PROCEEDINGS OF THE YEREVAN STATE UNIVERSITY

Physical and Mathematical Sciences 2026, **60**(1), p. 5–13

Mathematics

NEAR-INTERVAL EDGE-COLORINGS OF COMPLETE BIPARTITE GRAPHS

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A proper edge-coloring of a graph G is a mapping $\alpha : E(G) \longrightarrow \mathbb{N}$ such that $\alpha(e) \neq \alpha(e')$ for every pair of adjacent edges e and e' in G. If α is a proper edge-coloring of G and $v \in V(G)$, then $S_G(v,\alpha)$ denotes the set of colors appearing on edges incident to v. A proper edge-coloring α of a graph G with colors $1,\dots,t$ is called a *near-interval t-coloring* if all colors are used, and for each vertex $v \in V(G)$, $S_G(v,\alpha)$ is an interval of integers with no more than one gap. If a graph G has such a coloring, the minimum number of colors in a near-interval coloring of a graph G is denoted by $w^1(G)$. It is known that all complete bipartite graphs admit near-interval colorings. In this paper, we determine or bound the parameter $w^1(K_{m,n})$ $(m,n \in \mathbb{N})$ for complete bipartite graphs.

https://doi.org/10.46991/PYSUA.2026.60.1.005

MSC2020: 05C15.

Keywords: proper edge-coloring, near-interval coloring, interval coloring, complete bipartite graph.

Introduction. We use [1, 2] for terminology and notation not defined here. All graphs considered are finite, undirected, and contain no loops or multiple edges. $V(G)$ and $E(G)$ denote the sets of vertices and edges of a graph G, respectively. If $S \subseteq V(G)$, then $G[S]$ denotes the subgraph of G induced by S. The degree of a vertex $v \in V(G)$ is denoted by $d_G(v)$, the maximum degree of G by $\Delta(G)$, the chromatic index of G by $\chi'(G)$, and the diameter of G by $\mathrm{diam}(G)$. An (a,b)-biregular bipartite graph G is a bipartite graph G with the vertices in one part all having degree a and the vertices in the other part all having degree b.

A proper edge-coloring of a graph G is a mapping $\alpha : E(G) \longrightarrow \mathbb{N}$ such that $\alpha(e) \neq \alpha(e')$ for every pair of adjacent edges e and e' in G. If α is a proper edge-coloring of G and $v \in V(G)$, then $S_G(v,\alpha)$ (or $S(v,\alpha)$) denotes the set of colors appearing on edges incident to v. If α is a proper edge-coloring of a graph G and $v \in V(G)$, then the smallest and largest colors of $S(v,\alpha)$ are denoted by $\underline{S}(v,\alpha)$ and $\overline{S}(v,\alpha)$, respectively. A *near-interval t-coloring* (or *interval $(t,1)$-coloring*) [3, 4]

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of a graph G is a proper edge-coloring α with colors $1,\dots,t$ of G such that all colors are used, and the colors of the edges incident to each vertex v satisfy the following condition: $d_G(v)-1 \leq \overline{S}(v,\alpha)-\underline{S}(v,\alpha) \leq d_G(v)$. A graph G is *near-interval colorable*, if it has a near-interval t-coloring for some positive integer t. The set of all near-interval colorable graphs is denoted by $\mathfrak{N}^1$. For a graph $G \in \mathfrak{N}^1$, the minimum number of colors in a near-interval coloring of a graph G is denoted by $w^1(G)$. A proper edge-coloring α of a graph G with colors $1,\dots,t$ is called an *interval t-coloring* [5, 6], if all colors are used, and for each vertex $v \in V(G)$, $S_G(v,\alpha)$ is an integer interval. A graph G is *interval colorable*, if it has an interval t-coloring for some positive integer t. The set of all interval colorable graphs is denoted by $\mathfrak{N}$. For a graph $G \in \mathfrak{N}$, the minimum number of colors in an interval coloring of G is denoted by $w(G)$. Clearly, if $G \in \mathfrak{N}$, then $G \in \mathfrak{N}^1$ and $w^1(G) \leq w(G)$.

The concept of near-interval colorings of graphs was introduced by Petrosyan and Arakelyan in 2007 [4] as a natural generalization of interval colorings of graphs [5, 6]. There are many papers devoted to interval colorings of graphs, in particular, surveys on the topic can be found in some books (see, for example, [1, 7–9]). In [4], Petrosyan and Arakelyan proved, that if G is a connected graph and it has a near-interval t-coloring, then $t \leq (\operatorname{diam}(G)+1)\Delta(G)+1$; moreover, if G is also bipartite, then $t \leq \operatorname{diam}(G)\Delta(G)+1$. The authors also noted, that if G is a regular graph, then $G \in \mathfrak{N}^1$ and $w^1(G)=\chi'(G)$. In [3], Petrosyan, Arakelyan and Baghdasaryan provided upper bounds on the number of colors in near-interval colorings of simple graphs in terms of number of vertices. In particular, they proved that if G is a connected simple graph and it has a near-interval t-coloring, then $t \leq 2|V(G)|-1$. Moreover, they showed that the coefficient in the previous upper bound cannot be improved, in fact they proved that the complete graph K_n $(n \geq 2)$ has a near-interval t-coloring if and only if $\chi'(K_n) \leq t \leq 2n-3$. In [3], the authors also proved that all subcubic graphs are near-interval colorable, but for any positive integer $\Delta \geq 24$, there exists a bipartite graph G with maximum degree Δ and without near-interval colorings. In [10, 11], near-interval colorings of bipartite graphs were studied. In particular, Casselgren and Toft [11] proved that all $(3,5)$-biregular and $(4,6)$-biregular bipartite graphs have near-interval colorings. In [10], Petrosyan, Khachatrian and Mamikonyan showed that all bipartite graphs with maximum degree 4 have near-interval colorings. Moreover, they also proved that some classes of bipartite graphs with maximum degree 5 and 6 have near-interval colorings. In particular, they showed that if G is a bipartite graph with $\Delta(G)=5$ and without a vertex of degree 3, then $G \in \mathfrak{N}^1$ and $w^1(G)=5$, and if G is a bipartite graph with $\Delta(G)=6$ that has a 2-factor, then $G \in \mathfrak{N}^1$ and $w^1(G)=6$. Recently, Casselgren, Małafiejski, Pastuszak and Petrosyan [12] proved that all simple graphs with maximum degree at most 4 and all Class 1 graphs with maximum degree at most 5 and no vertex of degree 3 have near-interval colorings, but for any positive integer $\Delta \geq 18$, there exists a simple bipartite graph G with maximum degree Δ and without near-interval colorings. Moreover, they also proved that for subcubic graphs G, the problem of determining the exact value of $w^1(G)$ is NP-complete. For bipartite graphs, Casselgren, Małafiejski, Pastuszak and Petrosyan showed that all bipartite multigraphs with maximum degree at most 5 admit near-interval colorings, and for any positive integer $\Delta \geq 15$, there exists a bipartite multigraph G with maximum degree Δ and without near-interval colorings. In the same paper, they also gave a new upper bound on $w^1(K_{m,n})$ for complete bipartite graphs $K_{m,n}$, and suggested the problem of determining the exact value of $w^1(K_{m,n})$ for any $m,n \in \mathbb{N}$ as an open problem.

In this paper, we focus on the problem of determining the exact value of $w^1(K_{m,n})$ for any $m,n \in \mathbb{N}$. In particular, in the present paper, we determine or bound the parameter $w^1(K_{m,n})$ ($m,n \in \mathbb{N}$) for complete bipartite graphs.

Main Results. For two positive integers a and b with $a \leq b$, the set $\{a, a+1, \ldots, b\}$ is denoted by $[a,b]$. For two integers a and b, the greatest common divisor of a and b is denoted by $\gcd(a,b)$.

In 1989, Kamalian proved the following result.

Theorem 1. *[13]. For any $m,n \in \mathbb{N}$, we have*

$$w(K_{m,n}) = m+n-\gcd(m,n).$$

Recently, Casselgren, Małafiejski, Pastuszak and Petrosyan [12] obtained a similar result for near-interval colorings of complete bipartite graphs. In particular, the following result was derived.

Proposition 1. *[12]. For any $m,n \in \mathbb{N}$, we have*

$$w^1(K_{m,n}) \leq m+n+\min\{-\gcd(m,n), 1-\gcd(m+1,n), 1-\gcd(m,n+1)\}.$$

In the same paper [12], the authors suggested the problem of determining the parameter $w^1(K_{m,n})$ of complete bipartite graphs as an open problem. We begin our considerations with a slight improvement of Proposition 1.

Proposition 2. *For any $m,n \in \mathbb{N}$, we have*

$$w^1(K_{m,n}) \leq m+n+\min\{-\gcd(m,n), \quad 1-\gcd(m+1,n), \quad 1-\gcd(m,n+1), \quad 2-\gcd(m+1,n+1)\}.$$

Proof. Let us note that for the proof it is necessary to show that

$$w^1(K_{m,n}) \leq m+n+2-\gcd(m+1,n+1).$$

Let us consider the complete bipartite graph $K_{m+1,n+1}$. Clearly, $K_{m,n}$ can be obtained from $K_{m+1,n+1}$ by removing one vertex from each of the parts. Since, by Theorem 1, $w(K_{m+1,n+1}) = m+n+2-\gcd(m+1,n+1)$, we obtain

$$w^1(K_{m,n}) \leq m+n+2-\gcd(m+1,n+1).$$

Using this and taking into account the upper bound in Proposition 1, we have

$$w^1(K_{m,n}) \leq m+n+\min\{-\gcd(m,n), \quad 1-\gcd(m+1,n), \quad 1-\gcd(m,n+1), \quad 2-\gcd(m+1,n+1)\}.$$

□

Corollary 1. *For any $n,m \in \mathbb{N}$ with $m \leq n$ and $n+1 \equiv 0 \pmod{m+1}$, we have*

$$n \leq w^1(K_{m,n}) \leq n+1.$$

Our next result gives some bounds on $w^1(K_{m,n})$ for many cases of m and n.

Theorem 2. *For any $n,k \in \mathbb{N}$, we have*

$$2kn \leq w^1(K_{2n-2,2kn}) \leq 2kn+n-2.$$

Proof. First of all, clearly,

$$w^1(K_{2n-2,2kn}) \geq \chi'(K_{2n-2,2kn}) = \Delta(K_{2n-2,2kn}) = 2kn.$$

Let us now prove that $w^1(K_{2n-2,2kn}) \leq 2kn+n-2$.
Let $G = K_{2n-2,2kn}$ and denote its bipartition by

$$X = \{x_1,\dots,x_{2n-2}\}, \qquad Y = \left\{y^i_j : 1 \leq i \leq k,\ 1 \leq j \leq 2n\right\}.$$

We decompose the graph G into k subgraphs as follows:

$$G_1 = G\big[X \cup \{y^1_1, y^1_2, \dots, y^1_{2n}\}\big];$$
$$G_2 = G\big[X \cup \{y^2_1, y^2_2, \dots, y^2_{2n}\}\big];$$
$$\dots$$
$$G_k = G\big[X \cup \{y^k_1, y^k_2, \dots, y^k_{2n}\}\big].$$

Clearly, for each $r \in \{1,\dots,k\}$, G_r is isomorphic to $K_{2n-2,2n}$. For each $r \in \{1,\dots,k\}$, we define an edge-coloring α^r of G_r as follows: for $1 \leq i \leq 2n-2$ and $1 \leq j \leq 2n$, let

$$\alpha^r\left(x_i y^r_j\right) = \begin{cases} i+j-1, & \text{if } 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq 2n; \\ i+j, & \text{if } n \leq i \leq 2n-2 \text{ and } 1 \leq j \leq n; \\ i+j-2n, & \text{if } n \leq i \leq 2n-2 \text{ and } n+1 \leq j \leq 2n. \end{cases}$$

Let us show that for each $r \in \{1,\dots,k\}$, α^r is a near-interval $(3n-2)$-coloring of G_r.

By the definition of α^r $(1 \leq r \leq k)$, we have

a) for $1 \leq i \leq n-1$,

$$S_{G_r}(x_i,\ \alpha^r) = [i,\ i+2n-1];$$

b) for $n \leq i \leq 2n-2$,

$$S_{G_r}(x_i,\ \alpha^r) = [i+1,\ i+n] \cup [i-n+1,\ i] = [i-n+1,\ i+n];$$

c) for $1 \leq j \leq n$,

$$S_{G_r}\left(y^r_j,\ \alpha^r\right) = S_{G_r}\left(y^r_{n+j},\ \alpha^r\right)$$
$$= [j, j+n-2] \cup [j+n,\ j+2n-2] = [j,\ j+2n-2] \setminus \{j+n-1\}.$$

This implies that for each $r \in \{1,\dots,k\}$, α^r is a near-interval $(3n-2)$-coloring of G_r (Tab. 1).

Table 1

The near-interval 10*-coloring* α^1 *of* $K_{6,8}$

	y_1^1	y_2^1	y_3^1	y_4^1	y_5^1	y_6^1	y_7^1	y_8^1
x_1	1	2	3	4	5	6	7	8
x_2	2	3	4	5	6	7	8	9
x_3	3	4	5	6	7	8	9	10
x_4	5	6	7	8	1	2	3	4
x_5	6	7	8	9	2	3	4	5
x_6	7	8	9	10	3	4	5	6

We now define an edge-coloring α of G as follows: for each $r \in \{1,\dots,k\}$ and $e \in E(G_r)$, let

$$\alpha(e) = \alpha^r(e) + 2n(r-1).$$

It is easy to check that α is a near-interval $(2kn+n-2)$-coloring of G (Tab. 2).

Table 2

The near-interval 12*-coloring* α *of* $K_{4,12}$

	y_1^1	y_2^1	y_3^1	y_4^1	y_5^1	y_6^1	y_1^2	y_2^2	y_3^2	y_4^2	y_5^2	y_6^2
x_1	1	2	3	4	5	6	7	8	9	10	11	12
x_2	2	3	4	5	6	7	8	9	10	11	12	13
x_3	4	5	6	1	2	3	10	11	12	7	8	9
x_4	5	6	7	2	3	4	11	12	13	8	9	10

□

Corollary 2. *For any* $p \in \mathbb{N}$, *there exists a graph* G *such that* $G \in \mathfrak{N}$ *and*

$$w(G) - w^1(G) \geq p.$$

Proof. For a given $p \in \mathbb{N}$, let $G = K_{2p+2,2p+4}$. Since $\gcd(2p+2, 2p+4) = 2$, by Theorem 1, $G \in \mathfrak{N}$ and $w(G) = 4p+4$.

On the other hand, by Theorem 2, we obtain that

$$w^1(G) \leq 3(p+2) - 2 = 3p+4.$$

This implies that $w(G) - w^1(G) \geq p$. □

Our final result determines the exact value of $w^1(K_{m,n})$ for many cases of m and n.

Theorem 3. *For any* $n,k,c \in \mathbb{N}$ *with* $c \leq k$, *we have*

$$w^1\left(K_{n,(n+1)k-c}\right) = (n+1)k - c.$$

Proof. Let $G = K_{n,(n+1)k-c}$ and denote its bipartition by
$X = \{x_1,\dots,x_n\}, Y = \left\{y^i_j : 1 \leq i \leq c, 1 \leq j \leq n\right\} \cup \left\{y^i_j : c+1 \leq i \leq k, 1 \leq j \leq n+1\right\}$.

We decompose the graph G into k subgraphs as follows:

$$G_1 = G\big[X \cup \{y^1_1, y^1_2, \dots, y^1_n\}\big];$$
$$G_2 = G\big[X \cup \{y^2_1, y^2_2, \dots, y^2_n\}\big];$$
$$\dots$$
$$G_c = G\big[X \cup \{y^c_1, y^c_2, \dots, y^c_n\}\big];$$
$$G_{c+1} = G\big[X \cup \{y^{c+1}_1, y^{c+1}_2, \dots, y^{c+1}_{n+1}\}\big];$$
$$G_{c+2} = G\big[X \cup \{y^{c+2}_1, y^{c+2}_2, \dots, y^{c+2}_{n+1}\}\big];$$
$$\dots$$
$$G_k = G\big[X \cup \{y^k_1, y^k_2, \dots, y^k_{n+1}\}\big].$$

Clearly, for each $q \in \{1,\dots,c\}$, G_q is isomorphic to $K_{n,n}$ and for each $r \in \{c+1,\dots,k\}$, G_r is isomorphic to $K_{n,n+1}$. For each $q \in \{1,\dots,c\}$, we define an edge-coloring α^q of G_q as follows: for $1 \leq i \leq n$ and $1 \leq j \leq n$, let

$$\alpha^q\left(x_i y^q_j\right) = (i+j) \pmod{n} + 1.$$

Clearly, for each $q \in \{1,\dots,c\}$, α^q is an interval n-coloring of G_q, and for any vertex $v \in V(G_q)$,

$$S_{G_q}(v, \alpha^q) = \{1,2,\dots,n\} = [1,n].$$

For each $r \in \{c+1,\dots,k\}$, we define an edge-coloring β^r of G_r as follows: for $1 \leq i \leq n$ and $1 \leq j \leq n+1$, let

$$\beta^r\left(x_i y^r_j\right) = (i+j) \pmod{(n+1)} + 1.$$

Clearly, β^r is a proper edge-coloring with colors $1,2,\dots,n+1$.

By the definition of β^r ($c+1 \leq r \leq k$), we have

a) for $1 \leq i \leq n$,

$$S_{G_r}(x_i, \beta^r) = \{1,2,\dots,n+1\} = [1,n+1];$$

b) for $1 \leq j \leq n+1$,

$$\begin{aligned} S_{G_r}\left(y^r_j, \beta^r\right) &= \{1,2,\dots,n+1\} \setminus \{j \pmod{(n+1)} + 1\} \\ &= [1,n+1] \setminus \{j \pmod{(n+1)} + 1\}. \end{aligned}$$

This shows that for each $r \in \{c+1,\dots,k\}$, β^r is a near-interval $(n+1)$-coloring of G_r.

We now define an edge-coloring γ of G as follows: for each $e \in E(G)$, let

$$\gamma(e) = \begin{cases} \alpha^i(e) + (i-1)n, & \text{if } e \in E(G_i) \text{ and } 1 \leq i \leq c, \\ \beta^i(e) + nc + (i-c-1)(n+1), & \text{if } e \in E(G_i) \text{ and } c+1 \leq i \leq k. \end{cases}$$

It is not difficult to see that γ is a proper edge-coloring of G with colors $1,2,\dots,k(n+1)-c$ such that for any vertex $x_i \in V(G)$ $(1 \leq i \leq n)$

$$S(x_i,\gamma) = \{1,2,\dots,k(n+1)-c\} = [1,\ k(n+1)-c],$$

and for any vertex $y^l_j \in V(G)$ $(c+1 \leq l \leq k,\ 1 \leq j \leq n+1)$, $S\left(y^l_j,\gamma\right)$ is an interval of integers with no more than one gap (Tab. 3). □

Table 3

The near-interval 10*-coloring* γ *of* $K_{3,10}$

	G_1			G_2			G_3			
	y^1_1	y^1_2	y^1_3	y^2_1	y^2_2	y^2_3	y^3_1	y^3_2	y^3_3	y^3_4
x_1	3	1	2	6	4	5	9	10	7	8
x_2	1	2	3	4	5	6	10	7	8	9
x_3	2	3	1	5	6	4	7	8	9	10

Corollary 3. *For any $n \in \mathbb{N}$, there exists m' such that for any $m \in \mathbb{N}$ with $m \geq m'$, we have*

$$w^1(K_{m,n}) = m.$$

Conclusion. This work is devoted to the study of near-interval colorings of complete bipartite graphs. In particular, new upper bounds and exact values of $w^1(K_{m,n})$ were obtained for some families of complete bipartite graphs.

The main results obtained in this work are the following:

- for any $m,n \in \mathbb{N}$, we have

$$w^1(K_{m,n}) \leq m+n+\min\{-\gcd(m,n), 1-\gcd(m+1,n),$$
$$1-\gcd(m,n+1), 2-\gcd(m+1,n+1)\};$$

- for any $n,k \in \mathbb{N}$, we have

$$w^1(K_{2n-2,2kn}) \leq 2kn+n-2;$$

- for any $n,k,c \in \mathbb{N}$ with $c \leq k$, we have

$$w^1\left(K_{n,(n+1)k-c}\right) = (n+1)k-c.$$

Received 03.01.2026
Reviewed 03.03.2026
Accepted 26.03.2026

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ԼՐԻՎ ԵՐԿԿՈՂՄԱՆԻ ԳՐԱՖՆԵՐԻ ՀԱՄԱՐՅԱ ՄԻՋԱԿԱՅՔԱՅԻՆ ԿՈՂԱՅԻՆ ՆԵՐԿՈՒՄՆԵՐ

$\alpha : E(G) \longrightarrow \mathbb{N}$ արտապատկերումը կոչվում է G գրաֆի ճիշտ կողային ներկում, եթե ցանկացած երկու հարևան $e, e' \in E(G)$ կողերի համար $\alpha(e) \neq \alpha(e')$: G գրաֆի α ճիշտ կողային ներկման դեպքում v գագաթին կից կողերի գույների բազմությունը կնշանակենք $S_G(v, \alpha)$-ով: G գրաֆի α ճիշտ կողային ներկումը կոչվում է *համարյա միջակայքային ներկում*, եթե բոլոր գույները օգտագործված են, և կամայական $v \in V(G)$ գագաթի համար $S_G(v,\alpha)$ բազմությունը կազմում է առավելագույնը մեկ պակասող գույնով ամբողջ թվերի միջակայք: Եթե G գրաֆն ունի համարյա միջակայքային ներկում, ապա այդ ներկումներում մասնակցող գույների նվազագույն քանակը կնշանակենք $w^1(G)$-ով: Այս աշխատանքում ստացվել են $w^1(K_{m,n})$ $(m,n \in \mathbb{N})$ պարամետրի գնահատականներ և ճշգրիտ արժեքներ լրիվ երկկողմանի գրաֆների որոշ ընտանիքների համար:

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ПОЧТИ ИНТЕРВАЛЬНЫЕ РЕБЕРНЫЕ РАСКРАСКИ ПОЛНЫХ ДВУДОЛЬНЫХ ГРАФОВ

Отображение $\alpha : E(G) \longrightarrow \mathbb{N}$ называется правильной реберной раскраской графа G, если для любой пары смежных ребер e и e' графа G выполняется условие $\alpha(e) \neq \alpha(e')$. Если α – правильная реберная раскраска графа G и $v \in V(G)$, то через $S_G(v,\alpha)$ обозначим множество цветов ребер, инцидентных вершине v. Правильная реберная раскраска α графа G называется *почти интервальной t-раскраской графа G*, если каждый из t цветов использован и для любой вершины $v \in V(G)$ множество $S(v,\alpha)$ образует интервал целых чисел с не более чем одним пропуском. Если граф G обладает такой раскраской, то минимальное число цветов в почти интервальной реберной раскраске графа G обозначим через $w^1(G)$. Известно, что все полные двудольные графы обладают почти интервальной реберной раскраской. В настоящей работе получены некоторые точные значения и оценки параметра $w^1(K_{m,n})$ $(m,n \in \mathbb{N})$ полных двудольных графов.