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LOADS TRANSFER FROM THE SYSTEMS OF FINITE NUMBER FINITE-LENGTH STRINGERS TO AN INFINITE SHEET THROUGH ADHESIVE LAYERS

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The article considers the problem for an elastic infinite plate (sheet), which along of two parallel lines of its upper surface is strengthened by systems of finite number finite-length stringers having different elastic properties. The interaction between infinite sheet and stringers take place through thin, uniform, elastic adhesive layers having other physical-mechanical properties and geometric configuration. The stringers are deformed under the action of horizontal concentrated forces, which are applied at one end points of stringers. The problem of determining unknown contact forces acting between infinite sheet and stringers is reduced to the system of Fredholm integral equations of second kind with respect to arbitrary finite number of unknown functions, which are specified along of two parallel lines on different finite intervals. Further, are determined of the change regions of the problem characteristic parameters, for which this system of integral equations allows the exact solution and which can be solved by the method of successive approximations. Some particular cases are considered and the character and behavior of unknown shear contact forces near the end points of the stringers are investigated. For these cases numerical results depanding on the multiparameters of the problem are investigated in the previous article (A.V. Kerobyan, K.P. Sahakyan, Proc. YSU. Phys. Math. Sci. **57** (3) (2023), 86–100).

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Introduction. Investigation of problems which arise during load transfer from stringer to an elastic sheet through adhesive layer and the construction of exact and effective solutions for them have important meaning from both theoretical and applied aspects. Not stopping at the numerous studies devoted to this field, we note that some of them, which is closely associated with the considered problem. In article [1]

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considers the problem for an elastic infinite sheet, which is strengthened by three parallel finite stringers two of which are located on the same line, through adhesive layers. The problem of loads transfer from two parallel elastic stringers with finite lengths to an infinite sheet through adhesive layers is considered in [2]. In [3, 4] considers the problems of loads transfer from two finite stringers (overlays) to an infinite sheet (or half-plane) and infinite strip through adhesive layers when two finite stringers are arranged on the same line, with different approach to the solutions. The paper [5] considers the problem for an infinite sheet with two finite stringers when only one of the stringers is connected through an adhesive layer. In [6–9], using various approaches, problems are investigated for different elastic bodies, which are strengthened by finite-length stringer through adhesive layer. In [10, 11] transfer of loads from finite number of finite-length elastic stringers to an elastic half-plane and to an infinite strip through an adhesive layers are considered. Some problems for an elastic infinite sheet strengthened by parallel finite stringers without presence adhesive layer are considered in [12]. In this article, the problem is considered for an elastic infinite sheet, which on its finite parts along two parallel lines on its upper surface, is strengthened by systems of finite number finite-length stringers having different elastic properties. The interaction between sheet and stringers is assumed to be carried out through thin adhesive layers with different physical-mechanical properties and geometric configuration.

Main Results.

Statement of the Problem and Obtaining the System of Integral Equations. Let an elastic infinite plate (sheet) of small constant thickness h, the Young's modulus E and the Poisson's ratio ν, which is in a generalized plane stress state (xOy is its middle plane) on its upper surface along $y = b$ and $y = -d$ parallel lines being $l = b + d$ $(b, d > 0)$ distance from each other on the $[a_j, b_j]$, $b_j > a_j$, $j = \overline{1,n}$; $b_j < a_{j+1}$, $j = \overline{1,n-1}$, and $[c_k, d_k]$ $(d_k > c_k$, $k = \overline{1,m}$; $d_k < c_{k+1}$, $k = \overline{1,m-1})$ $n + m$, (where n, m are arbitrary finite numbers), finite number finite intervals, respectively, is strengthened by systems of finite number finite stringers, modulus of elasticity equal to E_j for $x \in [a_j, b_j]$, $j = \overline{1,n}$, and equal to E^*_k for $x \in [c_k, d_k]$, $k = \overline{1,m}$, respectively. It is supposed that the stringers have a rectangular cross-sections with small constant areas $F_1 = b^*_1 h_1$ for $x \in [a_j, b_j]$, $j = \overline{1,n}$, and $F_2 = b^*_2 h_2$ for $x \in [c_k, d_k]$, $k = \overline{1,m}$, respectively, where b^*_1 $(b^*_1 \ll b_j - a_j)$, b^*_2 $(b^*_2 \ll d_k - c_k)$ are the widths of the stringers, and h_1 and h_2 are their small constant thicknesses, respectively. The interaction between infinite sheet and stringers take place through thin, uniform, elastic adhesive layers with Young's modulus E_g, Poisson's ratio ν_g, and small constant thickness h_g. The problem is to specify the law of distribution of unknown contact forces acting between sheet and stringers when concentrated forces P_j and Q_k are applied at one end points of stringers $x = b_j$, $j = \overline{1,n}$, and $x = d_k$, $k = \overline{1,m}$, respectively, and are directed to parallel along the Ox-axis (see Figure).

It is assumed that during the deformation for the stringers the model of uniaxial strain state in combination with the model of contact along the line is realized [13],

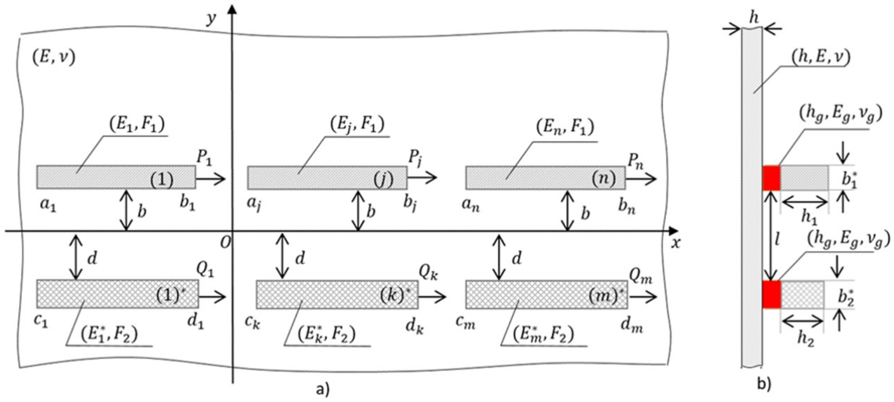

Fig. 1.

and for the adhesive layers there are the pure shear conditions [6], i.e. as [6, 12, 13] bending is neglected and the interaction between sheet and stringers is idealized as a line loading of the sheet [1, 2, 5–7].

Taking into account the above assumptions, according to the equilibrium conditions of an elements of finite-length stringers, which are defined on the $[a_j, b_j]$ and $[c_k, d_k]$ finite intervals, respectively, and Hooke's law, the differential equations for the equilibrium of the stringers on finite intervals $[a_j, b_j]$ $j = \overline{1,n}$, and $[c_k, d_k]$, $k = \overline{1,m}$, respectively, will be written in the following form:

$$\frac{d^2u^{(j)}}{dx^2} = \frac{p_j(x)}{E_jF_1}, \qquad a_j \le x \le b_j,\ j = \overline{1,n}, \tag{1}$$

$$\frac{d^2\bar{u}^{(k)}}{dx^2} = \frac{q_k(x)}{E_k^*F_2}, \qquad c_k \le x \le d_k,\ k = \overline{1,m}, \tag{2}$$

with the following boundary conditions:

$$\left.\frac{du^{(j)}}{dx}\right|_{x=a_j} = 0, \quad \left.\frac{du^{(j)}}{dx}\right|_{x=b_j} = \frac{P_j}{E_jF_1}, \quad j = \overline{1,n}, \tag{3}$$

$$\left.\frac{d\bar{u}^{(k)}}{dx}\right|_{x=c_k} = 0, \quad \left.\frac{d\bar{u}^{(k)}}{dx}\right|_{x=d_k} = \frac{Q_k}{E_k^*F_2}, \quad k = \overline{1,m}, \tag{4}$$

and where we have also of the stringers equilibrium conditions in the form:

$$\int_{a_j}^{b_j} p_j(s)ds = P_j,\ \ j = \overline{1,n}, \qquad \int_{c_k}^{d_k} q_k(v)dv = Q_k,\ \ k = \overline{1,m}. \tag{5}$$

Here $u^{(j)}(x) = u^{(j)}(x,b)$ and $\bar{u}^{(k)}(x) = \bar{u}^{(k)}(x,-d)$ are the horizontal displacements of the points of the stringers at $y = b$ and $y = -d$ parallel lines on the $[a_j, b_j]$, $j = \overline{1,n}$, and $[c_k, d_k]$, $k = \overline{1,m}$, finite intervals, respectively, $p_j(x) = b_1^*\tau_j^{(1)}(x,b)$, $\tau_j^{(1)}(x,b)$ are the shear stresses, acting under of the stringers on the $[a_j, b_j]$ finite parts, respectively, $q_k(x) = b_2^* \tau_k^{(2)}(x,-d)$, $\tau_k^{(2)}(x,-d)$ are the shear stresses, acting under of the stringers on the $[c_k, d_k]$ finite parts, respectively.

Now, assuming that each differential element of the adhesive layers is in a condition of pure shear [1, 2, 5–7], the following adhesive contact conditions are obtained:

$$u^{(j)}(x) - u_1(x,b) = k_1^* p_j(x), \qquad a_j \le x \le b_j,\ j = \overline{1,n}, \tag{6}$$

$$\bar{u}^{(k)}(x) - u_2(x,-d) = k_2^* q_k(x), \quad c_k \le x \le d_k,\ k = \overline{1,m}, \tag{7}$$

where $u_1(x,b)$ and $u_2(x,-d)$ are the horizontal displacements of the points of the elastic infinite sheet at $y = b$ and $y = -d$ parallel lines, respectively, $k_1^* = h_g/b_1^* G_g$, $k_2^* = h_g/b_2^* G_g$, $G_g = E_g/2(1+\nu_g)$, G_g is the shear modulus of adhesive layers, h_g is the thickness of the adhesive layers, $p_j(x) = b_1^* \tau_j^{(1)}(x,b) = b_1^* G_g \gamma_j^{(1)}(x,b)$, $q_k(x) = b_2^* \tau_k^{(2)}(x,-d) = b_2^* G_g \gamma_k^{(2)}(x,-d)$ and $\gamma_j^{(1)}(x,b)$, $\gamma_k^{(2)}(x,-d)$ are the shear deformations of the adhesive layers, on the $[a_j, b_j]$, $j = \overline{1,n}$, and $[c_k, d_k]$, $k = \overline{1,m}$, finite intervals, respectively.

On the other hand, in view of above assumptions, let write the horizontal displacements $u_1(x,b)$ and $u_2(x,-d)$ of the points of the elastic infinite sheet, when tangential (shear) forces with intensities $p_j(x)$, $j = \overline{1,n}$, and $q_k(x)$, $k = \overline{1,m}$, respectively, act on the $[a_j, b_j]$, $j = \overline{1,n}$, and $[c_k, d_k]$, $k = \overline{1,m}$, finite intervals respectively, of its upper surface along $y = b$ and $y = -d$ parallel lines, respectively, as in [1], in the following form:

$$u_1(x,b) = \frac{1}{\pi A^*} \sum_{i=1}^{n} \int_{a_i}^{b_i} \left(\ln \frac{1}{|x-s|} + C \right) p_i(s)\, ds + \frac{1}{\pi A^*} \sum_{\rho=1}^{m} \int_{c_\rho}^{d_\rho} (N(x-v) + C)\, q_\rho(v)\, dv, \tag{8}$$

$$u_2(x,-d) = \frac{1}{\pi A^*} \sum_{\rho=1}^{m} \int_{c_\rho}^{d_\rho} \left(\ln \frac{1}{|x-v|} + C \right) q_\rho(v)\, dv + \frac{1}{\pi A^*} \sum_{i=1}^{n} \int_{a_i}^{b_i} (N(x-s) + C)\, p_i(s)\, ds, \tag{9}$$

where

$$N(x) = \ln \frac{1}{\sqrt{x^2 + l^2}} - \frac{\kappa l^2}{x^2 + l^2},\ A^* = \frac{4Eh}{(1+\nu)(3-\nu)},\ \kappa = \frac{1+\nu}{3-\nu},\ l = b + d, \tag{10}$$

C is arbitrary constant.

Note that, the horizontal displacements $u(x,y)$ of the points of an infinite sheet, when shear forces act on its surface along the line $y = -d$ with intensity $q(x)$ $(-\infty < x < \infty)$ is given by the formula:

$$u(x,y) = \frac{1}{\pi A^*} \int_{-\infty}^{\infty} \left[\ln \frac{1}{\sqrt{(x-s)^2 + (y+d)^2}} - \frac{\kappa (y+d)^2}{(x-s)^2 + (y+d)^2} \right] q(s)\, ds + \text{const},$$

$$-\infty < x < \infty, \quad 0 < y < \infty.$$

Further, by virtue of (6) and (7), equations (1) and (2) can be written in the following form:

$$\frac{d^2u^{(j)}}{dx^2} - \gamma_j^2 u^{(j)}(x) = -\gamma_j^2 u_1(x,b), \qquad a_j \le x \le b_j,\ j = \overline{1,n}, \tag{11}$$

$$\frac{d^2\bar{u}^{(k)}}{dx^2} - \alpha_k^2 \bar{u}^{(k)}(x) = -\alpha_k^2 u_2(x,-d), \quad c_k \le x \le d_k, \quad k = \overline{1,m}, \tag{12}$$

with the boundary conditions (3) and (4), respectively.
Here $\gamma_j^2 = 1/k_1^* E_j F_1$, $j = \overline{1,n}$, and $\alpha_k^2 = 1/k_2^* E_k^* F_2$, $k = \overline{1,m}$.

The solutions to the boundary value problems (11) and (3), (12) and (4), respectively, we obtain in the form:

$$u^{(j)}(x) = u_0^{(j)}(x) + \gamma_j^2 \int_{a_j}^{b_j} G_j(x,s) u_1(s,b)\, ds, \qquad a_j \le x \le b_j,\ j = \overline{1,n}, \tag{13}$$

$$\bar{u}^{(k)}(x) = \bar{u}_0^{(k)}(x) + \alpha_k^2 \int_{c_k}^{d_k} K_k(x,v) u_2(v,-d)\, dv, \ \ c_k \le x \le d_k,\ k = \overline{1,m}, \tag{14}$$

where $u_0^{(j)}(x)$ and $\bar{u}_0^{(k)}(x)$ are the general solutions of the homogenous equations corresponding to equations (11) and (12), respectively, with the boundary conditions (3) and (4), respectively, and have the following form:

$$u_0^{(j)}(x) = \frac{P_j \cos h[\gamma_j(x-a_j)]}{\gamma_j E_j F_1 \sin h[\gamma_j(b_j - a_j)]}, \quad j = \overline{1,n},$$

$$\bar{u}_0^{(k)}(x) = \frac{Q_k \cos h[\alpha_k(x-c_k)]}{\alpha_k E_k^* F_2 \sin h[\alpha_k(d_k - c_k)]}, \quad k = \overline{1,m}.$$

In equations (13) and (14), $u_*^{(j)}(x) = \gamma_j^2 \int_{a_j}^{b_j} G_j(x,s) u_1(s,b)\, ds$, $j = \overline{1,n}$, and $\bar{u}_*^{(k)}(x) = \alpha_k^2 \int_{c_k}^{d_k} K_k(x,v) u_2(v,-d)\, dv$, $k = \overline{1,m}$, are the particular solutions of (11) and (12), respectively, with zero boundary conditions corresponding to conditions (3) and (4), respectively, $G_j(x,s)$, $j = \overline{1,n}$, and $K_k(x,v)$, $k = \overline{1,m}$, are Green's functions [14], and

$$G_j(x,s) = \frac{1}{\gamma_j \sin[\gamma_j(b_j - a_j)]} \begin{cases} \cos h[\gamma_j(x-b_j)] \cos h[\gamma_j(s-a_j)], & x > s, \\ \cos h[\gamma_j(x-a_j)] \cos h[\gamma_j(s-b_j)], & x < s, \end{cases}$$

$$K_k(x,v) = \frac{1}{\alpha_k \sin h[\alpha_k(d_k - c_k)]} \begin{cases} \cos h[\alpha_k(x-d_k)] \cos h[\alpha_k(v-c_k)], & x > v, \\ \cos h[\alpha_k(x-c_k)] \cos h[\alpha_k(v-d_k)], & x < v. \end{cases}$$

It is obvious, that the functions $G_j(x,s)$ and $K_k(x,v)$ are continuous functions and $G_j(x,s) = G_j(s,x)$ and $K_k(x,v) = K_k(v,x)$.

Further, by virtue of (13) and (14), according to (6) and (7), we obtain the following equations:

$$k_1^* p_j(x) + u_1(x,b) = \gamma_j^2 \int_{a_j}^{b_j} G_j(x,s) u_1(s,b)\, ds + u_0^{(j)}(x), \;\; a_j \le x \le b_j,\; j = \overline{1,n}, \quad (15)$$

$$k_2^* q_k(x) + u_2(x,-d) = \alpha_k^2 \int_{c_k}^{d_k} K_k(x,v) u_2(v,-d)\, dv + \bar{u}_0^{(k)}(x),$$

$$c_k \le x \le d_k,\; k = \overline{1,m}. \quad (16)$$

Now, by virtue of (8) and (9), from (15) and (16), we obtain the following system of integral equations:

$$p_j(x) + \frac{1}{\pi A^* k_1^*} \sum_{i=1}^{n} \int_{a_i}^{b_i} \left(\ln \frac{1}{|x-s|} + C \right) p_i(s)\, ds + \frac{1}{\pi A^* k_1^*} \sum_{\rho=1}^{m} \int_{c_\rho}^{d_\rho} (N(x-v)+C) q_\rho(v)\, dv$$

$$= \frac{\gamma_j^2}{\pi A^* k_1^*} \sum_{i=1}^{n} \int_{a_j}^{b_j} G_j(x,s) \int_{a_i}^{b_i} \left(\ln \frac{1}{|s-t|} + C \right) p_i(t)\, dt\, ds \quad (17)$$

$$+ \frac{\gamma_j^2}{\pi A^* k_1^*} \sum_{\rho=1}^{m} \int_{a_j}^{b_j} G_j(x,s) \int_{c_\rho}^{d_\rho} (N(s-\tau)+C)\, q_\rho(\tau)\, d\tau\, ds + \frac{u_0^{(j)}(x)}{k_1^*}, \quad a_j \le x \le b_j,\; j = \overline{1,n},$$

$$q_k(x) + \frac{1}{\pi A^* k_2^*} \sum_{\rho=1}^{m} \int_{c_\rho}^{d_\rho} \left(\ln \frac{1}{|x-v|} + C \right) q_\rho(v)\, dv + \frac{1}{\pi A^* k_2^*} \sum_{i=1}^{n} \int_{a_i}^{b_i} (N(x-s)+C)\, p_i(s)\, ds$$

$$= \frac{\alpha_k^2}{\pi A^* k_2^*} \sum_{\rho=1}^{m} \int_{c_k}^{d_k} K_k(x,v) \int_{c_\rho}^{d_\rho} \left(\ln \frac{1}{|v-\tau|} + C \right) q_\rho(\tau)\, d\tau\, dv$$

$$+ \frac{\alpha_k^2}{\pi A^* k_2^*} \sum_{i=1}^{n} \int_{c_k}^{d_k} K_k(x,v) \int_{a_i}^{b_i} (N(v-t)+C)\, p_i(t)\, dt\, dv + \frac{\bar{u}_0^{(k)}(x)}{k_2^*}, \quad c_k \le x \le d_k,\; k = \overline{1,m}.$$

It should be noted that the spectrum of the symmetric second-order differential operator $D = -d^2/dx^2 + \gamma^2 I$ with the domain of definition being twice continuous differentiating functions, satisfying the boundary conditions $\left(du^{(1)}/dx\right)_{x=a} = 0$ and $\left(du^{(1)}/dx\right)_{x=b} = 0$, are eigenvalues $\lambda_p = \gamma^2 + p^2\pi^2/(b-a)^2$ $(p = 0,1,2,\ldots)$, with corresponding eigenfunctions $\cos[p\pi(x-a)/(b-a)]$ $(p = 0,1,2,\ldots)$.

It is known [14], that symmetric quite continuous integral operator B:

$$B\varphi = \int_a^b G(x,s)\, \varphi(s)\, ds,$$

which acts in the space $L_2(a,b)$ is an inverse of the operator D.

Hence, we have:

$$\int_{a_j}^{b_j} G_j(x,s)\cos\left[\frac{p\pi(s-a_j)}{b_j-a_j}\right]ds = \frac{(b_j-a_j)^2}{(b_j-a_j)^2\gamma_j^2+p^2\pi^2}\cos\left[\frac{p\pi(x-a_j)}{b_j-a_j}\right], \quad p=0,1,2,\ldots, \tag{18}$$

$$\int_{c_k}^{d_k} K_k(x,v)\cos\left[\frac{q\pi(v-c_k)}{d_k-c_k}\right]dv = \frac{(d_k-c_k)^2}{(d_k-c_k)^2\alpha_k^2+q^2\pi^2}\cos\left[\frac{q\pi(x-c_k)}{d_k-c_k}\right], \quad q=0,1,2,\ldots, \tag{19}$$

where the functions $\cos\left[\frac{p\pi(x-a_j)}{b_j-a_j}\right]$ $(p=0,1,2,\ldots)$ and $\cos\left[\frac{q\pi(x-c_k)}{d_k-c_k}\right]$ $(q=0,1,2,\ldots)$, form full orthogonal systems in the spaces $L_2(a_j,b_j)$, $j=\overline{1,n}$, and $L_2(c_k,d_k)$, $k=\overline{1,m}$, respectively.

Further, after replacing the variables x, s, v, t, and τ by ax, as, av, at, and $a\tau$, respectively, where $a>0$ is the coordinate of one of the end points of stringers, we get the system (17) in the following form:

$$\varphi_j(x)+\delta^2\sum_{i=1}^{n}\int_{\alpha_i}^{\beta_i}\ln\frac{1}{|x-t|}\varphi_i(t)\,dt - a\gamma_j^2\delta^2\sum_{i=1}^{n}\int_{\alpha_j}^{\beta_j}G_j(ax,as)\int_{\alpha_i}^{\beta_i}\ln\frac{1}{|s-t|}\varphi_i(t)\,dt\,ds$$
$$+\delta^2\sum_{\rho=1}^{m}\int_{\xi_\rho}^{\eta_\rho}N_1(x-\tau)\psi_\rho(\tau)\,d\tau - a\gamma_j^2\delta^2\sum_{\rho=1}^{m}\int_{\alpha_j}^{\beta_j}G_j(ax,as)\int_{\xi_\rho}^{\eta_\rho}N_1(s-\tau)\,\psi_\rho(\tau)\,d\tau\,ds$$
$$-p_0^{(j)}(ax)=0, \quad \alpha_j\le x\le\beta_j,\ j=\overline{1,n}, \tag{20}$$

$$\psi_k(x)+\bar{\delta}^2\sum_{\rho=1}^{m}\int_{\xi_\rho}^{\eta_\rho}\ln\frac{1}{|x-\tau|}\psi_\rho(\tau)\,d\tau - a\alpha_k^2\bar{\delta}^2\sum_{\rho=1}^{m}\int_{\xi_k}^{\eta_k}K_k(ax,av)\int_{\xi_\rho}^{\eta_\rho}\ln\frac{1}{|v-\tau|}\psi_\rho(\tau)\,d\tau\,dv$$
$$+\bar{\delta}^2\sum_{i=1}^{n}\int_{\alpha_i}^{\beta_i}N_1(x-t)\varphi_i(t)\,dt - a\alpha_k^2\bar{\delta}^2\sum_{i=1}^{n}\int_{\xi_k}^{\eta_k}K_k(ax,av)\int_{\alpha_i}^{\beta_i}N_1(v-t)\varphi_i(t)\,dt\,dv$$
$$-\bar{q}_0^{(k)}(ax)=0, \quad xi_k\le x\le\eta_k,\ k=\overline{1,m},$$

since according to (18) and (19) we have also the following equalities:

$$\int_{\alpha_j}^{\beta_j}G_j(ax,as)\,ds=\frac{1}{a\gamma_j^2}, \quad \int_{\xi_k}^{\eta_k}K_k(ax,av)\,dv=\frac{1}{a\alpha_k^2}, \qquad j=\overline{1,n},\ \ k=\overline{1,m}. \tag{21}$$

Here
$\varphi_j(x) = p_j(ax),\ \psi_k(x) = q_k(ax),\ \delta^2 = a/\pi A^* k_1^*,\ \bar{\delta}^2 = a/\pi A^* k_2^*,\ l_* = l/a,$
$\alpha_j = a_j/a,\ \beta_j = b_j/a,\ \xi_k = c_k/a,\ \eta_k = d_k/a,\ j = \overline{1,n},\ k = \overline{1,m},\ b_* = b/a,\ d_* = d/a,$
$p_0^{(j)}(ax) = u_0^{(j)}(ax)/k_1^*,\ \bar{q}_0^{(k)}(ax) = \bar{u}_0^{(k)}(ax)/k_2^*,$
$N(ax) = \ln\frac{1}{a} + N_1(x),\ N_1(x) = \ln\frac{1}{\sqrt{x^2+l_*^2}} - \frac{\kappa l_*^2}{x^2+l_*^2}.$

One can represent the system of integral equations (20) in the following form:

$$\varphi_j(x) + \delta^2 \sum_{i=1}^{n} \int_{\alpha_i}^{\beta_i} M_j(x,t)\,\varphi_i(t)\,dt + \delta^2 \sum_{\rho=1}^{m} \int_{\xi_\rho}^{\eta_\rho} H_j(x,\tau)\,\psi_\rho(\tau)\,d\tau = f_0^{(j)}(x),$$
$$\alpha_j \le x \le \beta_j,\ j = \overline{1,n}, \tag{22}$$
$$\psi_k(x) + \bar{\delta}^2 \sum_{\rho=1}^{m} \int_{\xi_\rho}^{\eta_\rho} R_k(x,\tau)\,\psi_\rho(\tau)\,d\tau + \bar{\delta}^2 \sum_{i=1}^{n} \int_{\alpha_i}^{\beta_i} T_k(x,t)\varphi_i(t)\,dt = q_0^{(k)}(x),$$
$$\xi_k \le x \le \eta_k,\ k = \overline{1,m},$$

were

$$M_j(x,t) = \ln\frac{1}{|x-t|} - a\gamma_j^2 \int_{\alpha_j}^{\beta_j} G_j(ax,as) \ln\frac{1}{|s-t|}\,ds,$$
$$H_j(x,\tau) = N_1(x-\tau) - a\gamma_j^2 \int_{\alpha_j}^{\beta_j} G_j(ax,as) N_1(s-\tau)\,ds,\ j = \overline{1,n},$$
$$R_k(x,\tau) = \ln\frac{1}{|x-\tau|} - a\alpha_k^2 \int_{\xi_k}^{\eta_k} K_k(ax,av) \ln\frac{1}{|v-\tau|}\,dv,\ k = \overline{1,m},$$
$$T_k(x,t) = N_1(x-t) - a\alpha_k^2 \int_{\xi_k}^{\eta_k} K_k(ax,av) N_1(v-t)\,dv,\ k = \overline{1,m},$$
$$f_0^{(j)}(x) = p_0^{(j)}(ax) = \frac{u_0^{(j)}(ax)}{k_1^*} = \frac{P_j\gamma_j \cos h\,[a\gamma_j(x-\alpha_j)]}{\sin h\,[a\gamma_j(\beta_j-\alpha_j)]},$$
$$q_0^{(k)}(x) = \bar{q}_0^{(k)}(ax) = \frac{\bar{u}_0^{(k)}(ax)}{k_2^*} = \frac{Q_k\alpha_k \cos h\,[a\alpha_k(x-\xi_k)]}{\sin h\,[a\alpha_k(\eta_k-\xi_k)]}.$$

It is easy to see that the functions $f_0^{(j)}(x)$ and $q_0^{(k)}(x)$ are the adhesive contact forces in the case of a rigid sheet (i.e. when $E \to \infty$) and integrable functions on the segments $x \in [\alpha_j, \beta_j]$ and $x \in [\xi_k, \eta_k]$, respectively.

Note that the system of integral equations (22) is obtained by the changing the order of integration, the validity of which follows from the Fubini's theorem [14]. This theorem will often be used below without special mention.

Further, note that also, which the system of integral equations (22) was obtained without using the stringers equilibrium conditions (5) in the form:

$$\int\limits_{\alpha_j}^{\beta_j} p_j(as)\,ds = P_j/a, \quad j=\overline{1,n}, \qquad \int\limits_{\xi_k}^{\eta_k} q_k(av)\,dv = Q_k/a, \quad k=\overline{1,m}. \qquad (23)$$

In the system (22), the conditions (23) are satisfied automatically, since the following equalities take place:

$$\int\limits_{\alpha_j}^{\beta_j} f_0^{(j)}(x)\,dx = P_j/a, \quad j=\overline{1,n}, \qquad \int\limits_{\xi_k}^{\eta_k} q_0^{(k)}(x)\,dx = Q_k/a, \quad k=\overline{1,m}.$$

These can be easily verified by integrating the first n equations of (22) from α_j to β_j, $j=\overline{1,n}$, respectively, and the second m equations of (22) from ξ_k to η_k, $k=\overline{1,m}$, respectively, then changing the order of integration in the resulting double integrals and taking into account the equalities, which follow from (21):

$$\int\limits_{\alpha_j}^{\beta_j} M_j(x,t)\,dx = 0, \ \int\limits_{\alpha_j}^{\beta_j} H_j(x,\tau)\,dx = 0, \ j=\overline{1,n},$$

$$\int\limits_{\xi_k}^{\eta_k} R_k(x,\tau)\,dx = 0, \ \int\limits_{\xi_k}^{\eta_k} T_k(x,t)\,dx = 0, k=\overline{1,m}.$$

Some Particular Cases. Now let us consider several particular cases that are directly obtained from the system of integral equations (22). In the case $\delta^2 = \bar{\delta}^2 = 0$, from the system (22) we obtain the solution of the corresponding problem for the case of a rigid sheet (i.e. when $E \to \infty$) in the form $\varphi_j(x) = f_0^{(j)}(x)$, $x \in [\alpha_j, \beta_j]$, $j=\overline{1,n}$, and $\psi_k(x) = q_0^{(k)}(x)$, $x \in [\xi_k, \eta_k]$, $k=\overline{1,m}$, respectively. From these solutions, it is easy to see that the functions $\varphi_j(x)$ and $\psi_k(x)$ are bounded when $x \to \alpha_j$, $x \to \beta_j$, and $x \to \xi_k$, $x \to \eta_k$, respectively.

In the case of finite number finite stringers arranged on the finite intervals $[a_j, b_j]$, $j=\overline{1,n}$, or on the finite intervals $[c_k, d_k]$, $k=\overline{1,m}$, instead of the system (22), we obtain the system of Fredholm integral equations of the second kind with respect to an unknown functions $\varphi_j(x)$ defined on the segments $[\alpha_j, \beta_j]$, $j=\overline{1,n}$, in the following form:

$$\varphi_j(x) + \delta^2 \sum_{i=1}^{n} \int\limits_{\alpha_i}^{\beta_i} M_j(x,t)\varphi_i(t)\,dt = f_0^{(j)}(x), \quad \alpha_j \le x \le \beta_j, \ j=\overline{1,n}, \qquad (24)$$

or with respect to an unknown function $\psi_k(x)$ defined on the segments $[\xi_k, \eta_k]$, $k=\overline{1,m}$, in the form:

$$\psi_k(x) + \bar{\delta}^2 \sum_{\rho=1}^{m} \int\limits_{\xi_\rho}^{\eta_\rho} R_k(x,\tau)\psi_\rho(\tau)\,d\tau = q_0^{(k)}(x), \quad \xi_k \le x \le \eta_k, \ k=\overline{1,m}. \qquad (25)$$

Comparing the systems of integral equations (24) and (25), it is shown that, they have the same form.

Further, according to the system (24), in the case of one finite stringer arranged on the finite interval $[a_1, b_1]$ (i.e. when $n = 1$ in the system (24) and we have $j,\ i = 1$), instead of the system (24), we obtain the Fredholm integral equation of the second kind with respect to unknown function $\varphi_1(x)$, defined on the segment $[\alpha_1, \beta_1]$, in the following form:

$$\varphi_1(x) + \delta^2 \int_{\alpha_1}^{\beta_1} M_1(x,t)\,\varphi_1(t)\,dt = f_0^{(1)}(x), \quad \alpha_1 \le x \le \beta_1. \tag{24*}$$

On the other hand, according to (22), in the case of two parallel finite stringers arranged on the finite intervals $[a_1, b_1]$ and $[c_1, d_1]$ (i.e. when in the system (22) we have $n, m = 1$, and also we have $j, k = 1$, and $i, \rho = 1$), instead of the system (22), we obtain the system of Fredholm integral equations of the second kind with respect to unknown functions $\varphi_1(x)$ and $\psi_1(x)$ defined on the segments $[\alpha_1, \beta_1]$ and $[\xi_1, \eta_1]$, respectively, in the following form:

$$\varphi_1(x) + \delta^2 \int_{\alpha_1}^{\beta_1} M_1(x,t)\,\varphi_1(t)\,dt + \delta^2 \int_{\xi_1}^{\eta_1} H_1(x,\tau)\,\psi_1(\tau)\,d\tau = f_0^{(1)}(x),\ \alpha_1 \le x \le \beta_1,$$

$$\psi_1(x) + \bar{\delta}^2 \int_{\xi_1}^{\eta_1} R_1(x,\tau)\,\psi_1(\tau)\,d\tau + \bar{\delta}^2 \int_{\alpha_1}^{\beta_1} T_1(x,t)\,\varphi_1(t)\,dt = q_0^{(1)}(x),\ \xi_1 \le x \le \eta_1. \tag{22*}$$

In the case of three finite stringers, two of which are located on the same line, and defined on three finite intervals $[a_1, b_1]$, $[a_2, b_2]$ and $[c_1, d_1]$ (i.e. when in the system (22) we have $n = 2$, $m = 1$ and also we have $j, i = 1, 2$, and $k, \rho = 1$) respectively, instead of the system (22), we obtain the system of Fredholm integral equations of the second kind with respect to three unknown functions $\varphi_1(x)$, $\varphi_2(x)$ and $\psi_1(x)$ defined on the segments $[\alpha_1, \beta_1]$, $[\alpha_2, \beta_2]$ and $[\xi_1, \eta_1]$, respectively, in the following form:

$$\varphi_1(x) + \delta^2 \int_{\alpha_1}^{\beta_1} M_1(x,t)\varphi_1(t)\,dt + \delta^2 \int_{\alpha_2}^{\beta_2} M_1(x,t)\varphi_2(t)\,dt + \delta^2 \int_{\xi_1}^{\eta_1} H_1(x,\tau)\,\psi_1(\tau)\,d\tau = f_0^{(1)}(x), \quad \alpha_1 \le x \le \beta_1,$$

$$\varphi_2(x) + \delta^2 \int_{\alpha_1}^{\beta_1} M_2(x,t)\,\varphi_1(t)\,dt + \delta^2 \int_{\alpha_2}^{\beta_2} M_2(x,t)\varphi_2(t)\,dt + \delta^2 \int_{\xi_1}^{\eta_1} H_2(x,\tau)\,\psi_1(\tau)\,d\tau = f_0^{(2)}(x), \quad \alpha_2 \le x \le \beta_2, \tag{22**}$$

$$\psi_1(x)+\bar{\delta}^2\int\limits_{\xi_1}^{\eta_1} R_1(x,\tau)\psi_1(\tau)\,d\tau+\bar{\delta}^2\int\limits_{\alpha_1}^{\beta_1} T_1(x,t)\varphi_1(t)\,dt$$
$$+\bar{\delta}^2\int\limits_{\alpha_2}^{\beta_2} T_1(x,t)\varphi_2(t)\,dt=q_0^{(1)}(x),\quad \xi_1\le x\le\eta_1.$$

Thus, solving the problem is reduced to solving the system (22) of Fredholm integral equations of the second kind with squarely integrable kernels in two variables and with right-hand sides, which are the solutions of the problem in the case of rigid sheet. From the system (22), it is easy to see that at the end points of stringers $x=\alpha_j$, $x=\beta_j$ and $x=\xi_k$, $x=\eta_k$, the values of unknown shear contact forces $\varphi_j(x)$, $j=\overline{1,n}$, and $\psi_k(x)$, $k=\overline{1,m}$, respectively, are finite.

Also note that, which without presence of adhesive layer (i.e. in the case of ideal mechanical contact between sheet and stringer) in the same end points of the stringer the intensities of unknown contact shear forces (or stresses) have singularity of the square root power of integrable order [5, 12, 13, 15–17].

Investigation Solvability of the System of Integral Equations (22)***.*** Now write the system (22) in the following form:

$$\varphi+T\varphi=f_0, \tag{26}$$

where

$$\varphi=\begin{pmatrix}\varphi_1\\ \vdots\\ \varphi_n\\ \psi_1\\ \vdots\\ \psi_m\end{pmatrix},\quad f_0=\begin{pmatrix}f_0^{(1)}\\ \vdots\\ f_0^{(n)}\\ q_0^{(1)}\\ \vdots\\ q_0^{(m)}\end{pmatrix},\quad T=\begin{pmatrix}\delta^2k_{11} & \cdots & \delta^2k_{1n} & \delta^2s_{11} & \cdots & \delta^2s_{1m}\\ \vdots & & & & & \\ \delta^2k_{n1} & \cdots & \delta^2k_{nn} & \delta^2s_{n1} & \cdots & \delta^2s_{nm}\\ \bar{\delta}^2t_{11} & \cdots & \bar{\delta}^2t_{1n} & \bar{\delta}^2r_{11} & \cdots & \bar{\delta}^2r_{1m}\\ \vdots & & & & & \\ \bar{\delta}^2t_{m1} & \cdots & \bar{\delta}^2t_{mn} & \bar{\delta}^2r_{m1} & \cdots & \bar{\delta}^2r_{mm}\end{pmatrix},$$

$$k_{ji}\varphi_i=\int\limits_{\alpha_i}^{\beta_i} M_j(x,t)\varphi_i(t)\,dt,\ \ j,i=\overline{1,n},\ \ s_{j\rho}\psi_\rho=\int\limits_{\xi_\rho}^{\eta_\rho} H_j(x,\tau)\psi_\rho(\tau)\,d\tau,$$
$$j=\overline{1,n},\quad \rho=\overline{1,m}, \tag{27}$$

$$r_{k\rho}\psi_\rho=\int\limits_{\xi_\rho}^{\eta_\rho} R_k(x,\tau)\psi_\rho(\tau)\,d\tau,\ \ k,\rho=\overline{1,m},\ \ t_{ki}\varphi_i=\int\limits_{\alpha_i}^{\beta_i} T_k(x,t)\varphi_i(t)\,dt,$$
$$k=\overline{1,m},\quad i=\overline{1,n}.$$

Further, consider operator equation (26) in Banach space with elements

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \\ z_1 \\ \vdots \\ z_m \end{pmatrix}, \text{ where } y_j(x) \in L_2(\alpha_j, \beta_j) \ (j = \overline{1,n}), \ z_k(x) \in L_2(\xi_k, \eta_k) \ (k = \overline{1,m}),$$

and with norm:

$$\|y\| = \max\left\{\|y_1\|_{L_2(\alpha_1,\beta_1)}, \|y_2\|_{L_2(\alpha_2,\beta_2)}, \cdots, \|y_n\|_{L_2(\alpha_n,\beta_n)}, \|z_1\|_{L_2(\xi_1,\eta_1)}, \|z_2\|_{L_2(\xi_2,\eta_2)}, \cdots, \|z_m\|_{L_2(\xi_m,\eta_m)}\right\}.$$

$L_2(\alpha_j, \beta_j)$ and $L_2(\xi_k, \eta_k)$ are spaces of square integrable functions, specified on the intervals (α_j, β_j), $j = \overline{1,n}$, and (ξ_k, η_k), $k = \overline{1,m}$, respectively.

Operators k_{ji} and $r_{k\rho}$ act in the following form: $k_{ji}\colon L_2(\alpha_i, \beta_i) \to L_2(\alpha_j, \beta_j)$ $(j, i = \overline{1,n})$, and $r_{k\rho}\colon L_2(\xi_\rho, \eta_\rho) \to L_2(\xi_k, \eta_k)$ $(k, \rho = \overline{1,m})$, respectively, and operators $s_{j\rho}$ and t_{ki} act in the following form: $s_{j\rho}\colon L_2(\xi_\rho, \eta_\rho) \to L_2(\alpha_j, \beta_j)$ $(j = \overline{1,n}, \rho = \overline{1,m})$, $t_{ki}\colon L_2(\alpha_i, \beta_i) \to L_2(\xi_k, \eta_k)$ $(k = \overline{1,m}, i = \overline{1,n})$, respectively.

Obviously, the operator T acts in the Banach space and is a Fredholm operator. A sufficient condition for inversion of operator $I + T$ is the condition $\|T\| < 1$. Then operator equation (26) can be solved by the method of successive approximations, if $\|T\| < 1$, where

$$\|T\| = \max\left\{\delta^2\left(\sum_{i=1}^n \|k_{1i}\| + \sum_{\rho=1}^m \|s_{1\rho}\|\right), \delta^2\left(\sum_{i=1}^n \|k_{2i}\| + \sum_{\rho=1}^m \|s_{2\rho}\|\right), \ldots, \delta^2\left(\sum_{i=1}^n \|k_{ni}\| + \sum_{\rho=1}^m \|s_{n\rho}\|\right), \bar{\delta}^2\left(\sum_{i=1}^n \|t_{1i}\| + \sum_{\rho=1}^m \|r_{1\rho}\|\right), \bar{\delta}^2\left(\sum_{i=1}^n \|t_{2i}\| + \sum_{\rho=1}^m \|r_{2\rho}\|\right), \ldots, \bar{\delta}^2\left(\sum_{i=1}^n \|t_{mi}\| + \sum_{\rho=1}^m \|r_{m\rho}\|\right)\right\}.$$

Therefore, the condition $\|T\| < 1$ will be satisfied, if

$$\begin{aligned} &\delta^2\left(\sum_{i=1}^n \|k_{1i}\| + \sum_{\rho=1}^m \|s_{1\rho}\|\right) < 1, \quad \delta^2\left(\sum_{i=1}^n \|k_{2i}\| + \sum_{\rho=1}^m \|s_{2\rho}\|\right) < 1, \\ &\ldots, \delta^2\left(\sum_{i=1}^n \|k_{ni}\| + \sum_{\rho=1}^m \|s_{n\rho}\|\right) < 1, \quad \bar{\delta}^2\left(\sum_{i=1}^n \|t_{1i}\| + \sum_{\rho=1}^m \|r_{1\rho}\|\right) < 1, \\ &\bar{\delta}^2\left(\sum_{i=1}^n \|t_{2i}\| + \sum_{\rho=1}^m \|r_{2\rho}\|\right) < 1, \ldots, \quad \bar{\delta}^2\left(\sum_{i=1}^n \|t_{mi}\| + \sum_{\rho=1}^m \|r_{m\rho}\|\right) < 1. \end{aligned} \tag{28}$$

In this case, the solution of operator equation (26) is written in the form

$$\varphi = (\mathrm{I} + T)^{-1} f_0 = \sum_{\kappa=0}^{\infty} (-1)^\kappa T^\kappa f_0.$$

Now let's determine the values of δ^2 and $\bar{\delta}^2$ parameters of the problem, for which the conditions (28) will be satisfied. From (27), by virtue of Cauchy–Bunyakovski inequality, we get:

$$
\begin{aligned}
&\|k_{ji}\| \le c_{ji}, \quad c_{ji} = \left(\int_{\alpha_i}^{\beta_i} \int_{\alpha_j}^{\beta_j} M_j^2(x,t)\,dx\,dt \right)^{\frac{1}{2}}, \quad j,i = \overline{1,n}, \\
&\|s_{j\rho}\| \le e_{j\rho}, \quad e_{j\rho} = \left(\int_{\xi_\rho}^{\eta_\rho} \int_{\alpha_j}^{\beta_j} H_j^2(x,\tau)\,dx\,d\tau \right)^{\frac{1}{2}}, \quad j = \overline{1,n},\ \rho = \overline{1,m}, \\
&\|t_{ki}\| \le c_{ki}^*, \quad c_{ki}^* = \left(\int_{\alpha_i}^{\beta_i} \int_{\xi_k}^{\eta_k} T_k^2(x,t)\,dx\,dt \right)^{\frac{1}{2}}, \quad k = \overline{1,m},\ i = \overline{1,n}, \\
&\|r_{k\rho}\| \le e_{k\rho}^*, \quad e_{k\rho}^* = \left(\int_{\xi_\rho}^{\eta_\rho} \int_{\xi_k}^{\eta_k} R_k^2(x,\tau)\,dx\,d\tau \right)^{\frac{1}{2}}, \quad k,\rho = \overline{1,m}.
\end{aligned}
\tag{29}
$$

Obviously, the expressions for c_{ji}, $e_{j\rho}$, c_{ki}^* and $e_{k\rho}^*$ are difficult to calculate, but they can be estimated. Further, it was found out in [1], that the following estimates take place:

$$
\begin{aligned}
c_{ji} &< \left(\int_{\alpha_i}^{\beta_i} \int_{\alpha_j}^{\beta_j} \ln^2 |x-t|\,dx\,dt \right)^{\frac{1}{2}} \quad (j,i = \overline{1,n}), \\
e_{j\rho} &< \left(\int_{\xi_\rho}^{\eta_\rho} \int_{\alpha_j}^{\beta_j} N_1^2(x-\tau)\,dx\,d\tau \right)^{\frac{1}{2}} \quad (j = \overline{1,n},\ \ \rho = \overline{1,m}), \\
c_{ki}^* &< \left(\int_{\alpha_i}^{\beta_i} \int_{\xi_k}^{\eta_k} N_1^2(x-t)\,dx\,dt \right)^{\frac{1}{2}} \quad (k = \overline{1,m},\ \ i = \overline{1,n}), \\
e_{k\rho}^* &< \left(\int_{\xi_\rho}^{\eta_\rho} \int_{\xi_k}^{\eta_k} \ln^2 |x-\tau|\,dx\,d\tau \right)^{\frac{1}{2}} \quad (k,\rho = \overline{1,m}).
\end{aligned}
\tag{30}
$$

The estimates (30) for $e_{j\rho}$ and c_{ki}^* can be obtained also in the form:

$$
e_{j\rho} < \frac{1}{2} \left(\int_{\xi_\rho}^{\eta_\rho} \int_{\alpha_j}^{\beta_j} \ln^2 \left[(x-\tau)^2 + l_*^2 \right] dx\,d\tau \right)^{\frac{1}{2}} + \kappa l_*^2 \left(\int_{\xi_\rho}^{\eta_\rho} \int_{\alpha_j}^{\beta_j} \left[(x-\tau)^2 + l_*^2 \right]^{-2} dx\,d\tau \right)^{\frac{1}{2}}
$$
$$
(j = \overline{1,n},\ \rho = \overline{1,m}),
$$

$$c_{ki}^* < \frac{1}{2}\left(\int_{\alpha_i}^{\beta_i}\int_{\xi_k}^{\eta_k}\ln^2\left[(x-t)^2+l_*^2\right]dx\,dt\right)^{\frac{1}{2}} + \kappa l_*^2\left(\int_{\alpha_i}^{\beta_i}\int_{\xi_k}^{\eta_k}\left[(x-t)^2+l_*^2\right]^{-2}dx\,dt\right)^{\frac{1}{2}}$$
$$(k=\overline{1,m},\ i=\overline{1,n}).$$

Then the conditions (28) will be realized, if

$$\delta^2 < \left(\sum_{i=1}^{n} c_{ji} + \sum_{\rho=1}^{m} e_{j\rho}\right)^{-1} = c_j, \quad j=\overline{1,n},$$

$$\overline{\delta}^2 < \left(\sum_{i=1}^{n} c_{ki}^* + \sum_{\rho=1}^{m} e_{k\rho}^*\right)^{-1} = e_k, \quad k=\overline{1,m},$$

where c_j and e_k are positive numbers less than unity.

We also note that, from the condition of solvability of the system of Fredholm integral equations (24) and (25), we obtain the conditions of its solvability in the form:

$$\delta^2 < \left(\sum_{i=1}^{n} c_{ji}\right)^{-1} = c_j^*,\ j=\overline{1,n}, \qquad \text{and} \qquad \bar{\delta}^2 < \left(\sum_{\rho=1}^{m} e_{k\rho}^*\right)^{-1} = e_k^*,\ k=\overline{1,m},$$

respectively, where c_j^*, $j=\overline{1,n}$, and e_k^*, $k=\overline{1,m}$, are positive numbers less than unity.

The values of unknown shear forces $\varphi_j(x)$ and $\psi_k(x)$ at the end points $x=\alpha_j$, $x=\beta_j$, $j=\overline{1,n}$, and $x=\xi_k$, $x=\eta_k$, $k=\overline{1,m}$, of stringers, respectively, can be obtained from the system (22).

Conclusion. For investigation the changes in the law of distribution and behavior of unknown shear contact forces in this article an effective solution of considered problem is presented. The problem is reduced to solving arbitrary finite number system of Fredholm integral equations of the second kind with respect to unknown shear forces which are specified along two parallel lines on the finite number finite intervals and with right-hand sides which are the solutions of the considered problem in the case of rigid sheet. Further, are determined of the change regions of the problem characteristic parameters for which this system of integral equations allows the exact solutions. For some particular cases considered problem presented above, i.e. for the systems of Fredholm integral equations (22*), (22**) and as well as integral equation (24*) the multiple numerical results and its analysis are presented in [1], in the Supplementary Material.

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Ա. Վ. ՔԵՐՈԲՅԱՆ

ՎԵՐՋԱՎՈՐ ԹՎՈՎ ՎԵՐՋԱՎՈՐ ԵՐԿԱՐՈՒԹՅԱՄԲ ՍՏՐԻՆԳԵՐՆԵՐԻ ՀԱՄԱԿԱՐԳԵՐԻՑ ԲԵՌՆԱՎՈՐՈՒՄՆԵՐԻ ՓՈԽԱՆՑՈՒՄԸ ԱՆՎԵՐՋ ՍԱԼԻՆ ԿՊՉՈՒՆ ՇԵՐՏԵՐԻ ՄԻՋՈՑՈՎ

Աշխատանքում դիտարկված է խնդիր առաձգական անվերջ սալի (թիթեղի) համար, որն իր վերին մակերևույթի վրա երկու զուգահեռ գծերի երկարությամբ վերջավոր տեղամասերում ուժեղացված է վերջավոր թվով վերջավոր երկարությամբ ստրինգերների համակարգերով տարբեր առաձգական բնութագրերով: Փոխազդեցությունը անվերջ սալի և ստրինգերների միջև բոլոր տեղամասերում իրագործվում է բարակ, միատեսակ և այլ ֆիզիկամեխանիկական և երկրաչափական բնութագրեր ունեցող կպչուն շերտերի միջոցով: Ստրինգերները դեֆորմացիայի են ենթարկվում իրենց ծայրերում կիրառված հորիզոնական կենտրոնացած ուժերի ազդեցության տակ: Անհայտ կոնտակտային ուժերի որոշման խնդիրը հանգեցված է երկու զուգահեռ գծերի երկարությամբ տարբեր հատվածներում որոշված վերջավոր թվով անհայտ ֆունկցիաների նկատմամբ Ֆրեդհոլմի երկրորդ սեռի ինտեգրալ հավասարումների համակարգի լուծմանը: Այնուհետև որոշվում են խնդրին բնորոշ բնութագրիչ պարամետրերի փոփոխման այն տիրույթները, որոնց դեպքում այդ հավասարումների համակարգը թույլ է տալիս ճշգրիտ լուծում և որ այն կարելի է լուծել հաջորդական մոտավորությունների մեթոդով: Դիտարկված են մասնավոր դեպքեր և ուսումնասիրված են անհայտ շոշափող կոնտակտային ուժերի վարքը և բնույթը ստրինգերների ծայրակետերում: Այդ դեպքերի համար թվային արդյունքները կախված խնդրի բազմաթիվ պարամետրերից ուսումնասիրված են նախորդ հոդվածում (A.V. Kerobyan, K.P. Sahakyan, Proc. YSU. Phys. Math. Sci. **57** (3) (2023), 86–100):

А. В. КЕРОПЯН

ПЕРЕДАЧА НАГРУЗОК ОТ СИСТЕМ КОНЕЧНОГО ЧИСЛА СТРИНГЕРОВ КОНЕЧНЫХ ДЛИН К БЕСКОНЕЧНОЙ ПЛАСТИНЕ ПОСРЕДСТВОМ ЛИПКИХ СЛОЕВ

В работе рассматривается задача для упругой бесконечной пластины, которая на конечных участках вдоль двух параллельных линий своей верхней поверхности усилена системой из конечного числа стрингеров конечной длины с различными модулями упругости. Контактные связки между пластиной и стрингерами во всех участках осуществляются посредством одинаковых тонких липких слоев с другими физико-механическими и геометрическими характеристиками. Стрингеры деформируются под действием горизонтальных сосредоточенных сил, приложенных на их концах. В работе задача определения закона распределения неизвестных контактных сил, действующих между бесконечной пластиной н стрингерами, сведена к решению системы интегральных уравнений Фредгольма второго рода с конечным числом неизвестных функций, определенных вдоль двух параллельных линий на различных конечных интервалах. Затем определялись области изменения характерных параметров задачи, при которых полученная система интегральных уравнений допускает точное решение и может быть решена методом последовательных приближений. Рассмотрены некоторые частные случаи и исследованы характер и поведение неизвестных касательных контактных сил на концах стрингеров. Численные расчеты для этих случаев в зависимости от различных параметров задачи исследованы в предыдущей статье (A.V. Kerobyan, K.P. Sahakyan, Proc. YSU. Phys. Math. Sci. **57** (3) (2023), 86–100).