear forces act on its surface along the line $y = -d$ with intensity $\tau(x)$ $(-\infty < x < \infty)$ is given by the formula:

$$u(x,y) = \frac{1}{\pi A^*}\int_{-\infty}^{\infty}\left[\ln\frac{1}{\sqrt{(x-s)^2+(y+d)^2}} - \frac{\kappa(y+d)^2}{(x-s)^2+(y+d)^2}\right]\tau(s)\,ds + \text{const},$$

$$-\infty < x < \infty, \quad 0 < y < \infty.$$

Now, assuming that each differential element of the adhesive layers is in a condition of pure shear [1–10], the following contact conditions are obtained:

$$u^{(1)}(x)-u_1(x,b)=k_1^*p_1(x)\,,\qquad a_1\leqslant x\leqslant b_1\,,\tag{3}$$

$$u^{(2)}(x)-u_1(x,b)=k_1^*p_2(x)\,,\qquad a_2\leqslant x\leqslant b_2\,,\tag{4}$$

$$u^{(3)}(x)-u_2(x,-d)=k_2^*q(x)\,,\qquad c_1\leqslant x\leqslant d_1,\tag{5}$$

where $k_1^*=h_k/b_1^*G_k$, $k_2^*=h_k/b_3^*G_k$, $G_k=E_k/2(1+\nu_k)$, G_k is the shear modulus of adhesive layers, $u^{(1)}(x)=u^{(1)}(x,b)$, $u^{(2)}(x)=u^{(2)}(x,b)$ and $u^{(3)}(x)=u^{(3)}(x,-d)$ are the horizontal displacements of the points of the stringers at $y=b$ and $y=-d$ parallel lines, on the $[a_1,b_1]$, $[a_2,b_2]$ and $[c_1,d_1]$ finite intervals, respectively,

$$p_1(x)=b_1^*\tau^{(1)}(x,b)=b_1^*G_k\gamma_k^{(1)}(x,b),\qquad p_2(x)=b_1^*\tau^{(2)}(x,b)=b_1^*G_k\gamma_k^{(2)}(x,b)$$

and

$$q(x)=b_3^*\tau^{(3)}(x,-d)=b_3^*G_k\gamma_k^{(3)}(x,-d),\quad \gamma_k^{(1)}(x,b),\gamma_k^{(2)}(x,b)\ \text{ and }\ \gamma_k^{(3)}(x,-d)$$

are the shear deformations of the adhesive layers, on the $[a_1,b_1]$, $[a_2,b_2]$and $[c_1,d_1]$ finite intervals, respectively.

Further, taking into account the above assumptions, the differential equations for the equilibrium of the stringers on finite intervals $[a_1,b_1]$, $[a_2,b_2]$ and $[c_1,d_1]$ will be written in the following form:

$$\frac{d^2u^{(1)}}{dx^2}=\frac{p_1(x)}{E_1F_1}\,,\qquad a_1\leqslant x\leqslant b_1\,,\tag{6}$$

$$\frac{d^2u^{(2)}}{dx^2}=\frac{p_2(x)}{E_2F_2}\,,\qquad a_2\leqslant x\leqslant b_2\,,\tag{7}$$

$$\frac{d^2u^{(3)}}{dx^2}=\frac{q(x)}{E_3F_3},\qquad c_1\leqslant x\leqslant d_1\,,\tag{8}$$

which by virtue of (3), (4) and (5) can be written in the form:

$$\frac{d^2u^{(1)}}{dx^2}-\gamma_1^2u^{(1)}(x)=-\gamma_1^2u_1(x,b)\,,\qquad a_1\leqslant x\leqslant b_1\,,\tag{9}$$

$$\frac{d^2u^{(2)}}{dx^2}-\gamma_2^2u^{(2)}(x)=-\gamma_2^2u_1(x,b)\,,\qquad a_2\leqslant x\leqslant b_2\,,\tag{10}$$

$$\frac{d^2u^{(3)}}{dx^2}-\alpha^2u^{(3)}(x)=-\alpha^2u_2(x,-d)\,,\qquad c_1\leqslant x\leqslant d_1\,.\tag{11}$$

where we have also the following boundary conditions:

$$\left.\frac{du^{(1)}}{dx}\right|_{x=a_1}=0\,,\qquad \left.\frac{du^{(1)}}{dx}\right|_{x=b_1}=\frac{P_1}{E_1F_1}\,,\tag{12}$$

$$\left.\frac{du^{(2)}}{dx}\right|_{x=a_2}=0\,,\qquad \left.\frac{du^{(2)}}{dx}\right|_{x=b_2}=\frac{P_2}{E_2F_2}\,,\tag{13}$$

$$\left.\frac{du^{(3)}}{dx}\right|_{x=c_1}=0\,,\qquad \left.\frac{du^{(3)}}{dx}\right|_{x=d_1}=\frac{Q}{E_3F_3}\,.\tag{14}$$

Here $\gamma_1^2 = 1/k_1^* E_1 F_1$, $\gamma_2^2 = 1/k_1^* E_2 F_2$, $\alpha^2 = 1/k_2^* E_3 F_3$.

The solutions to the boundary value problems (9) and (12), (10) and (13), (11) and (14), respectively, we obtain in the form:

$$u^{(1)}(x) = u_0^{(1)}(x) + \gamma_1^2 \int_{a_1}^{b_1} G_1(x,s)\, u_1(s,b)\, ds\,, \qquad a_1 \leqslant x \leqslant b_1\,, \tag{15}$$

$$u^{(2)}(x) = u_0^{(2)}(x) + \gamma_2^2 \int_{a_2}^{b_2} G_2(x,s)\, u_1(s,b)\, ds\,, \qquad a_2 \leqslant x \leqslant b_2\,, \tag{16}$$

$$u^{(3)}(x) = u_0^{(3)}(x) + \alpha^2 \int_{c_1}^{d_1} K(x,v) u_2(v,-d) dv\,, \qquad c_1 \leqslant x \leqslant d_1\,, \tag{17}$$

where $u_0^{(1)}(x)$, $u_0^{(2)}(x)$ and $u_0^{(3)}(x)$ are the general solutions of the homogenous equations corresponding to Eqs. (9), (10) and (11), respectively, with the boundary conditions (12), (13) and (14), respectively, and have the following form:

$$u_0^{(j)}(x) = \frac{P_j \cos h[\gamma_j(x-a_j)]}{\gamma_j E_j F_j \sin h[\gamma_j(b_j-a_j)]} \;\; (j=1,2), \quad u_0^{(3)}(x) = \frac{Q \cos h[\alpha(x-c_1)]}{\alpha E_3 F_3 \sin h[\alpha(d_1-c_1)]}.$$

In Eqs. (15), (16) and (17), $u_*^{(j)}(x) = \gamma_j^2 \int_{a_j}^{b_j} G_j(x,s)\, u_1(s,b)\, ds$, $j=1,2$, and $u_*^{(3)}(x) = \alpha^2 \int_{c_1}^{d_1} K(x,v)\, u_2(v,-d) dv$ are the particular solutions of (9), (10) and (11) with zero boundary conditions corresponding to conditions (12), (13) and (14), respectively, $G_j(x,s)$ $(j=1,2)$ and $K(x,v)$ are Green's functions [13], and

$$G_j(x,s) = \frac{1}{\gamma_j \sin h[\gamma_j(b_j-a_j)]} \begin{cases} \cos h[\gamma_j(x-b_j)] \cos h[\gamma_j(s-a_j)]\,, & x>s, \\ \cos h[\gamma_j(x-a_j)] \cos h[\gamma_j(s-b_j)]\,, & x<s. \end{cases}$$

$$K(x,v) = \frac{1}{\alpha \sin h[\alpha(d_1-c_1)]} \begin{cases} \cos h[\alpha(x-d_1)] \cos h[\alpha(v-c_1)]\,, & x>v, \\ \cos h[\alpha(x-c_1)] \cos h[\alpha(v-d_1)]\,, & x<v. \end{cases}$$

It is obvious that the functions $G_j(x,s)$ and $K(x,v)$ are continuous functions and $G_j(x,s) = G_j(s,x)$, $K(x,v) = K(v,x)$.

Further, by virtue of (15), (16) and (17), according to (3), (4) and (5), we obtain the following equations:

$$k_1^* p_1(x) + u_1(x,b) = \gamma_1^2 \int_{a_1}^{b_1} G_1(x,s)\, u_1(s,b)\, ds + u_0^{(1)}(x)\,, \qquad a_1 \leqslant x \leqslant b_1\,, \tag{18}$$

$$k_1^* p_2(x) + u_1(x,b) = \gamma_2^2 \int_{a_2}^{b_2} G_2(x,s) u_1(s,b)\, ds + u_0^{(2)}(x)\,, \qquad a_2 \leqslant x \leqslant b_2\,, \tag{19}$$

$$k_2^* q(x) + u_2(x,-d) = \alpha^2 \int_{c_1}^{d_1} K(x,v)\, u_2(v,-d) dv + u_0^{(3)}(x)\,, \qquad c_1 \leqslant x \leqslant d_1\,. \tag{20}$$

Now, by virtue of (1) and (2), from (18), (19) and (20) we obtain the following system of integral equations:

$$
\begin{gathered}
p_1(x)+\frac{1}{\pi A^* k_1^*}\left[\int\limits_{a_1}^{b_1}\left(\ln\frac{1}{|x-s|}+C\right)p_1(s)\,ds+\int\limits_{a_2}^{b_2}\left(\ln\frac{1}{|x-s|}+C\right)p_2(s)\,ds\right.\\
\left.+\int\limits_{c_1}^{d_1}(N(x-v)+C)\,q(v)\,dv\right]=\frac{\gamma_1^2}{\pi A^* k_1^*}\int\limits_{a_1}^{b_1}G_1(x,s)\left[\int\limits_{a_1}^{b_1}\left(\ln\frac{1}{|s-t|}+C\right)p_1(t)\,dt\right.\\
\left.+\int\limits_{a_2}^{b_2}\left(\ln\frac{1}{|s-t|}+C\right)p_2(t)\,dt+\int\limits_{c_1}^{d_1}(N(s-\tau)+C)\,q(\tau)\,d\tau\right]ds+\frac{u_0^{(1)}(x)}{k_1^*},\ a_1\leqslant x\leqslant b_1,\\
p_2(x)+\frac{1}{\pi A^* k_1^*}\left[\int\limits_{a_1}^{b_1}\left(\ln\frac{1}{|x-s|}+C\right)p_1(s)\,ds+\int\limits_{a_2}^{b_2}\left(\ln\frac{1}{|x-s|}+C\right)p_2(s)\,ds\right.\\
\left.+\int\limits_{c_1}^{d_1}(N(x-v)+C)\,q(v)\,dv\right]=\frac{\gamma_2^2}{\pi A^* k_1^*}\int\limits_{a_2}^{b_2}G_2(x,s)\left[\int\limits_{a_1}^{b_1}\left(\ln\frac{1}{|s-t|}+C\right)p_1(t)\,dt\right.\\
\left.+\int\limits_{a_2}^{b_2}\left(\ln\frac{1}{|s-t|}+C\right)p_2(t)\,dt+\int\limits_{c_1}^{d_1}(N(s-\tau)+C)\,q(\tau)\,d\tau\right]ds+\frac{u_0^{(2)}(x)}{k_1^*},\ a_2\leqslant x\leqslant b_2,\\
q(x)+\frac{1}{\pi A^* k_2^*}\left[\int\limits_{c_1}^{d_1}\left(\ln\frac{1}{|x-v|}+C\right)q(v)\,dv+\int\limits_{a_1}^{b_1}(N(x-s)+C)\,p_1(s)\,ds\right.\\
\left.+\int\limits_{a_2}^{b_2}(N(x-s)+C)\,p_2(s)\,ds\right]=\frac{\alpha^2}{\pi A^* k_2^*}\int\limits_{c_1}^{d_1}K(x,v)\left[\int\limits_{c_1}^{d_1}\left(\ln\frac{1}{|v-\tau|}+C\right)q(\tau)\,d\tau\right.\\
\left.+\int\limits_{a_1}^{b_1}(N(v-t)+C)\,p_1(t)\,dt+\int\limits_{a_2}^{b_2}(N(v-t)+C)\,p_2(t)\,dt\right]dv+\frac{u_0^{(3)}(x)}{k_2^*},\ c_1\leqslant x\leqslant d_1.
\end{gathered}
\tag{21}
$$

It should be noted that the spectrum of the symmetric second-order differential operator $D=-d^2/dx^2+\gamma^2 I$, with the domain of definition being twice continuous differentiating functions, satisfying the boundary conditions $\left(du^{(1)}/dx\right)_{x=a}=0$ and $\left(du^{(1)}/dx\right)_{x=b}=0$, are eigenvalues $\lambda_n=\gamma^2+n^2\pi^2/(b-a)^2$ $(n=0,1,2,\ldots)$ with corresponding eigenfunctions $\cos[n\pi(x-a)/(b-a)]$ $(n=0,1,2,\ldots)$.

It is known [13], that symmetric quite continuous integral operator A:

$$
A\varphi=\int\limits_a^b G(x,s)\,\varphi(s)\,ds,
$$

which acts in the space $L_2(a,b)$ is an inverse of the operator D. Hence, we have:

$$\int_{a_j}^{b_j} G_j(x,s)\cos\left[\frac{n\pi(s-a_j)}{b_j-a_j}\right]ds = \frac{(b_j-a_j)^2}{(b_j-a_j)^2\gamma_j^2+n^2\pi^2}\cos\left[\frac{n\pi(x-a_j)}{b_j-a_j}\right], \quad n=0,1,2,\ldots,\quad j=1,2, \tag{22}$$

$$\int_{c_1}^{d_1} K(x,v)\cos\left[\frac{m\pi(v-c_1)}{d_1-c_1}\right]dv = \frac{(d_1-c_1)^2}{(d_1-c_1)^2\alpha^2+m^2\pi^2}\cos\left[\frac{m\pi(x-c_1)}{d_1-c_1}\right], \quad m=0,1,2,\ldots, \tag{23}$$

where the functions $\cos\left[\frac{n\pi(x-a_j)}{b_j-a_j}\right]$ $(j = 1,2)$ and $\cos\left[\frac{m\pi(x-c_1)}{d_1-c_1}\right]$ $(n,m=0,1,2,\ldots)$ form full orthogonal systems in the spaces $L_2(a_j,b_j)$ and $L_2(c_1,d_1)$, respectively.

Further, after replacing the variables x, s, v, t and τ by ax, as, av, at and $a\tau$, respectively, where $a>0$ is the coordinate of one of the end points of stringers, we will represent the system of integral Eq. (21) in the following form:

$$\begin{aligned}
&\varphi_1(x)+\delta_1^2\int_{\alpha_1}^{\beta_1} M_1(x,t)\,\varphi_1(t)\,dt+\delta_1^2\int_{\alpha_2}^{\beta_2} M_1(x,t)\,\varphi_2(t)\,dt \\
&\qquad +\delta_1^2\int_{\xi_1}^{\eta_1} H_1(x,\tau)\,\psi(\tau)\,d\tau = f_0^{(1)}(x), \quad \alpha_1\leqslant x\leqslant\beta_1, \\
&\varphi_2(x)+\delta_1^2\int_{\alpha_1}^{\beta_1} M_2(x,t)\,\varphi_1(t)\,dt+\delta_1^2\int_{\alpha_2}^{\beta_2} M_2(x,t)\,\varphi_2(t)\,dt \\
&\qquad +\delta_1^2\int_{\xi_1}^{\eta_1} H_2(x,\tau)\,\psi(\tau)\,d\tau = f_0^{(2)}(x), \quad \alpha_2\leqslant x\leqslant\beta_2, \\
&\psi(x)+\overline{\delta}_1^2\int_{\xi_1}^{\eta_1} R(x,\tau)\,\psi(\tau)\,d\tau+\overline{\delta}_1^2\int_{\alpha_1}^{\beta_1} T(x,t)\,\varphi_1(t)\,dt \\
&\qquad +\overline{\delta}_1^2\int_{\alpha_2}^{\beta_2} T(x,t)\,\varphi_2(t)\,dt = q_0(x), \quad \xi_1\leqslant x\leqslant\eta_1,
\end{aligned} \tag{24}$$

where

$$
\begin{aligned}
&\varphi_1(x)=p_1(ax),\ \varphi_2(x)=p_2(ax),\ \psi(x)=q(ax),\\
&N(ax)=\ln\frac{1}{a}+N_1(x),\ N_1(x)=\ln\frac{1}{\sqrt{x^2+\ell_*^2}}-\frac{\kappa\ell_*^2}{x^2+\ell_*^2},\\
&\alpha_1=a_1/a,\ \beta_1=b_1/a,\ \alpha_2=a_2/a,\ \beta_2=b_2/a,\ \xi_1=c_1/a,\ \eta_1=d_1/a,\\
&\ell_*=\ell/a,\ \delta_1^2=a/\pi k_1^*A^*,\ \overline{\delta}_1^2=a/\pi k_2^*A^*.\\
&M_j(x,t)=\ln\frac{1}{|x-t|}-a\gamma_j^2\int\limits_{\alpha_j}^{\beta_j}G_j(ax,as)\ln\frac{1}{|s-t|}\,ds,\ \ j=1,2,\\
&H_j(x,\tau)=N_1(x-\tau)-a\gamma_j^2\int\limits_{\alpha_j}^{\beta_j}G_j(ax,as)N_1(s-\tau)\,ds,\quad j=1,2,\\
&R(x,\tau)=\ln\frac{1}{|x-\tau|}-a\alpha^2\int\limits_{\xi_1}^{\eta_1}K(ax,av)\ln\frac{1}{|v-\tau|}\,dv,\\
&T(x,t)=N_1(x-t)-a\alpha^2\int\limits_{\xi_1}^{\eta_1}K(ax,av)N_1(v-t)\,dv,\\
&f_0^{(j)}(x)=p_0^{(j)}(ax)=\frac{u_0^{(j)}(ax)}{k_1^*}=\frac{P_j\gamma_j\cos h[a\gamma_j(x-\alpha_j)]}{\sin h[a\gamma_j(\beta_j-\alpha_j)]},\quad j=1,2,\\
&q_0(x)=q_0^{(3)}(ax)=\frac{u_0^{(3)}(ax)}{k_2^*}=\frac{Q\alpha\cos h[a\alpha(x-\xi_1)]}{\sin h[a\alpha(\eta_1-\xi_1)]}.
\end{aligned}
\tag{25}
$$

since according to (22) and (23) we have also the following equalities:

$$
\int\limits_{\alpha_j}^{\beta_j}G_j(ax,as)\,ds=\frac{1}{a\gamma_j^2},\ \ j=1,2,\qquad \int\limits_{\xi_1}^{\eta_1}K(ax,av)\,dv=\frac{1}{a\alpha^2}. \tag{26}
$$

It is obvious, that the functions $f_0^{(j)}(x)$ and $q_0(x)$ are integrable functions on the segments $x\in[\alpha_1,\beta_1]$, $x\in[\alpha_2,\beta_2]$ and $x\in[\xi_1,\eta_1]$, respectively.

Note that the system of integral equations (24) is obtained by the changing the order of integration, the validity of which follows from the Fubini's theorem [13]. This theorem will often be used below without special mention.

Now let us consider several particular cases that are directly obtained from the system (24). In the case $\delta_1^2=\overline{\delta}_1^2=0$, from the system (24) we obtain the solution of the corresponding problem for the case of a rigid sheet (i.e. when $E\to\infty$) in the form $\varphi_1(x)=f_0^{(1)}(x)$, $x\in[\alpha_1,\beta_1]$, $\varphi_2(x)=f_0^{(2)}(x)$, $x\in[\alpha_2,\beta_2]$ and $\psi(x)=q_0(x)$, $x\in[\xi_1,\eta_1]$, respectively. In the case of two parallel finite stringers arranged on the segments $[a_1,b_1]$ and $[c_1,d_1]$, instead of the system (24), we will obtain a system of Fredholm integral equations of the second kind with respect to an unknown functions $\varphi_1(x)$ and $\psi(x)$ defined on the segments $[\alpha_1,\beta_1]$ and $[\xi_1,\eta_1]$, respectively, in the following form:

$$\begin{aligned}&\varphi_1(x)+\delta_1^2\int\limits_{\alpha_1}^{\beta_1}M_1(x,t)\varphi_1(t)dt+\delta_1^2\int\limits_{\xi_1}^{\eta_1}H_1(x,\tau)\psi(\tau)d\tau=f_0^{(1)}(x),\ \alpha_1\leqslant x\leqslant\beta_1,\\&\psi(x)+\overline{\delta}_1^2\int\limits_{\xi_1}^{\eta_1}R(x,\tau)\psi(\tau)d\tau+\overline{\delta}_1^2\int\limits_{\alpha_1}^{\beta_1}T(x,t)\varphi_1(t)dt=q_0(x),\ \xi_1\leqslant x\leqslant\eta_1.\end{aligned}\tag{27}$$

In the case of two finite stringers arranged on the segments $[a_1,b_1]$ and $[a_2,b_2]$, instead of (24), we will obtain the system of Fredholm integral equations of the second kind with respect to an unknown functions $\varphi_1(x)$ and $\varphi_2(x)$ defined on the segments $[\alpha_1,\beta_1]$ and $[\alpha_2,\beta_2]$, respectively, in the following form:

$$\begin{aligned}&\varphi_1(x)+\delta_1^2\int\limits_{\alpha_1}^{\beta_1}M_1(x,t)\varphi_1(t)dt+\delta_1^2\int\limits_{\alpha_2}^{\beta_2}M_1(x,t)\varphi_2(t)dt=f_0^{(1)}(x),\ \alpha_1\leqslant x\leqslant\beta_1,\\&\varphi_2(x)+\delta_1^2\int\limits_{\alpha_1}^{\beta_1}M_2(x,t)\varphi_1(t)\,dt+\delta_1^2\int\limits_{\alpha_2}^{\beta_2}M_2(x,t)\varphi_2(t)\,dt=f_0^{(2)}(x),\ \alpha_2\leqslant x\leqslant\beta_2.\end{aligned}\tag{28}$$

In the case of one finite stringer defined on the segment $[a_1,b_1]$ or on the segment $[c_1,d_1]$, instead of system (24), we will obtain the Fredholm integral equation of the second kind with respect to an unknown function $\varphi_1(x)$ defined on the segment $[\alpha_1,\beta_1]$ in the following form:

$$\varphi_1(x)+\delta_1^2\int\limits_{\alpha_1}^{\beta_1}M_1(x,t)\,\varphi_1(t)\,dt=f_0^{(1)}(x),\quad \alpha_1\leqslant x\leqslant\beta_1,\tag{29}$$

or with respect to an unknown function $\psi(x)$ defined on the segment $[\xi_1,\eta_1]$, respectively, in the following form:

$$\psi(x)+\overline{\delta}_1^2\int\limits_{\xi_1}^{\eta_1}R(x,\tau)\,\psi(\tau)\,d\tau=q_0(x),\quad \xi_1\leqslant x\leqslant\eta_1.$$

Note that the system (24) was obtained without using the stringers equilibrium conditions:

$$\int\limits_{\alpha_1}^{\beta_1}p_1(ax)\,dx=P_1/a,\quad \int\limits_{\alpha_2}^{\beta_2}p_2(ax)\,dx=P_2/a,\quad \int\limits_{\xi_1}^{\eta_1}q(ax)\,dx=Q/a.\tag{30}$$

In the system (24), the conditions (30) are satisfied automatically, since the following equalities hold:

$$\int\limits_{\alpha_1}^{\beta_1}f_0^{(1)}(x)\,dx=P_1/a,\ \int\limits_{\alpha_2}^{\beta_2}f_0^{(2)}(x)\,dx=P_2/a,\ \int\limits_{\xi_1}^{\eta_1}q_0(x)\,dx=\mathrm{Q}/a.$$

These can be easily verified by integrating the first equation of (24) from α_1 to β_1, the second equation from α_2 to β_2, and the third equation from ξ_1 to η_1, then changing the order of integration in the resulting double integrals and taking into account the equalities

$\int\limits_{\alpha_1}^{\beta_1} M_1(x,t)\,dx = 0,\ \int\limits_{\alpha_2}^{\beta_2} M_2(x,t)\,dx = 0,\ \int\limits_{\alpha_1}^{\beta_1} H_1(x,\tau)\,dx = 0,\ \int\limits_{\alpha_2}^{\beta_2} H_2(x,\tau)\,dx = 0,$

$\int\limits_{\xi_1}^{\eta_1} R(x,\tau)\,dx = 0,\ \int\limits_{\xi_1}^{\eta_1} T(x,t)\,dx = 0$, which follow from (26).

Thus, solving the problem is reduced to solving the system (24) of Fredholm integral equations of the second kind with squarely integrable kernels in two variables and with right-hand sides, which are the solutions of the problem in the case of rigid sheet. From the system (24), it is easy to see that at the end points of the stringers $x=\alpha_1$, $x=\beta_1$, $x=\alpha_2$, $x=\beta_2$ and $x=\xi_1$, $x=\eta_1$, the values of unknown functions $\varphi_1(x)$, $\varphi_2(x)$ and $\psi(x)$, respectively, are finite.

Investigation of the Solvability of the System of Integral Equations (24). Now write the system (24) in the following form:

$$\varphi + K\varphi = y_0, \tag{31}$$

where

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{pmatrix}, \quad y_0 = \begin{pmatrix} f_0^{(1)} \\ f_0^{(2)} \\ q_0 \end{pmatrix}, \quad K = \begin{pmatrix} \delta_1^2 k_{11} & \delta_1^2 k_{12} & \delta_1^2 k_{13} \\ \delta_1^2 k_{21} & \delta_1^2 k_{22} & \delta_1^2 k_{23} \\ \overline{\delta}_1^2 k_{31} & \overline{\delta}_1^2 k_{32} & \overline{\delta}_1^2 k_{33} \end{pmatrix},$$

$$k_{11}\varphi_1 = \int\limits_{\alpha_1}^{\beta_1} M_1(x,t)\varphi_1(t)dt,\ k_{12}\varphi_2 = \int\limits_{\alpha_2}^{\beta_2} M_1(x,t)\varphi_2(t)dt,\ k_{13}\psi = \int\limits_{\xi_1}^{\eta_1} H_1(x,\tau)\psi(\tau)d\tau,$$

$$k_{21}\varphi_1 = \int\limits_{\alpha_1}^{\beta_1} M_2(x,t)\varphi_1(t)dt,\ k_{22}\varphi_2 = \int\limits_{\alpha_2}^{\beta_2} M_2(x,t)\varphi_2(t)dt,\ k_{23}\psi = \int\limits_{\xi_1}^{\eta_1} H_2(x,\tau)\psi(\tau)d\tau,$$

$$k_{31}\varphi_1 = \int\limits_{\alpha_1}^{\beta_1} T(x,t)\varphi_1(t)dt,\ k_{32}\varphi_2 = \int\limits_{\alpha_2}^{\beta_2} T(x,t)\varphi_2(t)dt,\ k_{33}\psi = \int\limits_{\xi_1}^{\eta_1} R(x,\tau)\psi(\tau)d\tau. \tag{32}$$

Further, consider operator Eq. (31) in Banach space with elements $y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$, where $y_1(x) \in L_2(\alpha_1,\beta_1)$, $y_2(x) \in L_2(\alpha_2,\beta_2)$, $y_3(x) \in L_2(\xi_1,\eta_1)$ and with norm: $\|y\| = \max\left\{\|y_1\|_{L_2(\alpha_1,\beta_1)}, \|y_2\|_{L_2(\alpha_2,\beta_2)}, \|y_3\|_{L_2(\xi_1,\eta_1)}\right\}$. $L_2(\alpha_j,\beta_j)$, $j=1,2$, and $L_2(\xi_1,\eta_1)$ are spaces of square integrable functions, specified on the intervals (α_j,β_j), $j=1,2$, and (ξ_1,η_1), respectively.

Operators k_{11}, k_{22} and k_{33} act in the spaces $L_2(\alpha_1,\beta_1)$, $L_2(\alpha_2,\beta_2)$ and $L_2(\xi_1,\eta_1)$, respectively, and operators k_{12}, k_{13}, k_{21}, k_{23}, k_{31} and k_{32} act in the following form: $k_{12}: L_2(\alpha_2,\beta_2)\to L_2(\alpha_1,\beta_1)$, $k_{13}: L_2(\xi_1,\eta_1)\to L_2(\alpha_1,\beta_1)$, $k_{21}: L_2(\alpha_1,\beta_1)\to L_2(\alpha_2,\beta_2)$, $k_{23}: L_2(\xi_1,\eta_1)\to L_2(\alpha_2,\beta_2)$, $k_{31}: L_2(\alpha_1,\beta_1)\to L_2(\xi_1,\eta_1)$, $k_{32}: L_2(\alpha_2,\beta_2)\to L_2(\xi_1,\eta_1)$.

Obviously, the operator K acts in the Banach space and is a Fredholm operator. A sufficient condition for inversion of operator $I+K$ is the condition $\|K\|<1$. Then (31) can be solved by the method of successive approximations, if $\|K\|<1$, where

$$\|K\| = \max\left\{\delta_1^2(\|k_{11}\|+\|k_{12}\|+\|k_{13}\|),\, \delta_1^2(\|k_{21}\|+\|k_{22}\|+\|k_{23}\|),\right.$$
$$\left.\overline{\delta}_1^2(\|k_{31}\|+\|k_{32}\|+\|k_{33}\|)\right\}.$$

Therefore, the condition $\|K\|<1$ will be satisfied, if

$$\delta_1^2(\|k_{11}\|+\|k_{12}\|+\|k_{13}\|)<1,\;\; \delta_1^2(\|k_{21}\|+\|k_{22}\|+\|k_{23}\|)<1,$$
$$\overline{\delta}_1^2(\|k_{31}\|+\|k_{32}\|+\|k_{33}\|)<1. \quad (33)$$

In this case, the solution of Eq. (31) is written in the form

$$\varphi = (\mathrm{I}+K)^{-1}y_0 = \sum_{m=0}^{\infty}(-1)^m K^m y_0.$$

Now let's determine the values of δ_1^2 and $\overline{\delta}_1^2$ parameters of the problem, for which the conditions (33) will be satisfied. From (32), by virtue of Cauchy–Bunyakovski inequality, we get:

$$\|k_{11}\|\leqslant c_{11},\, c_{11}=\left(\int_{\alpha_1}^{\beta_1}\int_{\alpha_1}^{\beta_1}M_1^2(x,t)dxdt\right)^{\frac{1}{2}},\, \|k_{12}\|\leqslant c_{12},\, c_{12}=\left(\int_{\alpha_2}^{\beta_2}\int_{\alpha_1}^{\beta_1}M_2^2(x,t)dxdt\right)^{\frac{1}{2}},$$

$$\|k_{13}\|\leqslant c_{13},\, c_{13}=\left(\int_{\xi_1}^{\eta_1}\int_{\alpha_1}^{\beta_1}H_1^2(x,\tau)dxd\tau\right)^{\frac{1}{2}},\, \|k_{21}\|\leqslant c_{21},\, c_{21}=\left(\int_{\alpha_1}^{\beta_1}\int_{\alpha_2}^{\beta_2}M_2^2(x,t)dxdt\right)^{\frac{1}{2}},$$

$$\|k_{22}\|\leqslant c_{22},\, c_{22}=\left(\int_{\alpha_2}^{\beta_2}\int_{\alpha_2}^{\beta_2}M_2^2(x,t)dxdt\right)^{\frac{1}{2}},\, \|k_{23}\|\leqslant c_{23},\, c_{23}=\left(\int_{\xi_1}^{\eta_1}\int_{\alpha_2}^{\beta_2}H_1^2(x,\tau)dxd\tau\right)^{\frac{1}{2}},$$

$$\|k_{31}\|\leqslant c_{31},\, c_{31}=\left(\int_{\alpha_1}^{\beta_1}\int_{\xi_1}^{\eta_1}T^2(x,t)dxdt\right)^{\frac{1}{2}},\, \|k_{32}\|\leqslant c_{32},\, c_{32}=\left(\int_{\alpha_2}^{\beta_2}\int_{\xi_1}^{\eta_1}T^2(x,t)dxdt\right)^{\frac{1}{2}},$$

$$\|k_{33}\|\leqslant c_{33},\, c_{33}=\left(\int_{\xi_1}^{\eta_1}\int_{\xi_1}^{\eta_1}R^2(x,\tau)dxd\tau\right)^{\frac{1}{2}}. \quad (34)$$

Obviously, the expressions for c_{ij} $(i,j=\overline{1,3})$, are difficult to calculate, but they can be estimated. It was found out in [1,4,10], that the following estimates take place:

$$c_{11} < \left(\int_{\alpha_1}^{\beta_1}\int_{\alpha_1}^{\beta_1} \ln^2|x-t|\,dxdt\right)^{\frac{1}{2}}, \quad c_{12} < \left(\int_{\alpha_2}^{\beta_2}\int_{\alpha_1}^{\beta_1} \ln^2|x-t|\,dxdt\right)^{\frac{1}{2}},$$

$$c_{21} < \left(\int_{\alpha_1}^{\beta_1}\int_{\alpha_2}^{\beta_2} \ln^2|x-t|\,dxdt\right)^{\frac{1}{2}}, \quad c_{22} < \left(\int_{\alpha_2}^{\beta_2}\int_{\alpha_2}^{\beta_2} \ln^2|x-t|\,dxdt\right)^{\frac{1}{2}},$$

$$c_{33} < \left(\int_{\xi_1}^{\eta_1}\int_{\xi_1}^{\eta_1} \ln^2|x-\tau|\,dxd\tau\right)^{\frac{1}{2}}, \quad c_{13} < \left(\int_{\xi_1}^{\eta_1}\int_{\alpha_1}^{\beta_1} N_1^2(x-\tau)\,dxd\tau\right)^{\frac{1}{2}}, \tag{35}$$

$$c_{23} < \left(\int_{\xi_1}^{\eta_1}\int_{\alpha_2}^{\beta_2} N_1^2(x-\tau)\,dxd\tau\right)^{\frac{1}{2}}, \quad c_{31} < \left(\int_{\alpha_1}^{\beta_1}\int_{\xi_1}^{\eta_1} N_1^2(x-t)\,dxdt\right)^{\frac{1}{2}},$$

$$c_{32} < \left(\int_{\alpha_2}^{\beta_2}\int_{\xi_1}^{\eta_1} N_1^2(x-t)\,dxdt\right)^{\frac{1}{2}}.$$

The estimates (35) for c_{13}, c_{23}, c_{31} and c_{32} can be obtained also in the form:

$$c_{13} < \frac{1}{2}\left(\int_{\xi_1}^{\eta_1}\int_{\alpha_1}^{\beta_1} \ln^2\left[(x-\tau)^2+\ell_*^2\right]dxd\tau\right)^{\frac{1}{2}} + \kappa\ell_*^2\left(\int_{\xi_1}^{\eta_1}\int_{\alpha_1}^{\beta_1}\left[(x-\tau)^2+\ell_*^2\right]^{-2}dxd\tau\right)^{\frac{1}{2}},$$

$$c_{23} < \frac{1}{2}\left(\int_{\xi_1}^{\eta_1}\int_{\alpha_2}^{\beta_2} \ln^2\left[(x-\tau)^2+\ell_*^2\right]dxd\tau\right)^{\frac{1}{2}} + \kappa\ell_*^2\left(\int_{\xi_1}^{\eta_1}\int_{\alpha_2}^{\beta_2}\left[(x-\tau)^2+\ell_*^2\right]^{-2}dxd\tau\right)^{\frac{1}{2}},$$

$$c_{31} < \frac{1}{2}\left(\int_{\alpha_1}^{\beta_1}\int_{\xi_1}^{\eta_1} \ln^2\left[(x-t)^2+\ell_*^2\right]dxdt\right)^{\frac{1}{2}} + \kappa\ell_*^2\left(\int_{\alpha_1}^{\beta_1}\int_{\xi_1}^{\eta_1}\left[(x-t)^2+\ell_*^2\right]^{-2}dxdt\right)^{\frac{1}{2}}, \tag{36}$$

$$c_{32} < \frac{1}{2}\left(\int_{\alpha_2}^{\beta_2}\int_{\xi_1}^{\eta_1} \ln^2\left[(x-t)^2+\ell_*^2\right]dxdt\right)^{\frac{1}{2}} + \kappa\ell_*^2\left(\int_{\alpha_2}^{\beta_2}\int_{\xi_1}^{\eta_1}\left[(x-t)^2+\ell_*^2\right]^{-2}dxdt\right)^{\frac{1}{2}}.$$

Then the conditions (33) will be realized, if

$$\delta_1^2 < (c_{11}+c_{12}+c_{13})^{-1} = c_1,\ \delta_1^2 < (c_{21}+c_{22}+c_{23})^{-1} = c_2,$$
$$\overline{\delta}_1^2 < (c_{31}+c_{32}+c_{33})^{-1} = c_3, \tag{37}$$

where c_1, c_2 and c_3 are positive numbers less than unity. The values of unknown functions $\varphi_1(x)$, $\varphi_2(x)$ and $\psi(x)$ in the end points of stringers $x=\alpha_1$, $x=\beta_1$, $x=\alpha_2$, $x=\beta_2$ and $x=\xi_1$, $x=\eta_1$, respectively, can be obtained from system (24).

We also note that, from the condition of solvability of the system of Fredholm integral equations (27), for δ_1^2, $\overline{\delta}_1^2$ parameters correspond to (33), (35) and (36) conditions, in this case, we can obtain the condition of solvability in the form:

$$\delta_1^2 < \left(c_{11}^* + c_{12}^*\right)^{-1} = c_1^*, \quad \overline{\delta}_1^2 < \left(c_{21}^* + c_{22}^*\right)^{-1} = c_2^*, \tag{38}$$

where also we have the following corresponding to the above notations: $c_{11}^* = c_{11}$, $c_{12}^* = c_{13}$ and $c_{21}^* = c_{31}$, $c_{22}^* = c_{33}$, respectively.

For the integral equation (29) we obtain the condition of solvability in the form:

$$\delta_1^2 < \left(\int_{\alpha_1}^{\beta_1}\int_{\alpha_1}^{\beta_1} \ln^2|x-t|\,dx\,dt\right)^{-\frac{1}{2}}. \tag{39}$$

Further, note that the multiple numerical results to solving of the system (24), (27) and the integral equation (29) and its analysis are presented on Appendix.

Conclusion. For investigation the changes in the law of distribution of unknown shear forces in this article an effective solution of considered problem is presented. The problem is reduced to solving a system of Fredholm integral equations of the second kind with respect to unknown shear forces which are specified o n three different finite intervals and with right-hand sides of which are the solutions of the problem in the case of a rigid sheet. Through this system of integral equations depending on the change of characteristic parameters of the problem the multiple numerical results and its analysis are presented. The law of distributions of shear forces is revealed depending on the changes in rigidity as from the materials of the stringers and sheet, as the changes of horizontal and also vertical distances between the parallel stringers, which are presented through its corresponding graphics.

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ՎԵՐՋԱՎՈՐ ԵՐԿԱՐՈՒԹՅԱՄԲ ԵՐԵՔ ՏԱՐԱՍԵՌ ԱՌԱՁԳԱԿԱՆ ՍՏՐԻՆԳԵՐՆԵՐԻՑ ԲԵՌՆԱՎՈՐՈՒՄՆԵՐԻ ՓՈԽԱՆՑՈՒՄԸ ԱՆՎԵՐՋ ՍԱԼԻՆ ԿՊՉՈՒՆ ՇԵՐՏԵՐԻ ՄԻՋՈՑՈՎ

Դիտարկված է խնդիր առաձգական անվերջ սալի համար, որն իր վերին մակերևույթի վրա իրար զուգահեռ երկու գծերի երկարությամբ վերջավոր պեղամասերում ուժեղացված է վերջավոր երկարությամբ երեք սպրինգերներով, պարբեր առաձգական և երկրաչափական բնութագրերով: Սպրինգերները դե-ֆորմացիայի են ենթարկվում իրենց ծայրերում կիրառված հորիզոնական ուժերի ազդեցության փակ: Փոխազդեցությունը անվերջ սալի և սպրինգերների միջև իրագործվում է այլ ֆիզիկամեխանիկական և երկրաչափական բնութագրեր ունեցող կպչուն շերտերի միջոցով: Անհայտ շոշափող լարումների որոշման խնդիրը հանգեցված է միմյանց զուգահեռ վերջավոր միջակայքերում երեք անհայտ ֆունկցիաների նկատմամբ Ֆրեդհոլմի երկրորդ սեռի ինտեգրալ հավասարմների համակարգի լուծմանը: Ցույց է տրված, որ խնդրին բնորոշ բնութագրիչ պարամետրերի փոփոխման որոշ տիրույթներում այդ հավասարումների համակարգը կարելի է լուծել հաջորդական մոտավորությունների մեթոդով: Դիտարկված են հնարավոր մասնավոր դեպքեր և արզաբանված են վերջավոր տեղամասերում գործող անհայտ շոշափող լարումների վարքը և բնույթը: Այնուհետև, խնդրին բնորոշ բնութագրիչ պարամետրերի արժեքների բազմաթիվ փոփոխություններից կախված ներկայացված են թվային հաշվարկներ և նրանց բազմակողմանի վերլուծություններ:

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ПЕРЕДАЧА НАГРУЗОК ОТ ТРЕХ РАЗНОРОДНЫХ УПРУГИХ СТРИНГЕРОВ КОНЕЧНЫХ ДЛИН К БЕСКОНЕЧНОЙ ПЛАСТИНЕ ПОСРЕДСТВОМ ЛИПКИХ СЛОЕВ

Рассматривается задача для упругой бесконечной пластины, которая на конечных участках вдоль двух параллельных линий своей верхней поверхности усилена тремя конечными стрингерами. Стрингеры деформируются под действием горизонтальных сил, приложенных на их концах. Контактные связки между пластиной и стрингерами осуществляются посредством одинаковых, тонких липких слоев с другими физико-механическими и геометрическими характеристиками. В работе задача определения закона распределения неизвестных касательных напряжений, действующих между бесконечной пластиной и стрингерами, сведена к решению системы интегральных уравнений Фредгольма второго рода с тремя неизвестными функциями, определенными на различных конечных интервалах. Показано, что в определенной области изменения характерных параметров задачи полученная система интегральных уравнений может быть решена методом последовательных приближений. Рассмотрены некоторые частные случаи и выяснены характер и поведение неизвестных касательных напряжений, действующих на трех конечных участках. Далее, в зависимости от изменения значений характерных параметров задачи осуществлены численный расчет и анализ полученных результатов.