

ADHESIVE CONTACT PROBLEM FOR AN INFINITE
SHEET STRENGTHENED BY THREE PARALLEL
FINITE-LENGTH STRINGERS

A. V. KEROPYAN *

Chair of Mechanics, YSU, Armenia

The article considers the problem for an elastic infinite sheet, which along of three parallel lines of its upper surface is strengthened by three parallel finite-length stringers having different elastic properties. The interaction between infinite sheet and three parallel stringers takes place through thin elastic adhesive layers having other physical-mechanical properties and geometric configuration. The stringers are deformed under the action of horizontal concentrated forces, which are applied at one end points of the stringers. The problem of determining the law of distribution of unknown contact forces acting between infinite sheet and stringers is reduced to the system of Fredholm integral equations of second kind with respect to three unknown functions, which are specified on three parallel finite intervals. Further, are determined of the change regions of the problem characteristic parameters, for which this system of integral equations allows the exact solutions and which can be solved by the method of successive approximations. Some special cases are considered and the character and behavior of unknown shear forces near the end points of the stringers are investigated. For these cases numerical results depending on the multiparameters of the problem are investigated in the article by A.V. Kerobyan and K.P. Sahakyan (Proc. YSU. Phys. Math. Sci., 57 (3) (2023), 86–100).

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Introduction. Investigation of problems, which arise during load transfer from stringer to an infinite sheet through adhesive layer and the construction of exact and effective solutions for them have important meaning from both theoretical and applied aspects. Without going into details, we note that some of them, which are closely associated with the considered problem. The problem of load transfer from two parallel elastic stringers with finite-length to an elastic infinite sheet through adhesive

* E-mail: agas50@ysu.am

layers is considered in article [1]. In [2] considers the problem for an elastic infinite sheet, which on its upper surface along two parallel lines is strengthened by three finite-length stringers, two of which are located on the same line, through adhesive layers. The problem for an elastic infinite sheet, which on its upper surface along two parallel lines is strengthened by the systems of finite number finite-length stringers through adhesive layers is considered in [3]. In [4, 5] considers the problems of load transfer from two finite stringers to an infinite sheet (or half-plane) and infinite strip through adhesive layers when two finite stringers are arranged on the same line with different approach to the solutions. In article [6] considers the problem for an infinite sheet with two finite stringers when only one of the stringers is connected through an adhesive layer. In [7–10], using various approaches, problems are investigated for different elastic bodies, which are strengthened by finite-length stringer through adhesive layer. In [11, 12], transfer of loads from finite number of finite-length stringers to an elastic half-plane and to an infinite strip through an adhesive layers are considered. Some problems for an elastic infinite sheet strengthened by parallel finite stringers without presence of adhesive layer are considered in [13]. In this article considers the problem for an elastic infinite sheet, which on its upper surface along three parallel lines is strengthened by three finite-length stringers having different elastic properties. The interaction between sheet and stringers is assumed to be carried out through thin adhesive layers with different physical-mechanical properties and geometric configuration.

Statement of the Problem and Obtaining the System of Integral Equations.

Let an elastic infinite plate of small constant thickness h , the Young's modulus E and the Poisson's ratio ν , which is in a generalized plane stress state (xOy is its middle plane), on its upper surface along three $y = b$, $y = c$ and $y = -d$ parallel lines being $l_1 = b + d$ ($b, d > 0$), $l_2 = b - c$ ($b > c > 0$), $l_3 = c + d$ distance from each other, respectively, on the $[a_1, b_1]$, $[\bar{a}_1, \bar{b}_1]$ and $[c_1, d_1]$, finite intervals, respectively, is reinforced by three parallel finite-length stringers modulus of elasticity equal to E_1 for $x \in [a_1, b_1]$ ($b_1 > a_1$), E_2 for $x \in [\bar{a}_1, \bar{b}_1]$ ($\bar{b}_1 > \bar{a}_1$) and equal to E_3 for $x \in [c_1, d_1]$ ($d_1 > c_1$), respectively. It is supposed that the stringers have a rectangular cross-sections with small constant areas $F = b_1^* h_1$, where b_1^* ($b_1^* \ll b_1 - a_1, \bar{b}_1 - \bar{a}_1$, and $d_1 - c_1$, respectively) are the width of the stringers (it is below also is the width of the adhesive layer), respectively, and h_1 are their small constant thicknesses. The interaction between infinite sheet and stringers takes place through thin, uniform, elastic adhesive layers with Young's modulus E_g , Poisson's ratio ν_g , and small constant thickness h_g . The problem is to specify the law of distribution of unknown contact forces acting between sheet and stringers, when the stringers are axially loaded by concentrated forces P , Q_1 and Q , which are applied at one end points of the stringers $x = b_1$, $x = \bar{b}_1$ and $x = d_1$, respectively, and are directed to parallel along the Ox -axis (see Figure).

It is assumed that during the deformation for the stringers the model of uniaxial strain state in combination with the model of contact along the line is realized [14], and for the adhesive layers there are the pure shear conditions [7], i.e. as [7, 13, 14]

bending is neglected and the interaction between sheet and stringers is idealized as a line loading of the sheet.

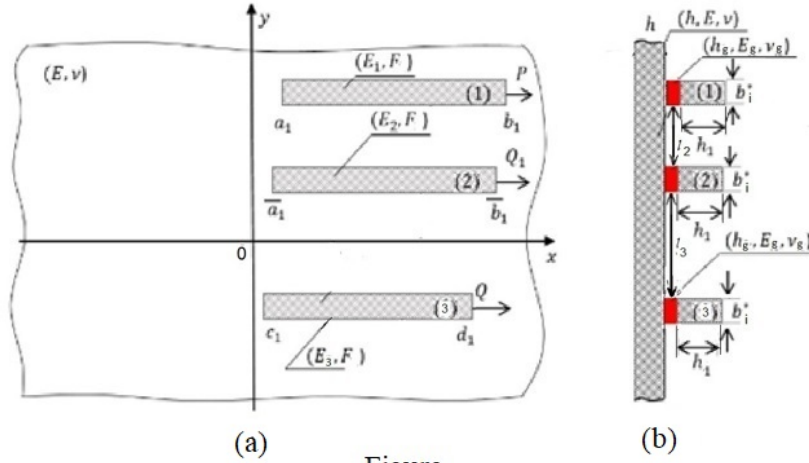


Figure.

Taking into account the above assumptions, according to the equilibrium conditions of an element of finite-length stringers, which are defined on the $[a_1, b_1]$, $[\bar{a}_1, \bar{b}_1]$ and $[c_1, d_1]$ finite intervals, respectively, and Hooke's law, the differential equations for the equilibrium of the stringers on finite intervals $[a_1, b_1]$, $[\bar{a}_1, \bar{b}_1]$ and $[c_1, d_1]$ will be written in the following form:

$$\frac{d^2 u^{(1)}}{dx^2} = \frac{p(x)}{E_1 F}, \quad a_1 \leq x \leq b_1, \quad (1)$$

$$\frac{d^2 u^{(2)}}{dx^2} = \frac{q_1(x)}{E_2 F}, \quad \bar{a}_1 \leq x \leq \bar{b}_1, \quad (2)$$

$$\frac{d^2 u^{(3)}}{dx^2} = \frac{q(x)}{E_3 F}, \quad c_1 \leq x \leq d_1, \quad (3)$$

with the following boundary conditions:

$$\left. \frac{du^{(1)}}{dx} \right|_{x=a_1} = 0, \quad \left. \frac{du^{(1)}}{dx} \right|_{x=b_1} = \frac{P}{E_1 F}, \quad (4)$$

$$\left. \frac{du^{(2)}}{dx} \right|_{x=\bar{a}_1} = 0, \quad \left. \frac{du^{(2)}}{dx} \right|_{x=\bar{b}_1} = \frac{Q_1}{E_2 F}, \quad (5)$$

$$\left. \frac{du^{(3)}}{dx} \right|_{x=c_1} = 0, \quad \left. \frac{du^{(3)}}{dx} \right|_{x=d_1} = \frac{Q}{E_3 F}, \quad (6)$$

and where we have also of the stringers equilibrium conditions in the form:

$$\int_{a_1}^{b_1} p(s) ds = P, \quad \int_{\bar{a}_1}^{\bar{b}_1} q_1(u) du = Q_1, \quad \int_{c_1}^{d_1} q(v) dv = Q. \quad (7)$$

Here $u^{(1)}(x) = u^{(1)}(x, b)$, $u^{(2)}(x) = u^{(2)}(x, c)$ and $u^{(3)}(x) = u^{(3)}(x, -d)$ are the horizontal displacements of the points of the stringers at $y = b$, $y = c$ and $y = -d$ parallel lines, respectively, on the $[a_1, b_1]$, $[\bar{a}_1, \bar{b}_1]$ and $[c_1, d_1]$ finite intervals, respectively; $p(x) = b_1^* \tau^{(1)}(x, b)$, $\tau^{(1)}(x, b)$ is the shear stresses, acting under the stringer on the $[a_1, b_1]$ finite part; $q_1(x) = b_1^* \tau^{(2)}(x, c)$, $\tau^{(2)}(x, c)$ is the shear stresses, acting under the stringer on the $[\bar{a}_1, \bar{b}_1]$ finite part, and $q(x) = b_1^* \tau^{(3)}(x, -d)$, $\tau^{(3)}(x, -d)$ is the shear stresses, acting under the stringer on the $[c_1, d_1]$ finite part.

Now, assuming that each differential element of the adhesive layers is in a condition of pure shear [1–12], the following adhesive contact conditions are obtained:

$$u^{(1)}(x) - u_1(x, b) = k_1^* p(x), \quad a_1 \leq x \leq b_1, \quad (8)$$

$$u^{(2)}(x) - u_2(x, c) = k_1^* q_1(x), \quad \bar{a}_1 \leq x \leq \bar{b}_1, \quad (9)$$

$$u^{(3)}(x) - u_3(x, -d) = k_1^* q(x), \quad c_1 \leq x \leq d_1, \quad (10)$$

where $k_1^* = h_g/b_1^* G_g$, $G_g = E_g/2(1 + \nu_g)$, G_g is the shear modulus of adhesive layers; $u_1(x, b)$, $u_2(x, c)$ and $u_3(x, -d)$ are the horizontal displacements of the points of the elastic infinite sheet at $y = b$, $y = c$ and $y = -d$ parallel lines, respectively; $p(x) = b_1^* \tau^{(1)}(x, b) = b_1^* G_g \gamma_g^{(1)}(x, b)$, $q_1(x) = b_1^* \tau^{(2)}(x, c) = b_1^* G_g \gamma_g^{(2)}(x, c)$ and $q(x) = b_1^* \tau^{(3)}(x, -d) = b_1^* G_g \gamma_g^{(3)}(x, -d)$, $\gamma_g^{(1)}(x, b)$, $\gamma_g^{(2)}(x, c)$ and $\gamma_g^{(3)}(x, -d)$ are the shear deformations of the adhesive layers, on the $[a_1, b_1]$, $[\bar{a}_1, \bar{b}_1]$ and $[c_1, d_1]$ finite intervals, respectively.

On the other hand, in view of above assumptions, let write the horizontal displacements $u_1(x, b)$, $u_2(x, c)$ and $u_3(x, -d)$ of the points of the elastic infinite sheet, when tangential (shear) forces with intensity $p(x)$, $q_1(x)$ and $q(x)$ act on the $[a_1, b_1]$, $[\bar{a}_1, \bar{b}_1]$ and $[c_1, d_1]$ finite intervals of its upper surface along $y = b$, $y = c$ and $y = -d$ parallel lines, respectively, in the following form [1, 2]:

$$u_1(x, b) = \frac{1}{\pi A^*} \int_{a_1}^{b_1} \left(\ln \frac{1}{|x-s|} + C \right) p(s) ds + \frac{1}{\pi A^*} \int_{\bar{a}_1}^{\bar{b}_1} (I(x-u) + C) q_1(u) du + \frac{1}{\pi A^*} \int_{c_1}^{d_1} (N(x-v) + C) q(v) dv, \quad (11)$$

$$u_2(x, c) = \frac{1}{\pi A^*} \int_{\bar{a}_1}^{\bar{b}_1} \left(\ln \frac{1}{|x-u|} + C \right) q_1(u) du + \frac{1}{\pi A^*} \int_{a_1}^{b_1} (I(x-s) + C) p(s) ds + \frac{1}{\pi A^*} \int_{c_1}^{d_1} (J(x-v) + C) q(v) dv, \quad (12)$$

$$u_3(x, -d) = \frac{1}{\pi A^*} \int_{c_1}^{d_1} \left(\ln \frac{1}{|x-v|} + C \right) q(v) dv + \frac{1}{\pi A^*} \int_{a_1}^{b_1} (N(x-s) + C) p(s) ds + \frac{1}{\pi A^*} \int_{\bar{a}_1}^{\bar{b}_1} (J(x-u) + C) q_1(u) du, \quad (13)$$

$$\text{where } N(x) = \ln \frac{1}{\sqrt{x^2 + l_1^2}} - \frac{\kappa l_1^2}{x^2 + l_1^2}, \quad I(x) = \ln \frac{1}{\sqrt{x^2 + l_2^2}} - \frac{\kappa l_2^2}{x^2 + l_2^2},$$

$$J(x) = \ln \frac{1}{\sqrt{x^2 + l_3^2}} - \frac{\kappa l_3^2}{x^2 + l_3^2}, \quad (14)$$

$$A^* = \frac{4Eh}{(1+\nu)(3-\nu)}, \quad \kappa = \frac{1+\nu}{3-\nu}, \quad l_1 = b+d, \quad l_2 = b-c, \quad l_3 = c+d,$$

C is arbitrary constant.

Note that the horizontal displacements $u(x, y)$ of the points of an infinite sheet, when shear forces act on its surface along the line $y = -d$ with intensity $q(x)$ ($-\infty < x < \infty$), is given by the formula

$$u(x, y) = \frac{1}{\pi A^*} \int_{-\infty}^{\infty} \left[\ln \frac{1}{\sqrt{(x-s)^2 + (y+d)^2}} - \frac{\kappa(y+d)^2}{(x-s)^2 + (y+d)^2} \right] q(s) ds + \text{const},$$

$-\infty < x, y < \infty.$

Further, by virtue of (8), (9) and (10), equations (1), (2) and (3) can be written in the form:

$$\frac{d^2 u^{(1)}}{dx^2} - \gamma_1^2 u^{(1)}(x) = -\gamma_1^2 u_1(x, b), \quad a_1 \leq x \leq b_1, \quad (15)$$

$$\frac{d^2 u^{(2)}}{dx^2} - \gamma_2^2 u^{(2)}(x) = -\gamma_2^2 u_2(x, c), \quad \bar{a}_1 \leq x \leq \bar{b}_1, \quad (16)$$

$$\frac{d^2 u^{(3)}}{dx^2} - \alpha^2 u^{(3)}(x) = -\alpha^2 u_3(x, -d), \quad c_1 \leq x \leq d_1, \quad (17)$$

with the boundary conditions (4), (5) and (6), respectively.

Here $\gamma_j^2 = 1/k_1^* E_j F$, $j = 1, 2$, $\alpha^2 = 1/k_1^* E_3 F$.

The solutions to the boundary value problems (15) and (4), (16) and (5), (17) and (6), respectively, we obtain in the form:

$$u^{(1)}(x) = u_0^{(1)}(x) + \gamma_1^2 \int_{a_1}^{b_1} G_1(x, s) u_1(s, b) ds, \quad a_1 \leq x \leq b_1, \quad (18)$$

$$u^{(2)}(x) = u_0^{(2)}(x) + \gamma_2^2 \int_{\bar{a}_1}^{\bar{b}_1} G_2(x, \xi) u_2(\xi, c) d\xi, \quad \bar{a}_1 \leq x \leq \bar{b}_1, \quad (19)$$

$$u^{(3)}(x) = u_0^{(3)}(x) + \alpha^2 \int_{c_1}^{d_1} K(x, v) u_3(v, -d) dv, \quad c_1 \leq x \leq d_1, \quad (20)$$

where $u_0^{(1)}(x)$, $u_0^{(2)}(x)$ and $u_0^{(3)}(x)$ are the general solutions of the homogenous equations corresponding to equations (14), (15) and (16), respectively, with the boundary conditions (4), (5) and (6), respectively, and have the following form:

$$u_0^{(1)}(x) = \frac{P \cosh[\gamma_1(x - a_1)]}{\gamma_1 E_1 F \sinh[\gamma_1(b_1 - a_1)]}, \quad u_0^{(2)}(x) = \frac{Q_1 \cosh[\gamma_2(x - \bar{a}_1)]}{\gamma_2 E_2 F \sinh[\gamma_2(\bar{b}_1 - \bar{a}_1)]},$$

$$u_0^{(3)}(x) = \frac{Q \cosh[\alpha(x - c_1)]}{\alpha E_3 F \sinh[\alpha(d_1 - c_1)]}.$$

In equations (18), (19) and (20), $u_*^{(1)}(x) = \gamma_1^2 \int_{a_1}^{b_1} G_1(x, s) u_1(s, b) ds$,

$$u_*^{(2)}(x) = \gamma_2^2 \int_{\bar{a}_1}^{\bar{b}_1} G_2(x, \xi) u_2(\xi, c) d\xi \quad \text{and} \quad u_*^{(3)}(x) = \alpha^2 \int_{c_1}^{d_1} K(x, v) u_3(v, -d) dv$$

are the particular solutions of equations (15), (16) and (17), respectively, with zero boundary conditions corresponding to conditions (4), (5) and (6), respectively; $G_1(x, s)$, $G_2(x, \xi)$ and $K(x, v)$ are Green's functions [15], and

$$G_1(x, s) = \frac{1}{\gamma_1 \sinh[\gamma_1(b_1 - a_1)]} \begin{cases} \cosh[\gamma_1(x - b_1)] \cosh[\gamma_1(s - a_1)], & x > s, \\ \cosh[\gamma_1(x - a_1)] \cosh[\gamma_1(s - b_1)], & x < s; \end{cases}$$

$$G_2(x, \xi) = \frac{1}{\gamma_2 \sinh[\gamma_2(\bar{b}_1 - \bar{a}_1)]} \begin{cases} \cosh[\gamma_2(x - \bar{b}_1)] \cosh[\gamma_2(\xi - \bar{a}_1)], & x > \xi, \\ \cosh[\gamma_2(x - \bar{a}_1)] \cosh[\gamma_2(\xi - \bar{b}_1)], & x < \xi; \end{cases}$$

$$K(x, v) = \frac{1}{\alpha \sinh[\alpha(d_1 - c_1)]} \begin{cases} \cosh[\alpha(x - d_1)] \cosh[\alpha(v - c_1)], & x > v, \\ \cosh[\alpha(x - c_1)] \cosh[\alpha(v - d_1)], & x < v. \end{cases}$$

It is obvious, that the functions $G_1(x, s)$, $G_2(x, \xi)$ and $K(x, v)$ are continuous functions and $G_1(x, s) = G_1(s, x)$, $G_2(x, \xi) = G_2(\xi, x)$, $K(x, v) = K(v, x)$.

Further, by virtue of (18), (19) and (20), according to (8), (9) and (10), we obtain the following equations:

$$k_1^* p(x) + u_1(x, b) = \gamma_1^2 \int_{a_1}^{b_1} G_1(x, s) u_1(s, b) ds + u_0^{(1)}(x), \quad a_1 \leq x \leq b_1, \quad (21)$$

$$k_1^* q_1(x) + u_2(x, c) = \gamma_2^2 \int_{\bar{a}_1}^{\bar{b}_1} G_2(x, \xi) u_2(\xi, c) d\xi + u_0^{(2)}(x), \quad \bar{a}_1 \leq x \leq \bar{b}_1, \quad (22)$$

$$k_1^* q(x) + u_3(x, -d) = \alpha^2 \int_{c_1}^{d_1} K(x, v) u_3(v, -d) dv + u_0^{(3)}(x), \quad c_1 \leq x \leq d_1. \quad (23)$$

Now, by virtue of (11), (12) and (13), from (21), (22) and (23), we obtain the following system of integral equations:

$$\begin{aligned}
& p(x) + \frac{1}{\pi A^* k_1^*} \left[\int_{a_1}^{b_1} \left(\ln \frac{1}{|x-s|} + C \right) p(s) ds + \int_{\bar{a}_1}^{\bar{b}_1} (I(x-u) + C) q_1(u) du \right. \\
& \left. + \int_{c_1}^{d_1} (N(x-v) + C) q(v) dv \right] = \frac{\gamma_1^2}{\pi A^* k_1^*} \int_{a_1}^{b_1} G_1(x, s) \left[\int_{a_1}^{b_1} \left(\ln \frac{1}{|s-t|} + C \right) p(t) dt \right. \\
& \left. + \int_{\bar{a}_1}^{\bar{b}_1} (I(s-\eta) + C) q_1(\eta) d\eta + \int_{c_1}^{d_1} (N(s-\tau) + C) q(\tau) d\tau \right] ds + \frac{u_0^{(1)}(x)}{k_1^*}, \quad a_1 \leq x \leq b_1; \\
& q_1(x) + \frac{1}{\pi A^* k_1^*} \left[\int_{\bar{a}_1}^{\bar{b}_1} \left(\ln \frac{1}{|x-u|} + C \right) q_1(u) du + \int_{a_1}^{b_1} (I(x-s) + C) p(s) ds \right. \\
& \left. + \int_{c_1}^{d_1} (J(x-v) + C) q(v) dv \right] = \frac{\gamma_2^2}{\pi A^* k_1^*} \int_{\bar{a}_1}^{\bar{b}_1} G_2(x, \xi) \left[\int_{\bar{a}_1}^{\bar{b}_1} \left(\ln \frac{1}{|\xi-\eta|} + C \right) q_1(\eta) d\eta \right. \\
& \left. + \int_{a_1}^{b_1} (I(\xi-t) + C) p(t) dt + \int_{c_1}^{d_1} (J(\xi-\tau) + C) q(\tau) d\tau \right] d\xi + \frac{u_0^{(2)}(x)}{k_1^*}, \quad \bar{a}_1 \leq x \leq \bar{b}_1; \\
& q(x) + \frac{1}{\pi A^* k_1^*} \left[\int_{c_1}^{d_1} \left(\ln \frac{1}{|x-v|} + C \right) q(v) dv + \int_{a_1}^{b_1} (N(x-s) + C) p(s) ds \right. \\
& \left. + \int_{\bar{a}_1}^{\bar{b}_1} (J(x-u) + C) q_1(u) du \right] = \frac{\alpha^2}{\pi A^* k_1^*} \int_{c_1}^{d_1} K(x, v) \left[\int_{c_1}^{d_1} \left(\ln \frac{1}{|v-\tau|} + C \right) q(\tau) d\tau \right. \\
& \left. + \int_{a_1}^{b_1} (N(v-t) + C) p(t) dt + \int_{\bar{a}_1}^{\bar{b}_1} (J(v-\eta) + C) q_1(\eta) d\eta \right] dv + \frac{u_0^{(3)}(x)}{k_1^*}, \quad c_1 \leq x \leq d_1.
\end{aligned} \tag{24}$$

It should be noted that the spectrum of the symmetric second-order differential operator $D = -d^2/dx^2 + \gamma^2 I$ with the domain of definition being twice continuous differentiating functions, satisfying the boundary conditions $(du^{(1)}/dx)_{x=a} = 0$ and $(du^{(1)}/dx)_{x=b} = 0$, are eigenvalues $\lambda_n = \gamma^2 + n^2 \pi^2 / (b-a)^2$ ($n = 0, 1, 2, \dots$) with corresponding eigenfunctions $\cos [n\pi(x-a)/(b-a)]$ ($n = 0, 1, 2, \dots$).

It is known [15], that symmetric quite continuous integral operator B :

$$B\varphi = \int_a^b G(x, s) \varphi(s) ds,$$

which acts in the space $L_2(a, b)$, is an inverse of the operator D .

Hence, we have:

$$\int_{a_1}^{b_1} G_1(x, s) \cos \left[\frac{n\pi(s-a_1)}{b_1-a_1} \right] ds = \frac{(b_1-a_1)^2}{(b_1-a_1)^2\gamma_1^2 + n^2\pi^2} \cos \left[\frac{n\pi(x-a_1)}{b_1-a_1} \right], \quad (25)$$

$$\int_{\bar{a}_1}^{\bar{b}_1} G_2(x, \xi) \cos \left[\frac{p\pi(\xi-\bar{a}_1)}{\bar{b}_1-\bar{a}_1} \right] d\xi = \frac{(\bar{b}_1-\bar{a}_1)^2}{(\bar{b}_1-\bar{a}_1)^2\gamma_2^2 + p^2\pi^2} \cos \left[\frac{p\pi(x-\bar{a}_1)}{\bar{b}_1-\bar{a}_1} \right], \quad (26)$$

$$\int_{c_1}^{d_1} K(x, v) \cos \left[\frac{m\pi(v-c_1)}{d_1-c_1} \right] dv = \frac{(d_1-c_1)^2}{(d_1-c_1)^2\alpha^2 + m^2\pi^2} \cos \left[\frac{m\pi(x-c_1)}{d_1-c_1} \right], \quad (27)$$

where the functions $\cos \left[\frac{n\pi(x-a_1)}{b_1-a_1} \right]$ ($n = 0, 1, 2, \dots$), $\cos \left[\frac{p\pi(x-\bar{a}_1)}{\bar{b}_1-\bar{a}_1} \right]$ ($p = 0, 1, 2, \dots$) and $\cos \left[\frac{m\pi(x-c_1)}{d_1-c_1} \right]$ ($m = 0, 1, 2, \dots$) form full orthogonal systems in the spaces $L_2(a_1, b_1)$, $L_2(\bar{a}_1, \bar{b}_1)$ and $L_2(c_1, d_1)$, respectively.

Further, after replacing the variables x, s, u, v, ξ, η, t and τ by $ax, as, au, av, a\xi, a\eta, at$ and $a\tau$, respectively, where $a > 0$ is the coordinate of one of the end points of stringers, we will represent the system of integral equations (24) as follows:

$$\begin{aligned} \varphi_1(x) + \delta^2 \int_{\alpha_1}^{\beta_1} \ln \frac{1}{|x-t|} \varphi_1(t) dt - a\gamma_1^2 \delta^2 \int_{\alpha_1}^{\beta_1} G_1(ax, as) \int_{\alpha_1}^{\beta_1} \ln \frac{1}{|s-t|} \varphi_1(t) dt ds \\ + \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} I_1(x-\eta) \psi_1(\eta) d\eta - a\gamma_1^2 \delta^2 \int_{\alpha_1}^{\beta_1} G_1(ax, as) \int_{\bar{\alpha}_1}^{\bar{\beta}_1} I_1(s-\eta) \psi_1(\eta) d\eta ds \\ + \delta^2 \int_{\xi_1}^{\eta_1} N_1(x-\tau) \psi_2(\tau) d\tau - a\gamma_1^2 \delta^2 \int_{\alpha_1}^{\beta_1} G_1(ax, as) \int_{\xi_1}^{\eta_1} N_1(s-\tau) \psi_2(\tau) d\tau ds \\ - ap_0^{(1)}(ax) = 0, \quad \alpha_1 \leq x \leq \beta_1; \\ \psi_1(x) + \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} \ln \frac{1}{|x-\eta|} \psi_1(\eta) d\eta - a\gamma_2^2 \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} G_2(ax, a\xi) \int_{\bar{\alpha}_1}^{\bar{\beta}_1} \ln \frac{1}{|\xi-\eta|} \psi_1(\eta) d\eta d\xi \\ + \delta^2 \int_{\alpha_1}^{\beta_1} I_1(x-t) \varphi_1(t) dt - a\gamma_2^2 \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} G_2(ax, a\xi) \int_{\alpha_1}^{\beta_1} I_1(\xi-t) \varphi_1(t) dt d\xi \\ + \delta^2 \int_{\xi_1}^{\eta_1} J_1(x-\tau) \psi_2(\tau) d\tau - a\gamma_2^2 \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} G_2(ax, a\xi) \int_{\xi_1}^{\eta_1} J_1(\xi-\tau) \psi_2(\tau) d\tau d\xi \\ - aq_0^{(2)}(ax) = 0, \quad \bar{\alpha}_1 \leq x \leq \bar{\beta}_1; \quad (28) \end{aligned}$$

$$\begin{aligned}
& \psi_2(x) + \delta^2 \int_{\xi_1}^{\eta_1} \ln \frac{1}{|x-\tau|} \psi_2(\tau) d\tau - a \alpha^2 \delta^2 \int_{\xi_1}^{\eta_1} K(ax, av) \int_{\xi_1}^{\eta_1} \ln \frac{1}{|v-\tau|} \psi_2(\tau) d\tau dv \\
& + \delta^2 \int_{\alpha_1}^{\beta_1} N_1(x-t) \varphi_1(t) dt - a \alpha^2 \delta^2 \int_{\xi_1}^{\eta_1} K(ax, av) \int_{\alpha_1}^{\beta_1} N_1(v-t) \varphi_1(t) dt dv \\
& + \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} J_1(x-\eta) \psi_1(\eta) d\eta - a \alpha^2 \delta^2 \int_{\xi_1}^{\eta_1} K(ax, av) \int_{\bar{\alpha}_1}^{\bar{\beta}_1} J_1(v-\eta) \psi_1(\eta) d\eta dv \\
& - a q_0^{(3)}(ax) = 0, \quad \xi_1 \leq x \leq \eta_1,
\end{aligned}$$

since according to (25), (26) and (27) we have also the following equalities :

$$\int_{\alpha_1}^{\beta_1} G_1(ax, as) ds = \frac{1}{a\gamma_1^2}, \quad \int_{\bar{\alpha}_1}^{\bar{\beta}_1} G_2(ax, a\xi) d\xi = \frac{1}{a\gamma_2^2}, \quad \int_{\xi_1}^{\eta_1} K(ax, av) dv = \frac{1}{a\alpha^2}. \quad (29)$$

Here

$$\alpha_1 = a_1/a, \quad \beta_1 = b_1/a, \quad \bar{\alpha}_1 = \bar{a}_1/a, \quad \bar{\beta}_1 = \bar{b}_1/a, \quad \xi_1 = c_1/a, \quad \eta_1 = d_1/a, \\
l_{1^*} = l_1/a, \quad l_{2^*} = l_2/a, \quad l_{3^*} = l_3/a, \quad \delta^2 = a/\pi k_1^* A^*,$$

$$\varphi_1(x) = ap(ax), \quad \psi_1(x) = aq_1(ax), \quad \psi_2(x) = aq(ax), \quad f_0(x) = ap_0^{(1)}(ax) = \frac{au_0^{(1)}(ax)}{k_1^*},$$

$$g_0(x) = aq_0^{(2)}(ax) = \frac{au_0^{(2)}(ax)}{k_1^*}, \quad q_0(x) = aq_0^{(3)}(ax) = \frac{au_0^{(3)}(ax)}{k_1^*}, \quad (30)$$

$$N(ax) = \ln \frac{1}{a} + N_1(x), \quad N_1(x) = \ln \frac{1}{\sqrt{x^2 + l_{1^*}^2}} - \frac{\kappa l_{1^*}^2}{x^2 + l_{1^*}^2},$$

$$I(ax) = \ln \frac{1}{a} + I_1(x), \quad I_1(x) = \ln \frac{1}{\sqrt{x^2 + l_{2^*}^2}} - \frac{\kappa l_{2^*}^2}{x^2 + l_{2^*}^2},$$

$$J(ax) = \ln \frac{1}{a} + J_1(x), \quad J_1(x) = \ln \frac{1}{\sqrt{x^2 + l_{3^*}^2}} - \frac{\kappa l_{3^*}^2}{x^2 + l_{3^*}^2}.$$

One can represent the system of integral equations (28) in the following form:

$$\begin{aligned}
\varphi_1(x) + \delta^2 \int_{\alpha_1}^{\beta_1} M_1(x, t) \varphi_1(t) dt + \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} S_1(x, \eta) \psi_1(\eta) d\eta \\
+ \delta^2 \int_{\xi_1}^{\eta_1} H_1(x, \tau) \psi_2(\tau) d\tau = f_0(x), \quad \alpha_1 \leq x \leq \beta_1,
\end{aligned}$$

$$\begin{aligned} \psi_1(x) + \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} M_2(x, \eta) \psi_1(\eta) d\eta + \delta^2 \int_{\alpha_1}^{\beta_1} S_2(x, t) \varphi_1(t) dt \\ + \delta^2 \int_{\xi_1}^{\eta_1} R_1(x, \tau) \psi_2(\tau) d\tau = g_0(x), \quad \bar{\alpha}_1 \leq x \leq \bar{\beta}_1, \end{aligned} \quad (31)$$

$$\begin{aligned} \psi_2(x) + \delta^2 \int_{\xi_1}^{\eta_1} R_2(x, \tau) \psi_2(\tau) d\tau + \delta^2 \int_{\alpha_1}^{\beta_1} H_2(x, t) \varphi_1(t) dt \\ + \delta^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} R_3(x, \eta) \psi_1(\eta) d\eta = q_0(x), \quad \xi_1 \leq x \leq \eta_1, \end{aligned}$$

where

$$\begin{aligned} M_1(x, t) &= \ln \frac{1}{|x-t|} - a\gamma_1^2 \int_{\alpha_1}^{\beta_1} G_1(ax, as) \ln \frac{1}{|s-t|} ds, \\ S_1(x, \eta) &= I_1(x-\eta) - a\gamma_1^2 \int_{\alpha_1}^{\beta_1} G_1(ax, as) I_1(s-\eta) ds, \\ H_1(x, \tau) &= N_1(x-\tau) - a\gamma_1^2 \int_{\alpha_1}^{\beta_1} G_1(ax, as) N_1(s-\tau) ds, \\ M_2(x, \eta) &= \ln \frac{1}{|x-\eta|} - a\gamma_2^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} G_2(ax, a\xi) \ln \frac{1}{|\xi-\eta|} d\xi, \\ S_2(x, t) &= I_1(x-t) - a\gamma_2^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} G_2(ax, a\xi) I_1(\xi-t) d\xi, \\ R_1(x, \tau) &= J_1(x-\tau) - a\gamma_2^2 \int_{\bar{\alpha}_1}^{\bar{\beta}_1} G_2(ax, a\xi) J_1(\xi-\tau) d\xi, \\ H_2(x, t) &= N_1(x-t) - a\alpha^2 \int_{\xi_1}^{\eta_1} K(ax, av) N_1(v-t) dv, \\ R_3(x, \eta) &= J_1(x-\eta) - a\alpha^2 \int_{\xi_1}^{\eta_1} K(ax, av) J_1(v-\eta) dv, \\ R_2(x, \tau) &= \ln \frac{1}{|x-\tau|} - a\alpha^2 \int_{\xi_1}^{\eta_1} K(ax, av) \ln \frac{1}{|v-\tau|} dv, \end{aligned} \quad (32)$$

$$\begin{aligned}
f_0(x) &= ap_0^{(1)}(ax) = \frac{au_0^{(1)}(ax)}{k_1^*} = \frac{Pa\gamma_1 \cosh[a\gamma_1(x - \alpha_1)]}{\sinh[a\gamma_1(\beta_1 - \alpha_1)]}, \\
g_0(x) &= aq_0^{(2)}(ax) = \frac{au_0^{(2)}(ax)}{k_1^*} = \frac{Q_1a\gamma_2 \cosh[a\gamma_2(x - \bar{\alpha}_1)]}{\sinh[a\gamma_2(\bar{\beta}_1 - \bar{\alpha}_1)]}, \\
q_0(x) &= aq_0^{(3)}(ax) = \frac{au_0^{(3)}(ax)}{k_1^*} = \frac{Qa\alpha \cosh[a\alpha(x - \xi_1)]}{\sinh[a\alpha(\eta_1 - \xi_1)]}.
\end{aligned}$$

Note that the system of integral equations (31) is obtained by the changing the order of integration, the validity of which follows from the Fubini's theorem [15]. This theorem will often be used below without special mention.

It is easy to see that the functions $f_0(x)$, $g_0(x)$ and $q_0(x)$ are the adhesive contact forces in the case of rigid sheet (i.e. when $E \rightarrow \infty$) and integrable functions on the segments $x \in [\alpha_1, \beta_1]$, $x \in [\bar{\alpha}_1, \bar{\beta}_1]$ and $x \in [\xi_1, \eta_1]$, respectively.

Note that also the system of integral equations (31) was obtained without using the stringers equilibrium conditions (7) in the form:

$$\int_{\alpha_1}^{\beta_1} p(as) ds = P/a, \quad \int_{\bar{\alpha}_1}^{\bar{\beta}_1} q_1(au) du = Q_1/a, \quad \int_{\xi_1}^{\eta_1} q(av) dv = Q/a. \quad (33)$$

In the system (31), the conditions (33) are satisfied automatically [1, 2], since the following equalities take place:

$$\int_{\alpha_1}^{\beta_1} f_0(x) dx = P, \quad \int_{\bar{\alpha}_1}^{\bar{\beta}_1} g_0(x) dx = Q_1, \quad \int_{\xi_1}^{\eta_1} q_0(x) dx = Q.$$

Some Special Cases. Now let us consider some special cases that are directly obtained from the system of integral equations (31). In the case $\delta^2 = 0$, from the system (31) we obtain the solution of the corresponding problem for the case of a rigid sheet (i.e. when $E \rightarrow \infty$) in the form $\varphi_1(x) = f_0(x)$, $x \in [\alpha_1, \beta_1]$, $\psi_1(x) = g_0(x)$, $x \in [\bar{\alpha}_1, \bar{\beta}_1]$ and $\psi_2(x) = q_0(x)$, $x \in [\xi_1, \eta_1]$, respectively. From these solutions, it is easy to see that the functions $\varphi_1(x)$, $\psi_1(x)$ and $\psi_2(x)$ are bounded when $x \rightarrow \alpha_1$, $x \rightarrow \beta_1$, $x \rightarrow \bar{\alpha}_1$, $x \rightarrow \bar{\beta}_1$, and $x \rightarrow \xi_1$, $x \rightarrow \eta_1$, respectively. Here note that this solutions are correspond also to the starting solutions (or the zero approximations) of the system (31), obtained by the method of successive approximations, depending on γ_1 , γ_2 and α parameters, respectively.

In the case of two parallel finite stringers arranged f. ex., on the finite intervals $[a_1, b_1]$ and $[c_1, d_1]$, instead of system (31), we obtain the system of Fredholm integral equations of the second kind with respect to an unknown contact forces $\varphi_1(x)$ and $\psi_2(x)$, defined on the segments $[\alpha_1, \beta_1]$ and $[\xi_1, \eta_1]$, respectively, in the following

form:

$$\begin{aligned} \varphi_1(x) + \delta^2 \int_{\alpha_1}^{\beta_1} M_1(x,t) \varphi_1(t) dt + \delta^2 \int_{\xi_1}^{\eta_1} H_1(x,\tau) \psi_2(\tau) d\tau &= f_0(x), \quad \alpha_1 \leq x \leq \beta_1, \\ \psi_2(x) + \delta^2 \int_{\xi_1}^{\eta_1} R_2(x,\tau) \psi_2(\tau) d\tau + \delta^2 \int_{\alpha_1}^{\beta_1} H_2(x,t) \varphi_1(t) dt &= q_0(x), \quad \xi_1 \leq x \leq \eta_1, \end{aligned} \quad (34)$$

and also with other combinations.

In the case of one finite stringer defined f. ex., on the finite interval $[a_1, b_1]$, or on the finite interval $[c_1, d_1]$, instead of system (31) or (34), we obtain the Fredholm integral equation of the second kind with respect to an unknown function $\varphi_1(x)$ defined on the segment $[\alpha_1, \beta_1]$ in the following form:

$$\varphi_1(x) + \delta^2 \int_{\alpha_1}^{\beta_1} M_1(x,t) \varphi_1(t) dt = f_0(x), \quad \alpha_1 \leq x \leq \beta_1, \quad (35)$$

or with respect to an unknown function $\psi_2(x)$ defined on the segment $[\xi_1, \eta_1]$ in the form:

$$\psi_2(x) + \delta^2 \int_{\xi_1}^{\eta_1} R_2(x,\tau) \psi_2(\tau) d\tau = q_0(x), \quad \xi_1 \leq x \leq \eta_1. \quad (36)$$

Comparing of the integral equations (35) and (36), it is shown that they have the same form. Thus, solving the problem is reduced to solving the system (31) of Fredholm integral equations of the second kind with squarely integrable kernels in two variables and with right-hand sides, which are the solutions of the problem in the case of rigid sheet. From the system (31), it is easy to see that at the end points $x = \alpha_1, x = \beta_1, x = \bar{\alpha}_1, x = \bar{\beta}_1$ and $x = \xi_1, x = \eta_1$ of stringers, respectively, the values of unknown contact forces $\varphi_1(x), \psi_1(x)$ and $\psi_2(x)$, respectively, are finite.

Note that also, which without presence of adhesive layer (i.e. in the case of ideal mechanical contact between sheet and stringer) in the same end points of the stringer the intensities of unknown contact shear forces (or stresses) have singularity of the square root power of integrable order [6, 13, 14, 16–18].

Investigation of the Solvability of the System of Integral Equations (31).

Now write the system (31) in the following form:

$$\psi + S\psi = y_0, \quad (37)$$

where

$$\psi = \begin{pmatrix} \varphi_1 \\ \psi_1 \\ \psi_2 \end{pmatrix}, \quad y_0 = \begin{pmatrix} f_0 \\ g_0 \\ q_0 \end{pmatrix}, \quad S = \begin{pmatrix} \delta^2 k_{11} & \delta^2 k_{12} & \delta^2 k_{13} \\ \delta^2 k_{21} & \delta^2 k_{22} & \delta^2 k_{23} \\ \delta^2 k_{31} & \delta^2 k_{32} & \delta^2 k_{33} \end{pmatrix},$$

$$\begin{aligned}
k_{11}\varphi_1 &= \int_{\alpha_1}^{\beta_1} M_1(x,t) \varphi_1(t) dt, & k_{12}\psi_1 &= \int_{\bar{\alpha}_1}^{\bar{\beta}_1} S_1(x,\eta) \psi_1(\eta) d\eta, \\
k_{13}\psi_2 &= \int_{\xi_1}^{\eta_1} H_1(x,\tau) \psi_2(\tau) d\tau, & k_{21}\varphi_1 &= \int_{\alpha_1}^{\beta_1} S_2(x,t) \varphi_1(t) dt, \\
k_{22}\psi_1 &= \int_{\bar{\alpha}_1}^{\bar{\beta}_1} M_2(x,\eta) \psi_1(\eta) d\eta, & k_{23}\psi_2 &= \int_{\xi_1}^{\eta_1} R_1(x,\tau) \psi_2(\tau) d\tau, \\
k_{31}\varphi_1 &= \int_{\alpha_1}^{\beta_1} H_2(x,t) \varphi_1(t) dt, & k_{32}\psi_1 &= \int_{\bar{\alpha}_1}^{\bar{\beta}_1} R_3(x,\eta) \psi_1(\eta) d\eta, \\
k_{33}\psi_2 &= \int_{\xi_1}^{\eta_1} R_2(x,\tau) \psi_2(\tau) d\tau.
\end{aligned} \tag{38}$$

Further, consider operator equation (37) in Banach space with elements $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$, where $y_1(x) \in L_2(\alpha_1, \beta_1)$, $y_2(x) \in L_2(\bar{\alpha}_1, \bar{\beta}_1)$ and $y_3(x) \in L_2(\xi_1, \eta_1)$

and with norm $\|Y\| = \max \left\{ \|y_1\|_{L_2(\alpha_1, \beta_1)}, \|y_2\|_{L_2(\bar{\alpha}_1, \bar{\beta}_1)}, \|y_3\|_{L_2(\xi_1, \eta_1)} \right\}$. $L_2(\alpha_1, \beta_1)$, $L_2(\bar{\alpha}_1, \bar{\beta}_1)$ and $L_2(\xi_1, \eta_1)$ are spaces of square integrable functions, specified on the intervals (α_1, β_1) , $(\bar{\alpha}_1, \bar{\beta}_1)$ and (ξ_1, η_1) , respectively.

Operators k_{11} , k_{22} and k_{33} act in the spaces $L_2(\alpha_1, \beta_1)$, $L_2(\bar{\alpha}_1, \bar{\beta}_1)$ and $L_2(\xi_1, \eta_1)$, respectively, and operators k_{12} , k_{13} , k_{21} , k_{23} , k_{31} , k_{32} act in the following form: $k_{12}: L_2(\bar{\alpha}_1, \bar{\beta}_1) \rightarrow L_2(\alpha_1, \beta_1)$, $k_{13}: L_2(\xi_1, \eta_1) \rightarrow L_2(\alpha_1, \beta_1)$, $k_{21}: L_2(\alpha_1, \beta_1) \rightarrow L_2(\bar{\alpha}_1, \bar{\beta}_1)$, $k_{23}: L_2(\xi_1, \eta_1) \rightarrow L_2(\bar{\alpha}_1, \bar{\beta}_1)$ and $k_{31}: L_2(\alpha_1, \beta_1) \rightarrow L_2(\xi_1, \eta_1)$, $k_{32}: L_2(\bar{\alpha}_1, \bar{\beta}_1) \rightarrow L_2(\xi_1, \eta_1)$.

Obviously, the operator S acts in the Banach space and is a Fredholm operator. A sufficient condition for inversion of operator $I + S$ is the condition $\|S\| < 1$. Then operational equation (37) can be solved by the method of successive approximations, if $\|S\| < 1$, where

$$\|S\| = \max \left\{ \delta^2 (\|k_{11}\| + \|k_{12}\| + \|k_{13}\|), \delta^2 (\|k_{21}\| + \|k_{22}\| + \|k_{23}\|), \delta^2 (\|k_{31}\| + \|k_{32}\| + \|k_{33}\|) \right\}.$$

Therefore, the condition $\|S\| < 1$ will be satisfied, if

$$\delta^2 (\|k_{11}\| + \|k_{12}\| + \|k_{13}\|) < 1, \quad \delta^2 (\|k_{21}\| + \|k_{22}\| + \|k_{23}\|) < 1, \\
\delta^2 (\|k_{31}\| + \|k_{32}\| + \|k_{33}\|) < 1. \tag{39}$$

In this case, the solution of equation (37) is written in the form

$$\psi = (I + S)^{-1} y_0 = \sum_{m=0}^{\infty} (-1)^m S^m y_0.$$

Now let's determine the values of δ^2 parameters of the problem, for which the conditions (39) will be satisfied. From (38), by virtue of Cauchy-Bunyakovski inequality, we get:

$$\begin{aligned}
\|k_{11}\| \leq c_{11}, c_{11} &= \left(\int_{\alpha_1}^{\beta_1} \int_{\alpha_1}^{\beta_1} M_1^2(x, t) dx dt \right)^{\frac{1}{2}}, \|k_{12}\| \leq c_{12}, c_{12} = \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\alpha_1}^{\beta_1} S_1^2(x, \eta) dx d\eta \right)^{\frac{1}{2}}, \\
\|k_{13}\| \leq c_{13}, c_{13} &= \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} H_1^2(x, \tau) dx d\tau \right)^{\frac{1}{2}}, \|k_{21}\| \leq c_{21}, c_{21} = \left(\int_{\alpha_1}^{\beta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} S_2^2(x, t) dx dt \right)^{\frac{1}{2}}, \\
\|k_{22}\| \leq c_{22}, c_{22} &= \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} M_2^2(x, \eta) dx d\eta \right)^{\frac{1}{2}}, \|k_{23}\| \leq c_{23}, c_{23} = \left(\int_{\xi_1}^{\eta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} R_1^2(x, \tau) dx d\tau \right)^{\frac{1}{2}}, \\
\|k_{31}\| \leq c_{31}, c_{31} &= \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} H_2^2(x, t) dx dt \right)^{\frac{1}{2}}, \|k_{32}\| \leq c_{32}, c_{32} = \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\xi_1}^{\eta_1} R_3^2(x, \eta) dx d\eta \right)^{\frac{1}{2}}, \\
\|k_{33}\| \leq c_{33}, c_{33} &= \left(\int_{\xi_1}^{\eta_1} \int_{\xi_1}^{\eta_1} R_2^2(x, \tau) dx d\tau \right)^{\frac{1}{2}}. \tag{40}
\end{aligned}$$

Obviously, the expressions for c_{ij} ($i, j = \overline{1, 3}$) are difficult to calculate, but they can be estimated. It was found out in [2], that the following estimates take place:

$$\begin{aligned}
c_{11} &< \left(\int_{\alpha_1}^{\beta_1} \int_{\alpha_1}^{\beta_1} \ln^2 |x - t| dx dt \right)^{\frac{1}{2}}, & c_{22} &< \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} \ln^2 |x - \eta| dx d\eta \right)^{\frac{1}{2}}, \\
c_{33} &< \left(\int_{\xi_1}^{\eta_1} \int_{\xi_1}^{\eta_1} \ln^2 |x - \tau| dx d\tau \right)^{\frac{1}{2}}, & c_{12} &< \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\alpha_1}^{\beta_1} I_1^2(x - \eta) dx d\eta \right)^{\frac{1}{2}}, \\
c_{13} &< \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} N_1^2(x - \tau) dx d\tau \right)^{\frac{1}{2}}, & c_{21} &< \left(\int_{\alpha_1}^{\beta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} I_1^2(x - t) dx dt \right)^{\frac{1}{2}}, \\
c_{23} &< \left(\int_{\xi_1}^{\eta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} J_1^2(x - \tau) dx d\tau \right)^{\frac{1}{2}}, & c_{31} &< \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} N_1^2(x - t) dx dt \right)^{\frac{1}{2}}, \\
c_{32} &< \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\xi_1}^{\eta_1} J_1^2(x - \eta) dx d\eta \right)^{\frac{1}{2}}. \tag{41}
\end{aligned}$$

The estimates (41) for c_{12} , c_{13} , c_{21} , c_{23} and c_{31} , c_{32} can be obtained also in the form:

$$\begin{aligned}
c_{12} &< \frac{1}{2} \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\alpha_1}^{\beta_1} \ln^2 [(x-\eta)^2 + l_{2*}^2] dx d\eta \right)^{\frac{1}{2}} + \kappa l_{2*}^2 \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\alpha_1}^{\beta_1} [(x-\eta)^2 + l_{2*}^2]^{-2} dx d\eta \right)^{\frac{1}{2}}, \\
c_{13} &< \frac{1}{2} \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} \ln^2 [(x-\tau)^2 + l_{1*}^2] dx d\tau \right)^{\frac{1}{2}} + \kappa l_{1*}^2 \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} [(x-\tau)^2 + l_{1*}^2]^{-2} dx d\tau \right)^{\frac{1}{2}}, \\
c_{21} &< \frac{1}{2} \left(\int_{\alpha_1}^{\beta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} \ln^2 [(x-t)^2 + l_{2*}^2] dx dt \right)^{\frac{1}{2}} + \kappa l_{2*}^2 \left(\int_{\alpha_1}^{\beta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} [(x-t)^2 + l_{2*}^2]^{-2} dx dt \right)^{\frac{1}{2}}, \\
c_{23} &< \frac{1}{2} \left(\int_{\xi_1}^{\eta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} \ln^2 [(x-\tau)^2 + l_{3*}^2] dx d\tau \right)^{\frac{1}{2}} + \kappa l_{3*}^2 \left(\int_{\xi_1}^{\eta_1} \int_{\bar{\alpha}_1}^{\bar{\beta}_1} [(x-\tau)^2 + l_{3*}^2]^{-2} dx d\tau \right)^{\frac{1}{2}}, \\
c_{31} &< \frac{1}{2} \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} \ln^2 [(x-t)^2 + l_{1*}^2] dx dt \right)^{\frac{1}{2}} + \kappa l_{1*}^2 \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} [(x-t)^2 + l_{1*}^2]^{-2} dx dt \right)^{\frac{1}{2}}, \\
c_{32} &< \frac{1}{2} \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\xi_1}^{\eta_1} \ln^2 [(x-\eta)^2 + l_{3*}^2] dx d\eta \right)^{\frac{1}{2}} + \kappa l_{3*}^2 \left(\int_{\bar{\alpha}_1}^{\bar{\beta}_1} \int_{\xi_1}^{\eta_1} [(x-\eta)^2 + l_{3*}^2]^{-2} dx d\eta \right)^{\frac{1}{2}}.
\end{aligned} \tag{42}$$

Then the conditions (39) will be realized, if

$$\begin{aligned}
\delta^2 &< (c_{11} + c_{12} + c_{13})^{-1} = c_1, & \delta^2 &< (c_{21} + c_{22} + c_{23})^{-1} = c_2, \\
\delta^2 &< (c_{31} + c_{32} + c_{33})^{-1} = c_3,
\end{aligned} \tag{43}$$

where c_1 , c_2 and c_3 are positive numbers less than unity.

Note that from the condition of solvability of the system of Fredholm integral equations (34) for δ^2 parameters, correspond to (39) and (43) conditions, in this case we obtain the condition of solvability in the form

$$\delta^2 < (c_{11}^* + c_{12}^*)^{-1} = c_1^*, \quad \delta^2 < (c_{21}^* + c_{22}^*)^{-1} = c_2^*, \tag{44}$$

where we have corresponding to the above the following notations, respectively:

$$c_{11}^* = c_{11}, \quad c_{12}^* = c_{13} \quad \text{and} \quad c_{21}^* = c_{31}, \quad c_{22}^* = c_{33}.$$

For the integral equation (35) we obtain the condition of solvability in the form

$$\delta^2 < \left(\int_{\alpha_1}^{\beta_1} \int_{\alpha_1}^{\beta_1} \ln^2 |x-t| dx dt \right)^{-\frac{1}{2}}.$$

The values of unknown shear forces $\varphi_1(x)$, $\psi_j(x)$ ($j = 1, 2$) at the end points of the stringers $x = \alpha_1$, $x = \beta_1$, $x = \bar{\alpha}_1$, $x = \bar{\beta}_1$, and $x = \xi_1$, $x = \eta_1$, respectively, can be obtained from the system (31).

Conclusion. For investigation the changes in the law of distribution and behavior of unknown contact forces in this article is presented an effective solution of considered problem. The problem is reduced to solving the system of Fredholm integral equations of the second kind with respect to three unknown functions, which are specified to on three each other parallel finite intervals and with right-hand sides, which are the solutions of the considered problem in the case of rigid sheet. Further, they are determined of the change regions of the problem characteristic parameters, for which this system of integral equations allows the exact solution and which can be solved by the method of successive approximations. Note that for some particular cases considered problem presented above, i. e. for the system of integral equations (34) and integral equation (35) the multiple numerical results and its analysis depending on the change of characteristic parameters of the problem are presented in article [2], in the Supplementary Material. Also note that, using analogous reasoning, we can obtain also the solutions to problems with a similar view in the case of four or more each other parallel finitely number finite-length stringers.

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REFERENCES

1. Kerobyan A.V. On a Problem for an Elastic Infinite Sheet Strengthened by Two Parallel Stringers with Finite Lengths through Adhesive Shear Layers. *Proc. of the YSU A. Phys. Math. Sci.* **54** (2020), 153–164.
<https://doi.org/10.46991/PYSU:A/2020.54.3.153>
2. Kerobyan A.V., Sahakyan K.P. Transfer of Loads from Three Heterogeneous Elastic Stringers with Finite Lengths to an Infinite Sheet through Adhesive Layers. *Proc. of the YSU A. Phys. Math. Sci.* **57** (2023), 86–100.
<https://doi.org/10.46991/PYSU:A/2023.57.3.086>
3. Kerobyan A.V. Loads Transfer from the Systems of Finite Number Finite Length Stringers to an Infinite Sheet through Adhesive Layers. *Proc. of the YSU A. Phys. Math. Sci.* **59** (2025), 46–62.
<https://doi.org/10.46991/PYSU:A/2025.59.2.046>
4. Kerobyan A.V. About Contact Problems for an Elastic Half-Plane and the Infinite Plate with Two Finite Elastic Overlays in the Presence of Shear Interlayers. *Proc. of the YSU A. Phys. Math. Sci.* **49** (2015), 30–38.
<https://doi.org/10.46991/PYSU:A/2015.49.2.030>

5. Kerobyan A.V. Contact Problems for an Elastic Strip and the Infinite Plate with Two Finite Elastic Overlays in the Presence of Shear Interlayers. *Proc. of the NAS of RA. Mechanics* **67** (2014), 22–34 (in Russian).
<http://doi.org/10.33018/67.1.2>
6. Grigoryan E.Kh., Kerobyan A.V., Shahinyan S.S. The Contact Problem for the Infinite Plate with Two Finite Stringers One from which is Glued, Other is Ideal Conducted. *Proc. of the NAS of RA. Mechanics* **55** (2002), 14–23 (in Russian).
7. Lubkin J.I., Lewis L.C. Adhesive Shear Flow for an Axially Loaded, Finite Stringer Bonded to an Infinite Sheet. *Quarterly Journal of Mechanics and Applied Mathematics* **23** (1970), 521–533.
<http://doi.org/10.1093/qjmam/23.4.521>
8. Kerobyan A.V., Sarkisyan V.S. The Solution of the Problem for an Anisotropic Half-plane on the Boundary of which Finite Length Stringer is Glued. *Proc. of the Scientific Conference, Dedicated to the 60th Anniversary of the Pedagogical Institute of Gyumri. Gyumri, Vysshaya Shkola* **1** (1994), 73–76 (in Russian).
9. Grigoryan E.Kh. On Solution of Problem for an Elastic Infinite Plate, one the Surface of which Finite Length Stringer is Glued. *Proc. of the NAS of RA. Mechanics* **53** (2000), 11–16 (in Russian).
10. Sarkisyan V.S., Kerobyan A.V. On the Solution of the Problem for Anisotropic Half-plane on the Edge of which a Nonlinear Deformable Stringer of Finite Length is Glued. *Proc. of the NAS of RA. Mechanics* **50** (1997), 17–26 (in Russian).
11. Kerobyan A.V., Sahakyan K.P. Loads Transfer from Finite Number Finite Stringers to an Elastic Half-plane through Adhesive Shear Layers. *Proc. of the NAS of RA. Mechanics* **70** (2017), 39–56 (in Russian).
<https://doi.org/10.33018/70.3.4>
12. Kerobyan A.V. Transfer of Loads from a Finite Number of Elastic Overlays with Finite Lengths to an Elastic Strip through Adhesive Shear Layers. *Proc. of the YSU A. Phys. Math. Sci.* **53** (2019), 109–118.
<https://doi.org/10.46991/PYSU:A/2019.53.2.109>
13. Aghayan K.L. Some Contact Problems for an Elastic Infinite Plate Strengthened by Elastic Overlays. *Proc. of the AS USSR. Mechanics of Solids* **5** (1972), 34–45 (in Russian).
14. Muki R., Sternberg E. On the Diffusion of Load from a Transverse Tension Bar into a Semi-infinite Elastic Sheet. *Transactions of the ASME* **4** (1968), 737–746.
<https://doi.org/10.1115/1.3601299>
15. Shilov G.E. *Mathematical Analysis. Special Course*. Moscow, State Publishing House of Physical and Mathematical Literature (1961), 442 (in Russian).
16. Aleksandrov B.M., Mkhitarian S.M. *Contact Problems for Bodies with Thin Coatings and Interlayers*. Moscow, Nauka (1983), 488 (in Russian).
17. Grigolyuk E.I., Tolkachev V.M. *Contact Problems of the Theory of Plates and Shells*. Moscow, Mashinostroenie (1980), 416 (in Russian).
18. Sarkisyan V.S. *Contact Problems for Semi-planes and Strips with Elastic Overlays*. Yerevan, YSU Publ. Press (1983), 260 (in Russian).

Ա. Վ. ՔԵՐՈՊՅԱՆ

ԿՊԶՈՒՆ ԿՈՆՏԱԿՏԱՅԻՆ ԽՆԴԻՐ ԱՆՎԵՐՋ ՍԱԼԻ ՆԱՄԱՐ,
ՈՐԸ ՈՒԺԵՂԱՑՎԱԾ Է ԵՐԵՔ ԶՈՒԳԱՆԵՌ ՎԵՐՋԱՎՈՐ
ԵՐԿԱՐՈՒԹՅԱՄԲ ԱՏՐԻՆԳԵՐՆԵՐՈՎ

Աշխատանքում դիտարկված է խնդիր առաձգական անվերջ սալի համար, որն իր վերին մակերևույթի վրա իրար զուգահեռ երեք գծերի երկարությամբ վերջավոր փեղամասերում ուժեղացված է երեք զուգահեռ վերջավոր երկարությամբ սարինգերներով փարբեր առաձգական բնութագրերով: Փոխազդեցությունը անվերջ սալի և սարինգերների միջև բոլոր փեղամասերում իրագործվում է բարակ, միապեսակ և այլ ֆիզիկամեխանիկական և երկրաչափական բնութագրեր ունեցող կաչուն շերտերի միջոցով: Սարինգերները դեֆորմացիայի են ենթարկվում իրենց ծայրերից մեկում կիրառված հորիզոնական կենտրոնացած ուժերի ազդեցության փակ: Անհայր կոնսոլիդացիայի ուժերի որոշման խնդիրը հանգեցված է երեք զուգահեռ վերջավոր հարվածներում որոշված երեք անհայր ֆունկցիաների նկատմամբ Ֆրեդհոլմի երկրորդ սեռի ինտեգրալ հավասարումների համակարգի լուծմանը: Այնուհետև որոշվում են խնդիրն բնորոշ բնութագրիչ պարամետրերի փոփոխման այն փիրույթները, որոնց դեպքում այդ հավասարումների համակարգը թույլ է փալիս ճշգրիտ լուծում և որ այն կարելի է լուծել հաջորդական մոտավորությունների մեթոդով: Դիտարկված են հարուկ հնարավոր դեպքեր և հերազոտված է վերջավոր փեղամասերում գործող անհայր շոշափող ուժերի վարքը և բնույթը: Այդ դեպքերի համար թվային արդյունքները կախված խնդրի բազմաթիվ պարամետրերից ուսումնասիրված են Ա. Վ. Քերոպյանի և Կ. Պ. Սահակյանի կողմից հեփելյալ հոդվածում (Proc. of the YSU. Phys. Math. Sci. 57 (3) (2023), 86–100).

А.В. КЕРОПЯН

АДГЕЗИОННАЯ КОНТАКТНАЯ ЗАДАЧА ДЛЯ БЕСКОНЕЧНОЙ
ПЛАСТИНЫ, УСИЛЕННОЙ ТРЕМЯ ПАРАЛЛЕЛЬНЫМИ
СТРИНГЕРАМИ КОНЕЧНОЙ ДЛИНЫ

В работе рассматривается задача для упругой бесконечной пластины, которая на конечных участках вдоль тремя параллельными линиями своей верхней поверхности усилена тремя параллельными стрингерами конечной длины с различными модулями упругости. Контактные связки между пластиной и тремя параллельными стрингерами во всех конечных участках осуществляются посредством одинаковых тонких лишкых слоев с другими физико-механическими и геометрическими характеристиками. Стрингеры

деформируется под действием горизонтальных сосредоточенных сил, приложенных на их концах. В работе задача определения закона распределения неизвестных контактных касательных сил, действующих между бесконечной пластиной и струнгами, сведена к решению системы интегральных уравнений Фредгольма второго рода с тремя неизвестными функциями, определенными на различных параллельных конечных интервалах. Далее определялись области изменения характерных параметров задачи, при которых полученная система интегральных уравнений допускает точное решение и может быть решена методом последовательных приближений. Рассмотрены некоторые возможные специальные случаи и исследованы характер и поведение неизвестных касательных сил на концах струнгов. Численные расчеты для этих случаев в зависимости от различных параметров задачи исследованы в работе А.В. Керобяна и К.П. Саакяна (Proc. YSU. Phys. Math. Sci. 57 (3) (2023), 86–100).